

Implementation of Vector Quantization for Image Compression - A Survey

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Abstract This paper presents a survey on vector quantization for image compression. Moreover it provides a means of decomposition of the signal in an approach which takes the improvement of inter and intra band correlation as more lithe partition for higher dimension vector spaces. Thus, the image is compressed without information loss using artificial neural networks (ANN). Since 1988, a growing body of research has examined the use of VQ for the image compression. This paper discusses about vector quantization, its principle and examples, its various techniques and image compression its advantages and applications. Additionally this paper also provides a comparative table in the view of simplicity, storage space, robustness and transfer time of various vector quantization methods. In addition the proposed paper also presents a survey on different methods of vector quantization for image compression.

Keywords: Artificial Neural Networks (ANN), Image compression, Vector Quantization and Multiple Codebooks.

I. INTRODUCTION

The technique of obtaining the compact representation of an image while maintaining all the necessary information without much data loss is referred to as Image Compression. It can be classified into two types, Lossless and Lossy compression. Lossy compression can be broadly classified into two types, namely Scalar Quantization (SQ) and Vector Quantization (VQ). A popular technique for source coding of image and speech data since 1980 is VQ [20]. VQ involves processing the input samples in groups into a set of well-defined vectors using some distortion measure.

Day by day the use of multimedia, images and the other picture formats are rapidly increasing in a variety application. Type of technique that is used to store in multimedia data is an important although storage is bigger than ever, however it is not enough. Hence, the data compression particularly the image compression plays a vital role.

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II. APPLICATIONS OF IMAGE COMPRESSION

Most of the benefits of image compression include less required storage space, quicker sending and receiving of images i.e., the transfer rate is high, and less time lost on image viewing and loading. A good example to illustrate this is the health industry, where the constant scanning and/or storage of medical images and documents take place. Image compression offers many other benefits, as information can be stored without placing large loads on system servers [21]. Depending on the type of compression applied, images can be compressed to save storage space, or to send to multiple physicians for examination. And conveniently, these images can uncompress when they are ready to be viewed, retaining the original high quality and detail that medical imagery demands.

Image compression is also plays an important role to any organization that requires the viewing and storing of images to be standardized, such as a chain of retail stores or a federal government agency. In the retail store example, the introduction and placement of new products or the removal of discontinued items can be much more easily completed when all employees receive, view and process images in the same way. Federal government agencies that standardize their image viewing, storage and transmitting processes can eliminate large amounts of time spent in explanation and problem solving. The time they save can then be applied to issues within the organization, such as the improvement of government and employee programs [25].

In the sanctuary and security based industries the image compression can greatly increase the efficiency of recording, processing and storage. For example, in a video networking or closed-circuit like the television application, several images at different frame rates may be required. Time is also a consideration, as different areas may need to be recorded for various lengths of time. Image resolution and quality also become considerations, as does network bandwidth, and the overall security of the system [22].

In many museums and galleries which considers the quality of reproductions to be of the extremely important. Image compression, therefore, can be very efficiently applied in cases where accurate representations of museum or gallery images are required, such as on a Web site. Detailed images that offer short download times and easy viewing benefit all types of visitors from the student to the discriminating collector Compressed images can also be used in museum or gallery kiosks for the education of that establishment's visitors [26]. In a library scenario, students and enthusiasts from around the world can view and enjoy a multitude of documents and texts without having to incur traveling or

lodging costs to do so. Regardless of industry, image compression has virtually endless benefits wherever improved storage, viewing and transmission of images are required [22].

III. VECTOR QUANTIZATION

The vector quantization is a classical quantization technique from signal processing and image compression which allows the modeling of probability density functions by the distribution of prototype vectors. It was originally used for data compression. It works by dividing a large set of points (vectors) into groups having approximately the same number of points closest to them. Each group is represented by its centroid point, as in k-means and some other algorithms [27].

The density matching property of vector quantization is powerful, especially for identifying the density of large and high-dimensional data. Since data points are represented by the index of their closest centroid, commonly occurring data have low error, and rare data high error. This is why VQ is suitable for lossy data compression. It can also be used for lossy data correction and density estimation.

This methodology of vector quantization is based on the competitive learning paradigm, so it is closely related to the self-organizing map model. Vector quantization is used for lossy data compression, lossy data correction and density estimation [3 and 12]. The lossy data correction, prediction and compression are used to recover data missing from some dimensions. This is done by finding the nearest group with the data dimensions available by vector quantization, and then predicting the result based on the values for the missing dimensions, assuming that they will have the same value as the group's centroid.

A. Principal of Vector Quantization

Vector Quantization assist to project a continuous input space on a discrete output space, while minimizing the loss of information [24]. To define zones in the space, the set of points contained in each zone being projected on a representative vector.

B. Use of Vector Quantization in Image compression

The methodology of vector quantization is also called "block quantization" or "pattern matching quantization" is often used in lossy image compression.

It works by encoding values from a multidimensional vector space into a finite set of values from a discrete subspace of lower dimension [27]. The lower-space vector requires less storage space, so the image can be easily compressed. Due to the density matching property of vector quantization, the compressed data have errors that are inversely proportional to their density.

The renovation is usually done by projection or by using a codebook [1]. In some cases, the codebook can be also used to entropy code the discrete value in the same step, by

generating a prefix coded variable-length encoded value as its output.

The set of distinct amplitude levels is quantized jointly rather than each sample being quantized separately. Figure 1 describes the basic block diagram of vector quantization. Consider a K-dimensional vector $[x_1, x_2, \dots, x_k]$ of amplitude levels. It is compressed by choosing the nearest matching vector from a set of N-dimensional vectors $[y_1, y_2, \dots, y_n]$. All possible combinations of the N-dimensional vector $[y_1, y_2, \dots, y_n]$ form the codebook. Only the index of the codeword in the codebook is sent instead of the quantized values. This conserves space and achieves more compression.

IV. BACK GROUND STUDY ON VECTOR QUANTIZATION ALGORITHMS

This section of the paper confers the most common methodologies and algorithms used in vector quantization for image compression along with its algorithm in brief and all the algorithms are overall discussed.

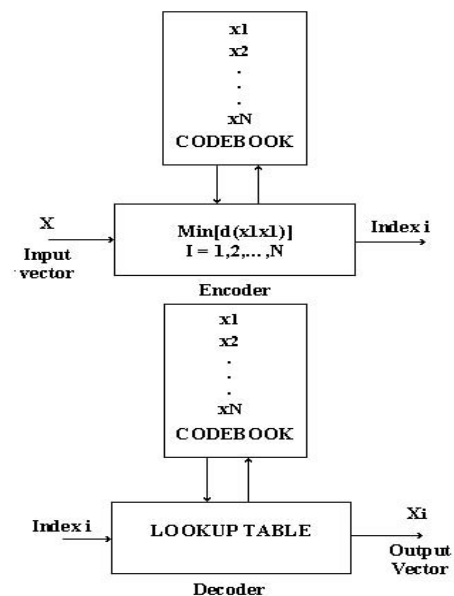


Figure 1. Block diagram of VQ

C. Generalized Lloyd Vector Quantization

Kaukoranta et. al., in [15] projected the vector quantization methodology can be applied using GLA algorithm for its simplicity in image compression. The essential conditions for minimizing the expected distortion D in general

situations are embodied in the two conditions of the generalized Lloyd algorithm:

1. Given the input vector x , choose the code $c = c(x)$ to minimize the squared error distortion $|x - x'(c)|^2$
2. Given the code c , compute the reconstruction vector $x'(c)$ as the centroid of those input vectors x that satisfy condition 1.

To employ the vector quantization, the algorithm mechanism are done in batch mode by alternately optimizing the encoder $c(x)$ in accordance with condition 1, and then optimizing the decoder in accordance with condition 2, until D reaches a minimum. Figure 2 represents the model of generalized Lloyd vector quantization algorithm. To conquer the problem of local minima, it may be necessary to run the algorithm several times with different initial code vectors.

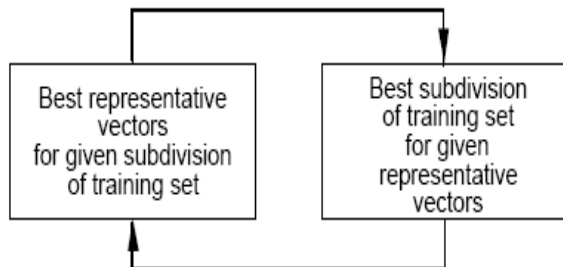


Figure 2. Generalized Lloyd Vector Quantization Algorithm

Each of the iteration in the GLA consists of two phases: the codebook assignment phase and the codebook adjustment phase. During the former, each of the vectors from the input set is assigned to the nearest vector from the codebook. During the latter, each of the codebook vectors is replaced with the centroid of all input vectors assigned to it. This procedure is convergent, and minimizes the mean square error of quantization.

There are two problems with GLA: the "empty cells" and how to choose the initial codebook. Empty cell is created when some vector from the codebook gets no input vectors assigned to it during the assignment phase. It will not move during the adjustment phase and will therefore the cell will probably remain empty in all subsequent operations. In this case, it should be removed from the codebook.

D. Entropy-Constrained VQ

V.K. Jain et. al., discussed in [8], Entropy-Constrained Vector Quantization approach for image compression is

presented which incorporates the concept of activity map in image compression. The explanatory issue behind the vector sub band coding is the efficient compression of the little energy-albeit perceptually significant, upper-bands of the decomposed images. As the components of the noteworthy vectors in the upper sub bands display the contour information of the original images that has to be compressed, therefore, the locations and orientations of such edge information can be determined from the original image itself.

The code words in the state codebook are sorted by the distortion measure, the code vectors produced smaller distortion are in the lower index parts of the state codebook. If the distortion measure can match the property of the input image, the output indices often lies in lower parts. Therefore, entropy coding, such as Huffman code, can be used to reduce the necessary bit rate of the indices. Shorter symbols are assigned to encode the lower index, because their appearance probability is higher. Hence, the code words in the state codebook have different cost in the transmitting of indices. In general coding process, the nearest code vectors are always selected. Whether the improvement of nearest code words over the other code words is worth the cost in transmitting the channel symbol is not considered.

This information can be used to generate the activity map, which in turn is used to obtain transmission control maps. The control maps are used to turn on/off the transmission of vectors. Most importantly, the approach is extended and applied to color image coding.

E. Wavelet Based Vector Quantization

According to F. Madeiro et. al., [7] Wavelet Based VQ is the best way of quantizing and compressing images. This methodology takes multiple stage discrete wavelet transform of code words and uses them in both search and design processes for the image compression. Accordingly, the codebook consists of a table, which includes only the wavelet coefficients [6]. The key idea in the mechanism of this algorithm is finding Representative code vectors for each stage are found by first combining n code words in k groups, where k_n gives the codebook size.

Following the process obtaining the k groups, one may take the centroids of the clusters as their representative vectors. Processing as before, can decrease the number of representative vectors this procedure enables us to obtain correct code vectors in $2n$ comparisons for the two stage design instead of nk computations [13]. In the design stage, obtaining the clusters plays an important role, and many times one can end up with non-optimum centroids, which results in incorrect code vector correspondences. In order to decrease the computation time, after the standard design procedure, it is liable to replace the representative vectors by low bands of their wavelet transforms [4 and 5]. It can be started with the original code vectors and combine them into the lowest group clusters. After obtaining the centroids of the clusters, the last stages are represented. Then it can take the first stage wavelet transforms of the representative

vectors and use only the lower bands as new representatives. The same procedure can again be implemented to obtain the above stage clusters which include wavelet transforms. The degree of image degradation after VQ is detected by judging the matching distances (distortions) in VQ processing. By using the quad tree block alteration, the optimum VQ coding can be performed. It has a major drawback in the amount of computations during the search for optimum code vector in encoding. This complexity can be reduced by using an efficient codebook design and wavelet based tree structure

F. Multiple-VQ Based Image Compression

One of the methods in vector quantization, called MVQ discussed by Noritaka Shigei et. al., in [19] generates multiple independent codebooks to compress an image. In the image restoration, MVQ restores low quality images from the multiple codebooks, and then combines the low quality ones into a high quality one. It uses an effective coding scheme for codebook indexes to overcome the inefficiency in compression rate. The MVQ method outperforms a conventional single-VQ method when the compression rate is smaller than some values. In the compression phase, C codebooks are generated by performing VQ C times, and for each codebook an index sequence is generated. In the restoration phase, a restored image is generated from C pairs of codebook and index sequence generated in the compression phase. In the following, the number of VQs is assumed to be two, that is $C = 2$, because this choice is the most effective in terms of trade-off between accuracy and compression rate. Although it describe only the case of $C = 2$, the methods can be easily extended to the general case of an arbitrary number of C. The use of different multiple data sets provide a better performance than the one of the same data set.

G. Multistage Vector Quantization

B.H. Juang et. al., discussed in [2], multistage vector quantization (MSVQ) and their variants have been recently proposed. Before MSVQ is designed, the user must artificially determine the number of code words in each VQ stage. However, the users usually have no idea regarding the number of code words in each VQ stage, and thus doubt whether the resulting MSVQ is optimal [14 and 9]. This algorithm can automatically find the number of code words to optimize each VQ stage according to the rate-distortion performance. Furthermore, using a sharing codebook can further reduce the storage size of MSVQ. Combining numerous similar code words in the VQ stages of MSVQ produces the code words of the sharing codebook.

A structured VQ scheme which can achieve very low encoding and storage complexity is MSVQ. This appealing property of MSVQ motivated us to use MSVQ in the quantization stage. The basic idea of multistage vector quantization is to divide the encoding task into successive stages, where the first stage performs a relatively crude quantization of the input vector. Then a second-stage

quantizer operates on the error vector between the original and the quantized first-stage output. The quantized error vector then provides a second approximation to the original input vector thereby leading to a refined or more accurate representation of the input. A third stage quantizer may then be used to quantize the second-stage error to provide a further refinement and so on.

H. Pyramid Vector Quantization Algorithm

One of the key problems for the basic VQ method according to Lee et. al., in [16] full search algorithm, is that it is computationally intensive and is difficult for real time processing. Many fast encoding algorithms have been developed for this reason. Pyramid Vector Quantization Algorithm of searching and processing code words to significantly speed up the searching process especially for high dimensional vectors and codebook with large size; reduce the actual requirement for memory, which is preferred in hardware implementation system, e.g., SOC and produce the same encoded image quality as full search algorithm. The algorithm of Pyramidal Vector Quantization is given below

In the first step, All the vectors (having dimension of L) taken from the wavelet transformed images are projected on the surface of the pyramid such that the projection yields the least mean squared error. Then it proceeds with the vectors lying on the surface of the pyramid are then scaled to an inner pyramid with a scaling factor where the inner pyramid is chosen based on the rate criterion 'R' bits/dimension. The next step continues with choosing the inner pyramid: To encode the L dimensional vectors obtained from the sub-band images at a specified rate per dimension namely R bits/dimension, the largest value of K should be found. Then it follows with finding the code vector (the set of code vectors is given by $N(L, K)$) nearest to the scaled vector on the S (L, K) lattice. Geometrically S (L, K) is the surface of the hyper pyramid in L-dimensional space. Finally it will find the index of the code vectors generated, based on the magnitude enumeration algorithm.

I. Neuquant Vector Quantization Method

Mark Nelson et. al., [17] discussed Neuquant VQ method as a post-clustering vector quantization method, which uses a neural network to achieve a high quality color reduction. The algorithm used is a self organizing, which assigns a neuron to each palette entry. In the learning phase of the neural network every neuron acquires a value. To this value a cluster of pixels from the source image is assigned, which matches the RGB value as close as possible. After this phase every pixel is assigned the value of palette entry of the corresponding value.

Neuquant is named after Anthony Dekker for his contribution in the applications of Kohonen's self-organizing maps. It is quite suitable for quantization of vectors of higher dimensions. Each neuron has two

neighbors in this string. Each of the vector in the input set, "stimulate" the string: to find the neuron closest to the stimulus vector and pull it in the direction of the stimulus, one-half the distance towards it. Its neighbors in the string also get pulled towards the stimulus, but by a lesser amount, say, one-fourth. Their neighbors are influenced only by one-eighth and so on, until at some distance along the string the reaction to the stimulus stops. When it is feed the input set as stimuli, the string moves, turns, and stretches itself to occupy these parts of the vector space where the input data is dense—which is precisely what is wanted from a good VQ codebook. Neuquant with larger codebooks sometimes wastes codebook entries. Table 1 shows the comparison of various vector quantization techniques for image compression.

J. Tree Structured VQ

Hui et al [11] brought the Tree Structured VQ techniques in order to decrease the table storage needed for en-coding and decoding along with unstructured vector quantization (UVQ) or Tree-Structured Vector Quantization [23] (TSVQ). Particularly, a low-storage Secondary Quantizer is employed to squeeze the code vectors of the primary quantizer. The absolute benefits of uniform and non-uniform Secondary Quantization are examined.

Quantization levels are put up on a binary or Quad tree structure (sub-optimal). It is to set vectors in different quadrant. Signs of vectors are only needed to be evaluated. This will reduce the number of links by 2^L for L-d vector problem. It will work fine for symmetric distribution

K. Hybrid Vector Quantization

Vector quantization has been lucratively applied to the wavelet transformed coefficients, effecting in high quality image compression.

Vector Quantization Approach	Simplicity	Robustness	Storage Space	Transfer Time
Generalized Lloyd VQ	H	H	L	M
Entropy - Constrained VQ	M	H	M	U
Wavelet based VQ Approach	H	M	L	L
Multiple VQ based Image Compression	M	H	M	M
Multistage VQ Approach	L	M	M	U

Pyramid VQ Algorithm	M	H	L	L
Neuquant VQ Approach	H	H	L	M
Tree structured VQ	M	M	L	U
Hybrid VQ Approach	H	M	M	L

Table 1 Comparison of various Vector Quantization Techniques used for Image Compression. H-High, M-Medium, L-Low and U-Undetermined.

According to Mitra, S et al., [18] the hybrid VQ algorithm is explained as follows, first the correlation in the test image is taken away by wavelet transform. Wavelet transform is employed in the course of lifting scheme. The wavelet employed during lifting scheme is HAAR wavelet. Then it will lead with the primary level of decomposition gives way four components namely LL, LH, HL and HH correspondingly. Multistage VQ is implemented to LL band. The pointed coefficients in the sub band LH, HL and HH are pyramidal vector quantized by capturing vectors of stated measurement. Finally an entropy coding algorithm like Huffman coding is implemented as the final stage of the compression system to code the indices. The proposed algorithm groups the benefit of lessening of codebook searches and storage complexity which is intrinsic to MSVQ [18]. Additionally pyramid vector quantization may not need great codebook storage having simple encoding and decoding algorithm. Hence high compression ratio can be accomplished by including PVQ along with MSVQ.

V. FUTURE ENHANCEMENTS

The upcoming standard will incorporate many of the research works in vector quantization and will address many important aspects in image compression for the next millennium. However, the current data and image compression techniques might be far away from the ultimate limits that are imposed by the underlying structure of specific data sources such as images. Hence future work determines to enhance the wavelet based vector quantization algorithm that performs well for any format of the image. By implementing an effective code book and the wavelet based tree structure the computation complexity can be reduced. Interesting issues like obtaining accurate models of images, optimal representations of such models, and rapidly computing such optimal representations are the real challenges facing the data compression community. The future enhancement primarily focuses on developing vector quantization algorithms that may ensure less storage space, minimum transfer time and minimum image loading time.

The interaction of analysis with data and image compression, such as the joint source-channel coding, image coding and compression based on models of human perception, scalability, robustness, error resilience, and complexity are a few of the many outstanding challenges in image compression with vector quantization field to be fully resolved and may affect image data compression performance in the years to come.

VI. CONCLUSION

This paper takes a detailed survey on the existing most significant and commonly used methodologies in vector quantization for image compression. Every approach has its own advantages and limitations. The vector quantization methodology extends scalar quantization to higher dimensional space. By grouping input samples into vectors and using a vector quantizer, a lower bit rate and higher performance can be achieved. This paper takes a deep look into the vector quantization, its principal, vector quantization in image compression and the applications of image compression are made a detailed study with examples.

Further, all the common methodologies in vector quantization for image compression are been discussed in detail. In some cases, the complexity increase exponentially as the rate increases for a given vector size. Full-search VQ such as Entropy-Constrained VQ (ECVQ) has only small quantization distortion. However, it has long compression time, and may not be well suited for real time signal compression systems. Tree-Structured VQ (TSVQ) although can significantly reduce the compression time, has the disadvantage that the storage size required for the VQ is usually very large and cannot be controlled during the design process. The direct use of VQ suffers from a serious complexity barrier. The Generalized Lloyd Vector Quantization is used for its simplicity in image compression and essentially minimizing the expected distortion in image compression. Moreover, wavelet based vector quantization approach for image compression may perform image

quantizing and compression significantly well. As this methodology takes multiple stages discrete wavelet transform of code words and uses them in both search and design processes for the image compression. This algorithm can be enhanced to improve image compression efficiency. There is no single algorithm or methodology that satisfies all the requirements that use the vector quantization for image compression as every algorithm has its own merits and demerits.

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