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Wavelets, its Application and Technique in signal and image processing

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WAVELETS, ITS APPLICATION AND TECHNIQUE IN SIGNAL AND IMAGE PROCESSING

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Abstract - Wavelets are functions that satisfy certain mathematical requirement and used in representing data or functions. Wavelets allow complex information such as data compression, signal recognition, signal and image processing, music and computer graphics etc. The wavelet decomposition analysis is used most often in wavelet signal processing. It is used in signal compression as well as in signal identification, although in the latter case, reconstruction of the original is not always required. The decomposition separates a signal into components at various scales corresponding to successive octave frequencies. Each component can be processed individually by a different algorithm. In this work we first try to introduce wavelet and then some of its applications and technique in signal and image processing. Here we also approach a new filtering technique.

I. Introduction

The word "*wavelet*" is due to Morlet and Grossmann [9] in the early 1980s influenced by ideas from both pure and applied mathematics. They used the French word *ondelette*, meaning "small wave". Soon it was transferred to English by translating "*onde*" into "wave", giving "wavelet". Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, seismic geology and medical technology etc.

Before 1930, the main branch of mathematics leading to wavelets began with Joseph Fourier at 1807 with his theories of frequency analysis. The first mention of wavelets appeared in the Ph.D. thesis of A. Haar at 1909. A physicist Paul Levy investigated Brownian motion at 1930, a type of random signal. Another 1930s research effort by Little-wood, Paley and Stein involved computing the energy of a function. The development of wavelets was starting with Haar's work in the early 20th century. Pierre Goupillaud, Grossmann and Morlet's [9] formulation of what is now known as the continuous wavelet transformation(*CWT)*, Jan-Olov Stromberg's early 1983s work on discrete wavelets. Stephane Mallat gave wavelets in digital signal processing at 1985 and multiresolution framework at 1989. In 1988, Daubechies referred orthogonal wavelets with compact support, Nathalie Delprat introduced time-frequency interpretation of the *CWT* at 1991, Newland exhibited harmonic wavelet transformation at 1993 and many others since.

Theory of wavelets has been developed essentially in last twenty years. Approximation by wavelet polynomials is progressing rapidly. A wavelet is a wave-like oscillation with amplitude that starts out at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph or heart monitor. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "shift, multiply and sum" technique called convolution, with portions of an unknown signal to extract information from the unknown signal [21].

The Fourier transform shows up in a remarkable number of areas outside of classic signal processing. Now a day the mathematics of wavelets is much larger than that of the Fourier transform. Initial wavelet applications involved signal processing and filtering. However, wavelets have been applied in many other areas including non-linear regression, image compression, turbulence, human vision, radar earthquake prediction and seismic wave etc.

Wavelets are mathematical functions that cut up data into different frequency components and then study each component with a resolution matched to its scale. A wavelet transform is the representation of a function by wavelets. More technically, a wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale. The wavelet will resonate if the unknown signal contains information of similar frequency - just as a tuning fork physically resonates with sound waves of its specific tuning frequency. This concept of resonance is at the core of many practical applications of wavelet theory.

A recent literature on wavelet signal processing shows the focus on using the wavelet algorithms for processing one-dimensional and two-dimensional signals. Acoustic, speech, music and electrical transient signals are popular in 1-D wavelet signal processing. The 2-D wavelet signal processing involves mainly noise reduction, signature identification, target detection, signal and image compression and interference suppression.

In this work we have tried to show the technique how we use the wavelet in signal and image processing.

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We also suggest a new filtering technique. In this technique we consider only the wavelet coefficients which are mainly contribute to the given signal or which are having a special behavior.

II. Basic Definitions About Wavelets

1) *Wavelets*

Wavelets are localized waves functions that are confined in finite domains and used to represent data or a function. Instead of oscillating forever, they drop to zero. They come from the iteration of filters (with recalling). Fourier analysis has a serious draw back. In transforming to the frequency domain time information is lost. When looking at a Fourier transformation of a signal, it is impossible to tell when a particular event took place. In an analogous way to Fourier analysis, which analyzes the frequency content in a function using sines and cosines, wavelet analysis analyzes the scale of a function's content with special basis functions called wavelets [14].

Equivalent mathematical conditions for wavelets are:

$$\int_{-\infty}^{\infty} \left|\psi(x)\right|^2 dx < \infty \tag{2.1}$$

$$\int_{-\infty}^{\infty} \left|\psi(x)\right| dx = 0 \tag{2.2}$$

$$\int_{-\infty}^{\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega < \infty \tag{2.3}$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(x)$ and *(2.3)* is called the admissibility condition. The wavelet $\psi(t)$ and its shifts $\psi(t-k)$ are at unit scale. The wavelets $\psi(2^j t)$ and $\psi(2^j t-k)$ are at 2^j scale. Wavelet produces a natural multiresolution for every image.

2) *Wavelet Transform*

Wavelets are used to transform the signal under investigation into another representation, which presents the signal information in a more useful form. This transformation of signal is known as the wavelet transform. Mathematically speaking, the wavelet transform is a convolution of the wavelet function with the signals. Jean Morlet in 1982, first considered wavelets as a family of functions constructed from translations and dilations of a single function called the "mother wavelet", *ψ(x)*. They are defined by

$$\psi_{j,k}(x) = \frac{1}{\sqrt{|j|}}\psi\left(\frac{x-k}{j}\right), \quad j,k \in Z,\ j \neq 0 \tag{2.4}$$

The parameter 'j' is the scaling parameter or scale and it measures the degree of compression. The parameter 'k' is the translation parameter which determines the time location of the wavelet. If $|j|<1$, then the wavelet in *(2.4)* is the compressed version (smaller support in time- domain) of the mother wavelet and corresponds mainly to higher frequencies. On the other hand, when $|j|>1$, then $\psi_{j,k}(x)$ has a larger time-width than $\psi(x)$ and corresponds to lower frequencies. Thus, wavelets have time-widths adapted to their frequencies. Wavelet transforms are in both the frequency and time domain. As the frequency of the wavelet is doubled, there are twice as many wavelets required to fill up the same time span. Therefore, one actually gets frequency information at different times in the signal.

3) *Dilation and Translation*

We consider, a function $\psi(x) = 2\pi\left(\frac{1}{2\pi} - x^2\right)e^{-\pi x^2}$ which is shown by the *Figure-2.1* and associated a family of functions $\psi_{j,k}(t)$ defined by $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$, $x \in \square$ & $j,k \in \square$. The function must be translated and dilated for the different values of $k \in \square$ & $j \in \square$ respectively.

To show the translation of the given function $\psi(x)$, we first consider $j\ \ 0$. Then, for $k=0,\ \psi_{0,0}(x) = 2^0\psi(2^0 x - 0) = \psi(x)$. Thus, the function $\psi_{0,0}(x)$ is $\psi(x)$ itself. Also for any $k \in \square$ then $\psi_{0,k}(x) = 2^0\psi(2^0 x - k) = \psi(x-k)$. The graph of the function $\psi_{0,k}(x)$ or translating (shifting) graph of $\psi(x)$ for k units to the right side of the axis shown by the *figure- 2.2 – figure -2.3.* Again, to show the dilation of the given function $\psi(x)$ then putting $k\ \ 0$ and get $\psi_{j,0}(x)\ \ 2^{j/2}\psi(2^j x)$. As *figure-2.4– figure-2.7* demonstrate, the functions $\psi_{j,0}(x)$ are scaled (dilated) versions of $\psi(x)$, when j is a large positive number, the graph of $\psi_{j,0}(x)$ is a compressed version of $\psi(x)$, while negative values of j lead to less localized versions of $\psi(x)$. In figure we show that the functions $\psi_{j,k}(x)$ are translated (shifted) and scaled (dilated) versions of $\psi(x)$, for details see [8].]

Figure 2.1: The function $\psi_{0,0}(x)$.

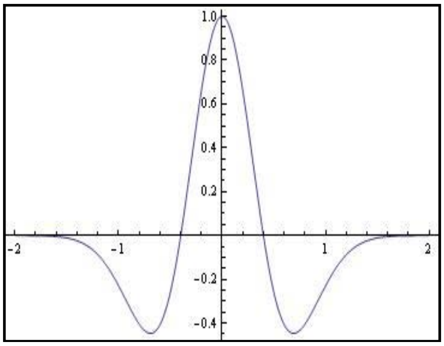


Figure 2.2:The function $\psi_{0,1}(x)$

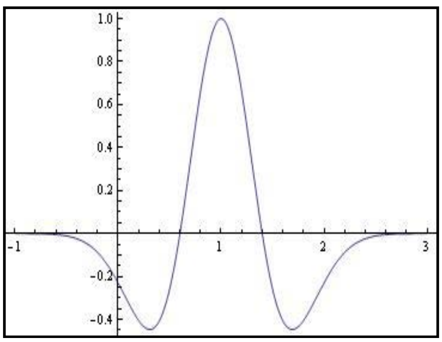


Figure 2.3: The function $\psi_{0,2}(x)$

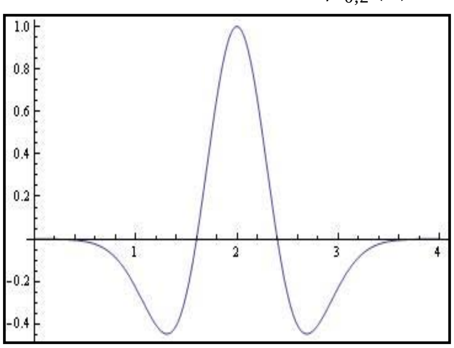


Figure 2.4: The function $\psi_{0,2}(x)$

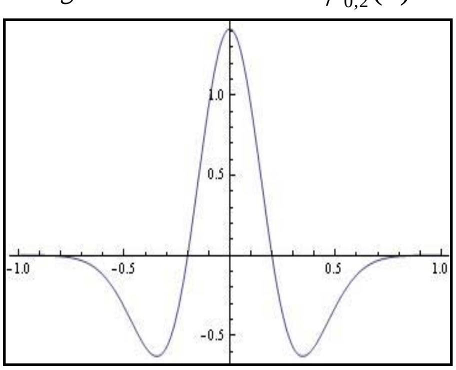


Figure 2.5: The function $\psi_{0,2}(x)$

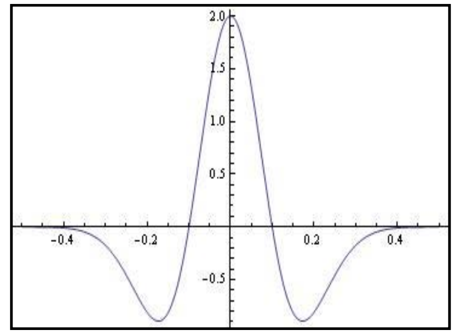


Figure 2.6: The function $\psi_{-1,0}(x)$

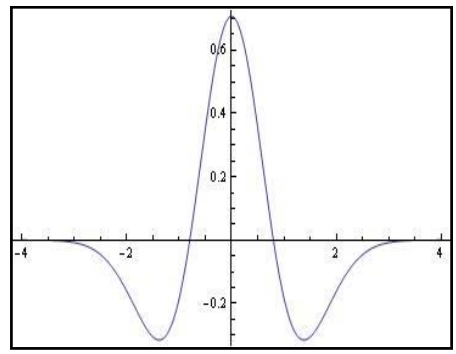
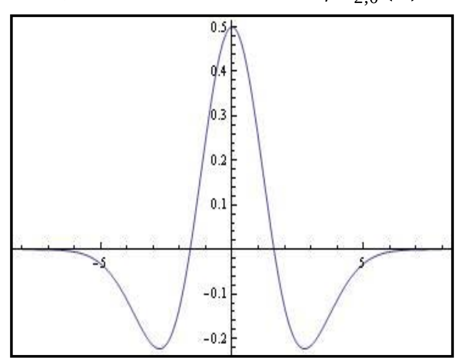


Figure 2.7: The function $\psi_{-2,0}(x)$



4) *Wavelet Representation of function*

In many applications, especially in signal processing, a finite number of values represent data, so it is important and often useful to consider discrete version of the continuous representation of a function of two continuous parameters j,k can be converted into a discrete one by assuming that *j* and *k* take only integral values. Our discrete wavelets are not time-discrete, only the translation and the scale step are discrete. Consider a family of function

$$\psi_{j,k}(x) = a_0^{-j/2}\psi(a_0^{-j}x - kb_0), \quad j,k \in [R] \qquad (2.5)$$

where a_0, b_0 are two positive constants.

The continuous wavelet transform of a given function $f(x) \in$[R] fined by

$$(w_\psi f)(j,k) = \langle f, \psi_{j,k} \rangle = \int_{-\infty}^{\infty} f(t)\overline{\psi_{j,k}}(t)\,dt \tag{2.6}$$

$$\therefore \langle f, \psi_{j,k} \rangle = a_0^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(t)\overline{\psi}\left(a_0^{-j}t - kb_0\right)dt$$

where both f and ψ are continuous, $\overline{\psi}_{j,k}(x)$ is the complex conjugate of $\psi_{j,k}(x)$. For computational efficiency, $a_0 = 2$ and $b_0 = 1$ are commonly used so that results lead to a binary dilation of 2^j and a dyadic translation of $k2^j$. From *(2.5)* we get,

$$\psi_{j,k}(x) = 2^{-\frac{j}{2}}\psi(2^{-j}x - k) \tag{2.7}$$

then equation *(2.6)* become

$$(W_\psi f)(j,k) = \langle f, \psi_{j,k} \rangle = 2^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(x)\overline{\psi}\left(2^{-j}x - k\right)dx \tag{2.8}$$

The variables j and k are integers that scale dilate the mother function $\psi(x)$ to generate wavelets. The scale index j indicates the wavelet's width and the location index k gives its position. For the discrete wavelet transform we can define in the following way

$$W_\varphi(j_0,k) = c_{j_0,k} = 2^{-j/2}\sum_{k=0}^{2^{j_0-1}} f(x)\varphi_{j_0,k}(x) \tag{2.9}$$

$$W_\psi(j,k) = d_{j,k} = 2^{-j/2}\sum_{k=0}^{2^{j-1}} f(x)\psi_{j,k}(x) \tag{2. 10}$$

for $j \geq j_0$ and then the wavelet series of *f(x)* can be written as

$$f(x) = 2^{j/2}\sum_{k=0}^{2^{j_0-1}} c_{j_0,k}\varphi_{j_0,k}(x) + 2^{j/2}\sum_{j=j_0}^{\infty}\sum_{k=0}^{2^j-1} d_{j,k}\psi_{j,k}(x) \tag{2. 11}$$

where, $f(x)$, $\varphi_{j_0,k}(x)$ and $\psi_{j,k}(x)$ are functions of the discrete variable, for details see [11].

III. Wavelets Decomposition and Reconstruction

1) *Definition*

Let A_j be generated by the bases $\{\phi_{j,k} : 2^{j/2}\phi(2^j t - k), k \in$[R]$\}$ and W_j by $\{\psi_{j,k} : 2^{j/2}\phi(2^j t - k), k \in$[R]$\}$, where j represents individual scales. Then any function $p_j(t)$ and $q_j(t)$ can be represented as the linear combination of $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ respectively. We must have a finite energy function $\phi(t) \in L_2$([R]) called scaling function that generates a nested sequence $\{A_j\}$ and satisfies a dilation equation:

$$\phi(t) = \sum_k g_0[k]\phi(at - k) \tag{3.1}$$

Here $a = 2$ and $g_0[k] \in l_2$ be the filters. Thus the function $\phi(t)$ is represented as a superposition of a scaled and translated version of itself. For details one can see [7].

More precisely, A_0 is generated by $\{\phi(t-k); k \in$[R]$\}$ and in general, A_j by $\left\{\phi_{j,k} : 2^{\frac{j}{2}}\varphi(2^j t - k); j,k \in\text{[R]}\right\}$.

$$\text{So, } p(t) \in A_j \Leftrightarrow p(2t) \in A_{j+1} \tag{3.2}$$

$$\text{Similarly, } p(t) \in A_j \Leftrightarrow p(2^{-j}t) \in A_j \tag{3.3}$$

There are many functions that generate sequences of subspaces. But the properties (3.2) and (3.3) and the dilation equations are unique to multi resolution analysis. For each j, since A_j is a proper subspaces of A_{j+1}, there are some space left in A_j called W_j which we combined with A_j gives us A_{j+1}. This subspaces $\{W_j\}$ are called the wavelet subspace and is complementary to A_j in A_{j+1}, *i.e.*

$$A_j \cap W_j = \{0\}, \quad j \in \text{[R]} \tag{3.4}$$

$$A_j \oplus W_j = A_{j+1} \tag{3.5}$$

With the condition (3.4), the summation in (3.5) is referred to as a direct sum and the decomposition in (3.5) as direct sum decomposition. Subspaces $\{W_j\}$ are generated by $\phi(t) \in L_2$([R]) called the wavelet, in the same way $\{A_j\}$ is generated by $\phi(t)$. In other words, $p_j(t) \in A_j$ can be written as

$$p_j(t) = \sum_k a_{j,k}\phi(2^j t - k) \quad (3.6)$$

and, any function $q_j(t) \in W_j$ can be written as

$$q_j(t) = \sum_k w_{j,k}\psi(2^j t - k) \quad (3.7)$$

for some coefficients $\{a_{j,k}\}_{k\in[R]}, \{w_{j,k}\}_{k\in[R]} \in l_2$. Since $A_{j+1} = W_j \oplus A_j = W_j \oplus W_{j-1} \oplus A_{j-1}$;

$\therefore A_{j+1} = W_j \oplus W_{j-1} \oplus W_{j-2} \oplus W_{j-3} \oplus ......$ where, $A_j = \overset{j-1}{\underset{l=-\infty}{\oplus}} W_l$. For the properties of orthogonality, we know $W_j \perp A_j$ which implies that every subspace carried on different information. Such that

$$\int_{-\infty}^{\infty} \phi(t)\psi(t-l)\,dt = 0, \ \forall\, l \in [R] \, and \ j = 0 \quad (3.8)$$

Therefore, we can be decomposed of any function in different smoothness spaces (like L_p-space) by using the formulas (3.6) and (3.7).

2) *Decomposition Algorithm*

We consider, the expression of the *CWT* of a signal $p(t)$ is

$$w_\psi p(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} p(t)\overline{\psi\left(\frac{t-b}{a}\right)} d\iota \quad (3.9)$$

where, $a = 1/2^j$ and $b = k/2^j \quad j,k \in [R]$ are the scale and translation parameter respectively . The *CWT* of $p(t)$ is a number at $(k/2^j, 1/2^j)$ on the time scale plane. It represents the correlation between $p(t)$ and $\overline{\psi}(t)$ at that time-scale point. It is called the discrete wavelet transform (DWT), which generates a sparse set of values on the time scale plane. We consider,

$$W_\psi = W_\psi p\left(\frac{k}{2^j}, \frac{1}{2^j}\right) = \int_{-\infty}^{\infty} p(t)\overline{\psi\left(\frac{t-k/2^j}{1/2^j}\right)} dt \quad (3.10)$$

to represent a signal by the wavelet coefficients at $(k/2^j, 1/2^j)$. A discrete time scale map represents the signal $p(t)$. It is known that the *CWT* generates redundant information about the signal on the time-scale plane. By choosing $(b = k/2^j, a = 1/2^j)$, it is much more efficient using the *DWT* to process a signal. It has been shown that the *DWT* keeps enough information of the signal such that it reconstructs the signal perfectly from the wavelet coefficients. In fact the number of coefficients need for perfect reconstruction is the same as the number of data samples. The decomposition analysis is used most often in wavelet signal processing. It is used in signal compression as well as in signal identification, although in the latter case, reconstruction of the original is not always required. The decomposition separates a signal into components at various scales corresponding to successive octave frequencies. Each component can be processed individually by a different algorithm. In each cancellation, for example, each component is processed with an adaptive filter of a different filter length to improve convergence. The mportant issue of this decomposition is to retain all pertinent information so that the user may recover the original signal. This algorithm is based on the decomposition relation in MRA discussion. We rewrite several of these relations here for easy reference.

Let, $p_j(t) \in A_j$; then, $p_j(t) = \sum_k a_{j,k}\phi_{j,k}(t)$ &

$$p_{j+1}(t) = \sum a_{j+1,k}\phi_{j+1,k}(t)$$

and $q_j(t) \in W_j$; then, $q_j(t) = \sum_k w_{j,k}\psi_{j,k}(t)$

The MRA requires that $A_{j+1} = A_j \oplus W_j$.

(3.11)

But we have, $p_{j+1}(t) = p_j(t) + q_j(t)$

$$\therefore \sum_k a_{j+1,k}\phi_{j+1,k}(t) = \sum_k a_{j,k}\phi_{j,k}(t) + \sum_k w_{j,k}\psi_{j,k}(t)$$

(3.12)

We substitute the decomposition relation

$$\phi(2^{j+1}t - l) = \sum_k \left[h_0[2k-l]\phi(2^j t - k) + h_1[2k-l]\psi(2^j t - k)\right]$$

(3.13)

into (3.12) to yield an equation in which all bases are at resolution *j*. After inter changing the order of summations and comparing the coefficients of $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ on both sides of the equation, we obtained

$$a_{j,k} = \sum_l h_0[2k-l]a_{j+1,l}, \quad w_{j,k} = \sum_l h_1[2k-l]a_{j+1,l}$$

where the right hand side of the equations corresponds to decimation by *2* after convolution. These formulas relate the coefficients of the scaling functions and wavelets at any scale to coefficients at the next higher scale. By repeating this algorithm is depicted in following figure, where we used the vector notation

$$a_j = \{a_{k,j}\}, \quad w_j = \{w_{k,j}\}, \quad h_0 = \{h_0[k]\}, \quad and \ h_1 = \{h_1[k]\}$$

(3.14)

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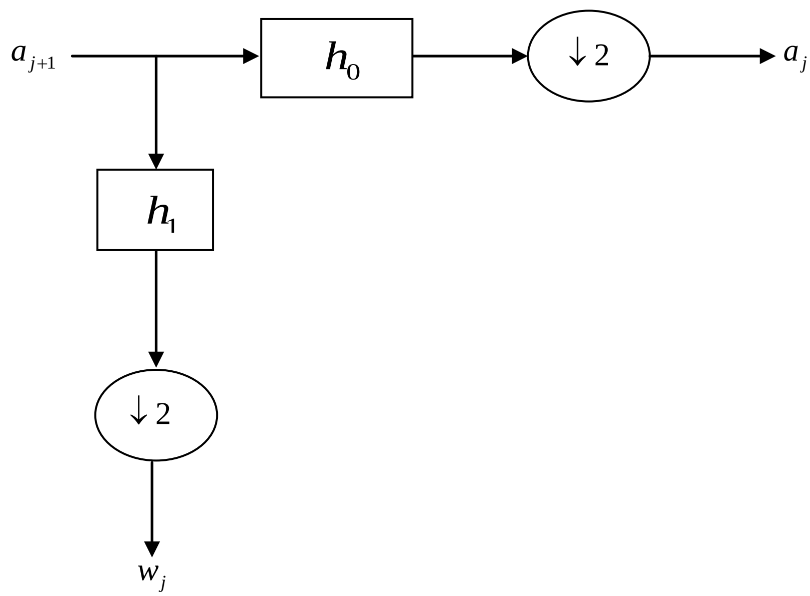


Figure 3.1: Single level wavelet decomposition process

where, $h_0[k]$ and $h_1[k]$ are decomposition sequences. These sequences hold the properties of the given signal. The decomposition block can be applied repeatedly to the scaling function coefficients at lower resolution to build a wavelet decomposition tree which shown by the following figure 3.2, for details see [4].

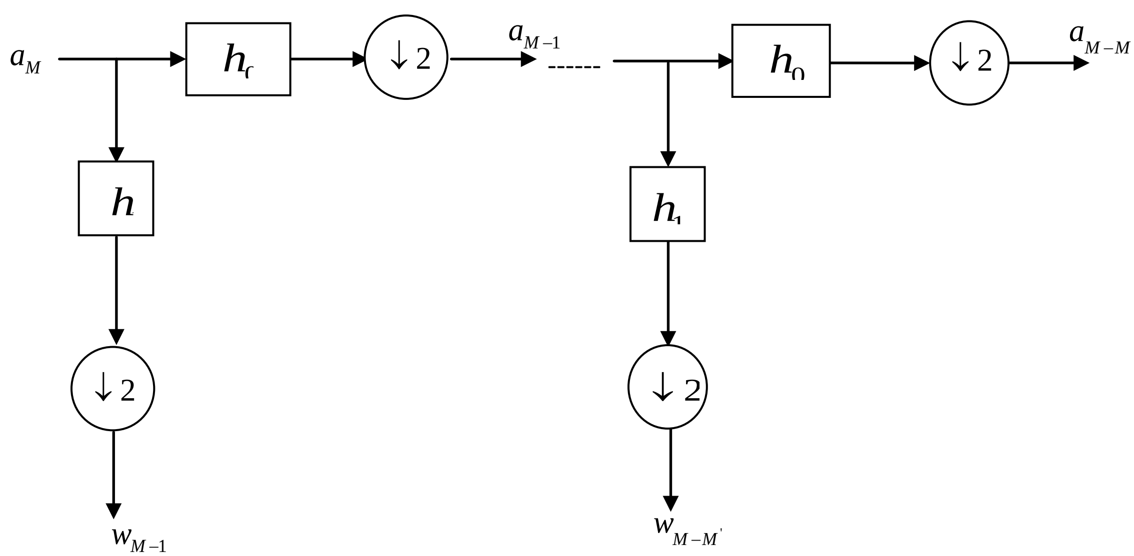


Figure 3.2: Wavelet decomposition tree

3) *Reconstruction Algorithm*

If the original data can be recovered perfectly then it is called transformation of unique inverse which is very important for any transform. For continuous signals, some transformations have their unique inverses in theory but cannot be implemented in actuality. There exist a unique inverse *DWT* (or the synthesis transform) such that the original function can be recovered perfectly from its components at different scales.

The reconstruction algorithm is based on the refinement equation of the Scaling and Wavelet function. We take a sum of these components at the *j* resolution is

$$p_j(t)+q_j(t)=\sum_k a_{j,k}\phi_{j,k}(t)+\sum_k w_{j,k}\psi_{j,k}(t)=p_{j+1}(t)$$

(3.15)

By a substitution of the two scale relations in to *(3.15)*, then

$$\sum_k a_{j,k}\sum_l g_0[l]\phi(2^{j+1}t-2k-l)+\sum_k w_{j,k}\sum_l g_1[l]\phi(2^{j+1}t-2k-l)=\sum_l a_{j+1,l}\phi(2^{j+1}t-l)$$

(3.16)

Comparing the coefficients of $\phi(2^{j+1}t-l)$ on both sides of *(3.16)* yields,

$$a_{j+1,l}=\sum_l\left\{g_0[l-2k]a_{j,k}+g_1[l-2k]w_{j,k}\right\}\quad(3.17)$$

where the right side of the equations correspond to interpolation followed by convolution.

We emphasize here that although the mechanics of computation are carried out in digital signal processing fashion, the decomposition and reconstruction algorithms are actually processing analog signals. The fundamental idea is to represent an analog signal by its components at different scales for efficient processing [8,10].

The algorithm of reconstruction is shown by the following figure 3.3:

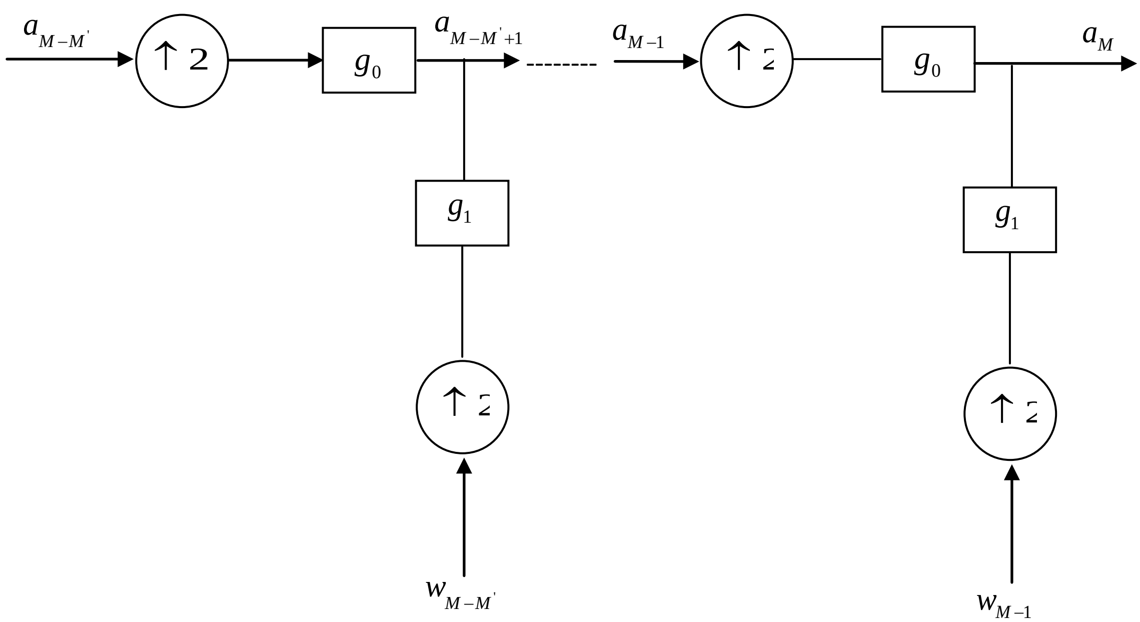


Figure 3.3: Wavelet Reconstruction tree

4) *Applications of Wavelet*

Wavelets are a powerful statistical tool which can be used for a wide range of applications. Wavelet transforms are now being adopted for a vast number of applications, often replacing the conventional Fourier Transform. Many areas of physics have seen this paradigm shift, including molecular dynamics, astrophysics, density-matrix localization, seismic geophysics, optics, turbulence and quantum mechanics. This change has also occurred in image processing, blood-pressure, heart-rate and ECG analysis, DNA analysis, protein analysis, climatology, general signal processing, speech recognition, computer graphics and multifractal analysis.

One use of wavelet approximation is in data compression. Like some other transforms, wavelet transforms can be used to transform data, and then encode the transformed data, resulting in effective compression.

5) *FBI Fingerprint Compression*

Fingerprint verification is one of the most reliable personal identification methods and it plays a very important role in forensic and civilian applications. Between 1924 and today, the US Federal Bureau of Investigation (FBI) has collected about 30 million sets of fingerprints [2]. The archive consists mainly of inked impressions on paper cards. Facsimile scans of the impressions are distributed among law enforcement agencies, but the digitization quality is often low. Because a number of jurisdictions are experimenting with digital storage of the prints, incompatibilities between data formats have recently become a problem. This problem led to a demand in the criminal justice community for a digitization and compression standard.

For the above case the data storage problem is perspective. Fingerprint images are digitized at a resolution of 500 pixels per inch with 256 levels of gray-scale information per pixel. A single fingerprint is about 700,000 pixels and needs about 0.6 Mbytes to store. A pair of hands, then, requires about 6 Mbytes of storage.

So digitizing the FBI's current archive would result in about 200 terabytes of data. (Notice that at today's prices of about $900 per Giga byte for hard-disk storage, the cost of storing these uncompressed images would be about 200 million dollars.) Obviously, data compression is important to bring these numbers down. In order for a system of this type of work in practice, it is necessary to be able to store the scanned data using as few bits as possible and it is necessary to have fast algorithms to extract the essential data from the finger being scanned. Wavelets fulfill both the conditions [3].

Figure 4.1. An FBI-digitized left thumb fingerprint. The image on the left is the original; the one on the right is reconstructed image through wavelet compression.

6) *Denoising Noisy Data*

In diverse fields scientists are faced with the problem of recovering a true signal from incomplete, indirect or noisy data. Can wavelets help solve this problem? The answer is certainly "yes," through a technique called ***wavelet shrinkage and thresholding method that*** David Donoho has worked on the area [7].

The technique works in the following way. When we decompose a data set using wavelets, we use filters that act as ***averaging*** filters. Some of the resulting wavelet coefficients correspond to details in the data set. If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of ***thresholding***, then, is to set to zero all coefficients that are less than a particular threshold. These coefficients are used in an inverse wavelet transformation to reconstruct the data set. Figure 4.2 is a pair of "before" and "after" illustrations of a nuclear magnetic resonance (NMR) signal. The signal is transformed, thresholded and inverse-transformed. The technique is a significant step forward in handling noisy data because the denoising is carried out without smoothing out the sharp structures. The result is cleaned-up signal that still shows important details.

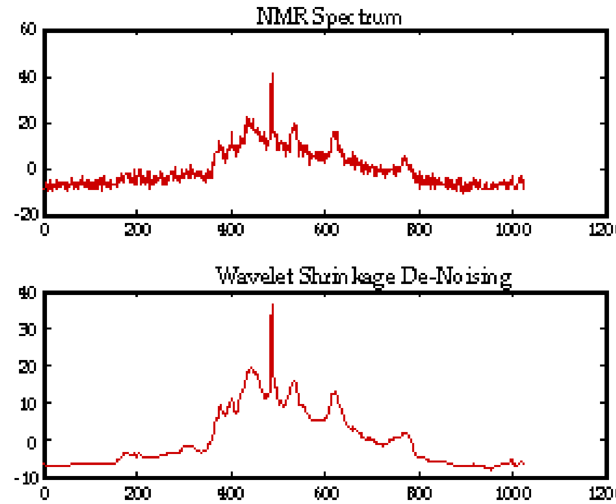


Figure 4.2 "Before" and "after" illustrations of a nuclear magnetic resonance signal. The original signal is at the top, the denoised signal at the bottom.

Figure 4.3 displays an image created by Donoho of Ingrid Daubechies and then several close-up images of her eye. The following an original image, a close-up image, close-up image with noise added and finally denoised image. To denoise the image Donoho done the following steps: [For details one can see [16]].

a. transformed the image to the wavelet domain using Coiflets with three vanishing moments,
b. applied a threshold at two standard deviations, and
c. inverse-transformed the image to the signal domain.

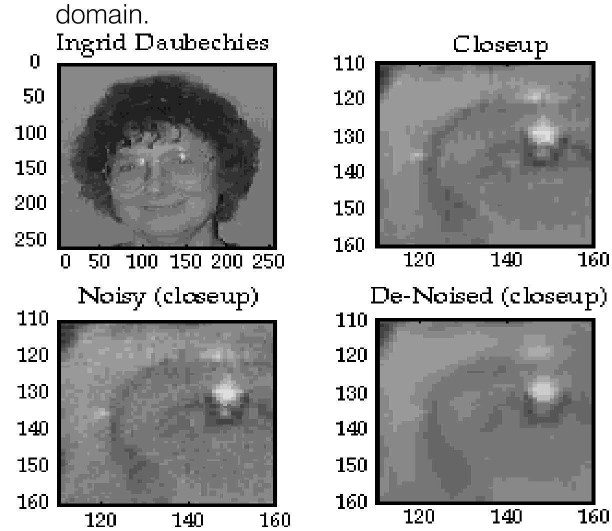


Figure 4.3 Denoising an image of Ingrid Daubechies' left eye. The top left image is the original. At top right is a close-up image of her left eye. At bottom left is a close-up image with noise added. At bottom right is a close-up image, denoised.

7) *Computer and Human Vision*

In the early 1980s, David Marr began work at MIT's Artificial Intelligence Laboratory on artificial vision for robots. He is an expert on the human visual system.

Marr believed that it was important to establish scientific foundations for vision. He then developed working algorithmic solutions. Marr's theory was that image processing in the human visual system has a complicated hierarchical structure that involves several layers of processing. At each processing level, the retinal system provides a visual representation that scales progressively in a geometrical manner. His arguments hinged on the detection of intensity changes. He theorized that intensity changes occur at different scales in an image, so that their optimal detection requires the use of operators of different sizes. He also theorized that sudden intensity changes produce a peak or trough in the first derivative of the image. These two hypotheses require that a vision filter have two characteristics: it should be a differential operator and it should be capable of being tuned to act at any desired scale. Marr's operator was a wavelet that today is referred to as a "Marr wavelet" [12].

8) *Wavelet Analysis of Medical Signals*

Over recent years, wavelet transforms have played an increasingly important role in the medical signal analysis. Wavelet transform analysis has been applied to a wide variety of biomedical signals including the ECG, EEG, EMG, Echocardiograms, MRI Images, clinical sounds-arterial bruits, heart sounds, breath sounds, respiratory patterns, blood pressure trends and DNA sequences. Research by P.S. Addison, Jamic Watson and Nopadol Uchaipichat and a number of collaborators from the UK and Europe has led to new insights into the underlying structure of a number of cardiac arrhythmias including ventricular fibrillation (VF) and atrial fibrillation (AF).

The two plots below show a short segment of ECG containing normal sinus rhythm together with its associated wavelet transform scalogram. The QRS complex of the waveform is evident from the conical structures in the scalogram, converging to the high frequency components of the RS spike. The P and T waves are also labeled in the plot. This figure highlights the wavelet transforms ability to pick out short duration, high frequency components in the time-frequency plane. An equivalent short time Fourier transform (STFT) spectrogram smears this short duration information due to its fixed width window.

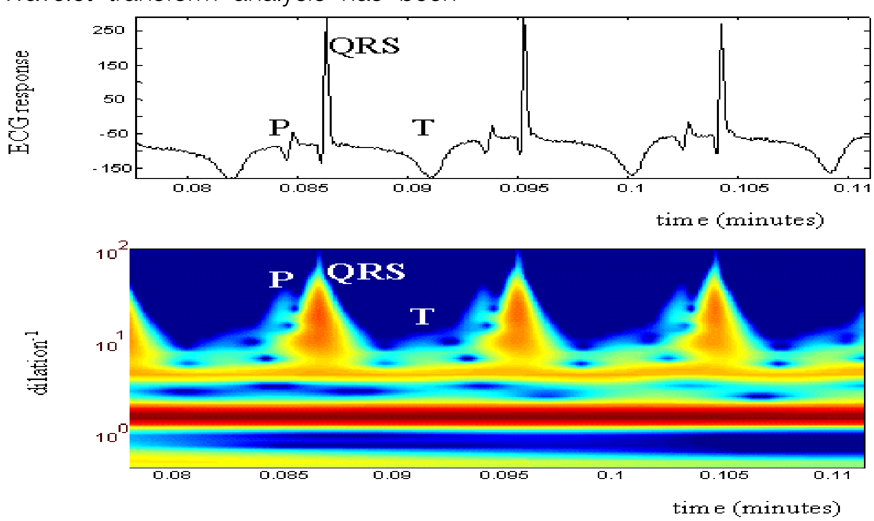


Figure 4.5 A short segment of ECG containing normal sinus rhythm together with its associated wavelet transform scalogram.

Ventricular fibrillation (VF) is the primary cardiac rhythm associated with sudden cardiac death. In the literature, VF is often described as a signal which is 'uncoordinated', 'random', 'chaotic', 'noisy' etc. The two plots below show the ECG and corresponding wavelet transform for a segment of ventricular fibrillation. High frequency spiking is evident in the scalogram plot.

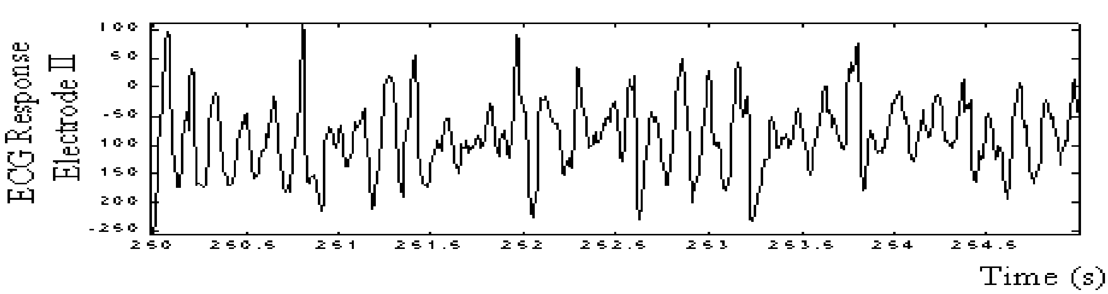


Figure4.6 ECG for a segment of ventricular fibrillation.

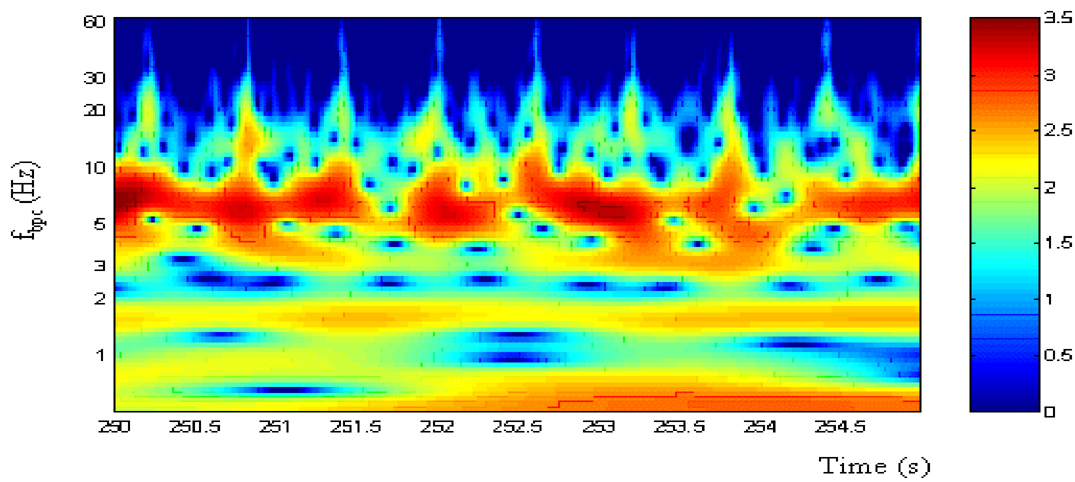


Figure 4.7 ECG corresponding wavelet transform for a segment of ventricular fibrillation.

The figure below shows a 7 minute sequence of ventricular fibrillation (VF) with CPR initiated after 5 minutes (evident by the red, high energy band appearing at 5 minutes in the lower right hand quadrant). Distinct banding can also be seen in the pre-CPR sequence. Closer inspection of the trace over smaller time windows reveals a rich structure within the VF signal.

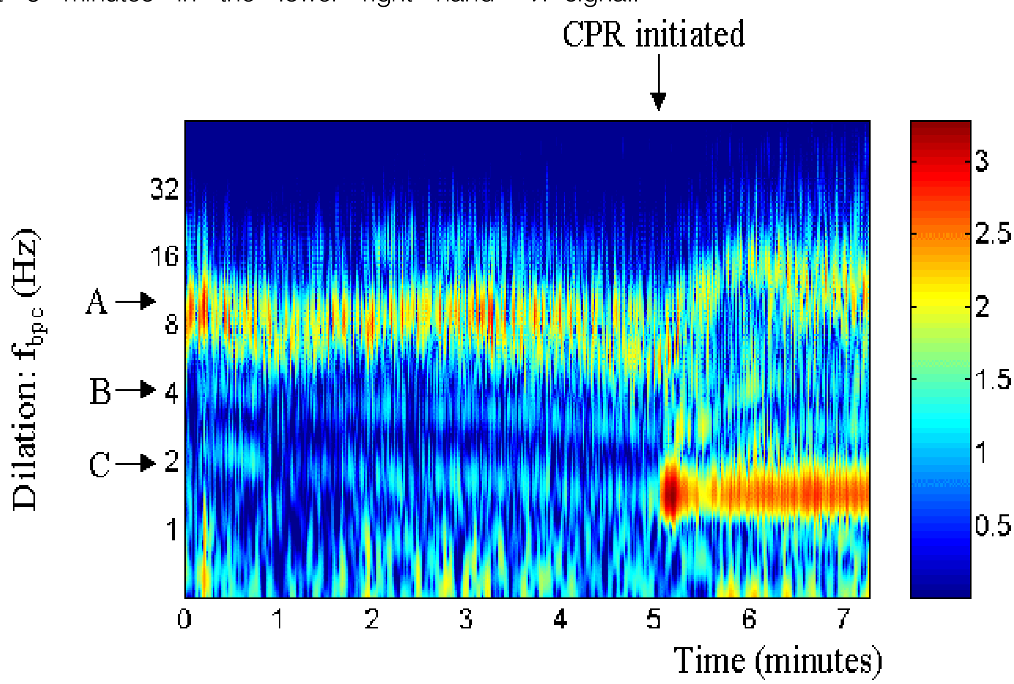
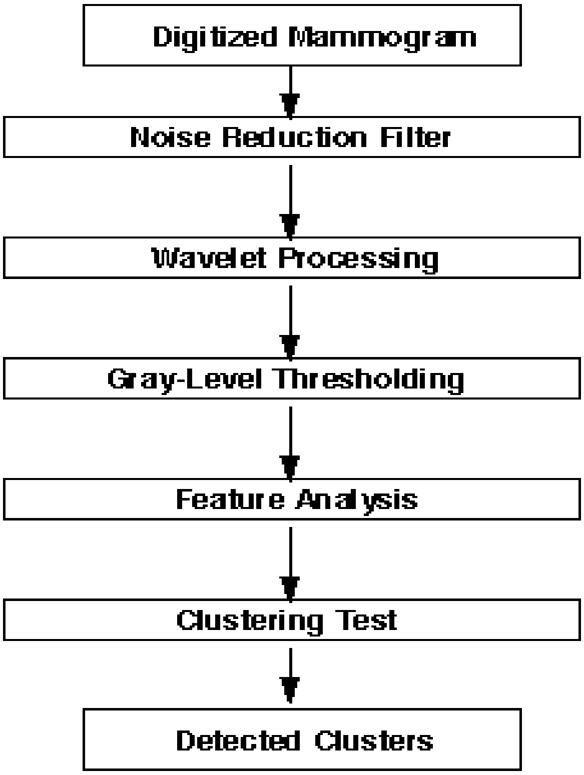


Figure 4.8 A 7 minutes sequence of VF with CPR initiated after 5 minutes

9) *Wavelet-Based Computer-Aided Diagnosis (CAD) Scheme for the Detection of Clustered Micro Calcifications in Digital Mammograms*

Breast cancer is a major cause of death for women; it causes an estimated 46,000 deaths per year in the United States. Mammography has been proved to be the primary radiologic procedure for early detection of breast cancer. Between 60 percent and 70 percent of no palpable breast carcinomas demonstrate micro calcifications on mammograms. Therefore, clustered micro calcifications on mammograms are an important indicator of breast carcinoma. However, 10 percent to 30 percent of women who have breast cancer and undergo mammography have negative mammograms. In about two-thirds of these false negative cases, the radiologist failed to detect a cancer that was evident retrospectively.

Yoshida et al. has recently introduced the wavelet transform into the computer-aided diagnosis (CAD) scheme in an attempt to improve the sensitivity and specificity of the system. Below is a block diagram of the wavelet-based CAD system.



The wavelet transform is employed as a preprocessing step whose goal is to enhance the micro calcifications and suppress the background structure in the mammogram.

10) *Wavelet Analysis of Turbulent Flow Fields*

The turbulent flow fields downstream of a variety of flow obstacles placed in an open channel flow are measured using laser Doppler anemometry. The time series obtained are analyzed using a variety of wavelets. P.S. Addison and Kevin Murray's project seeks to develop an iterated transform which should adapt to the flow structures at various resolution levels in the flow. The advantage of such a transform would be that the analysis of time series would not be confined to a single transform used over all scales which must, at present, be selected using an a priori knowledge of the flow field.

The plots below show the velocity time series taken from the vortex shedding flow behind a cylinder in an open channel and its corresponding wavelet transform scalogram plot (Mexican Hat). The scalogram shows up both the vortex shedding (as regular peaks and troughs) in the lower part of the plot and larger coherent flow structures towards the top of the plot.

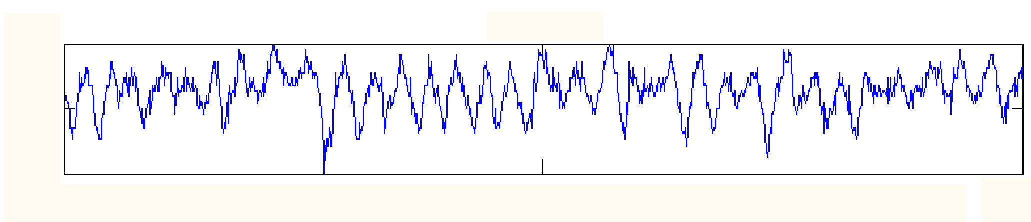


Figure 4.9 The velocity time series taken from vortex shedding in an open channel

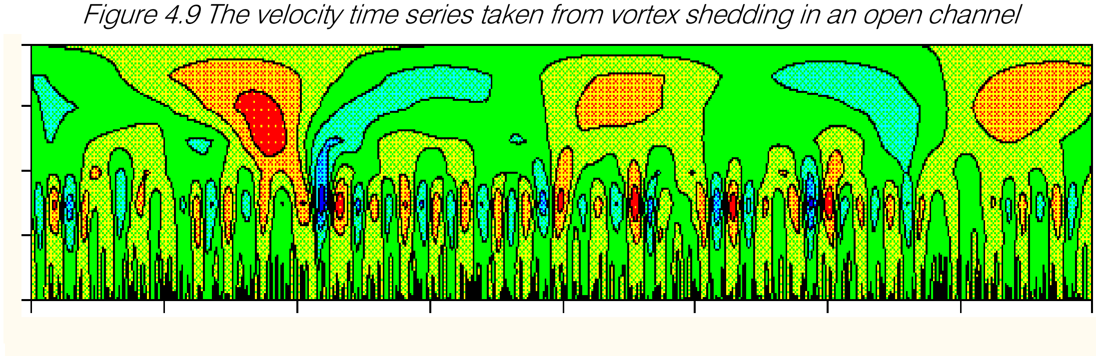


Figure 4.10 The velocity time series taken from vortex shedding in an open channel corresponding wavelet transform scalogram plot.

IV. Conclusion

The basic idea of wavelet analysis is to use a cluster of wavelet functions to express a signal. It has a high time-frequency resolution in low frequency bands, a high time resolution and low frequency resolution in high frequency bands. The main information of discrete wavelet transform locates in low frequency domain. By using the above characteristics the applications of wavelet increased tremendously in different areas of science, engineering, Medical science etc. Wavelet transforms are now being adopted for a vast number of applications, often replacing the conventional Fourier Transform. It can reflect the characteristic of signal more effectively and process a more detailed decomposition for high frequency bands. As a result, the decomposition sequence has a high time-frequency resolution and same bandwidth in the whole time-frequency domain. This indicates the special feature of the given signal. If we filtering the decomposed signal (3.14) for some special character of an image, like colour of an eye or visible distinguishing marks etc, then the modified decomposed signal will be very short, time saving and improve the storage capacity.

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