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On Topological Sets and Spaces

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ON TOPOLOGICAL SETS AND SPACES

Strictly as per the compliance and regulations of:

On Topological Sets and Spaces

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Abstract: In this research paper we are introducing the concept of m-closed set and m-$T_{1/3}$ space,s discussed their properties, relation with other spaces and functions. Also, we would like to discuss the applications of topology in industries through different areas of sciences such as Biology, Chemistry, Physics, Computer Science, Business Economics and Engineering.

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I. Introduction

Throughout this paper (X,τ), (Y,τ) and (Z,τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset a of a topological space (X, τ), int (A), cl(A) and C(A) represents the interior of A, the closure of A, and the complement of A in X respectively. In present time topology is an important branch of pure mathematics. But it is difficult to fix a date for the starting of topology as a subject in its own right. The first time use of term TOPOLOGY that we know of appeared in title of a book written by J.W.Listing in 1847. Great mathematician Riemann used exclusively used the term "Analysis Situs". An important work was done by Pioncare in 1985 when he published his first paper on "Analysis Situs". However, historians of science regard cantor's research on Fourier series from 1879 to 1884 as the beginning of General Topology. N. Levine [1] and M.E. Abd El. Monsef *et al*[2] introduced semi-open sets and β-sets . D. Andrijevic[3] used notation semi preopen sets for β-sets. Again N. Levine[4] generalized the concept of closed sets to generalized closed sets. P. Bhattacharya and B.K. Lahiri[5] generalized the concept of closed sets to semi-generalized closed sets via semi-open sets. N. Biswas[6] studied that the complement of a semi-open set is called a semi-closed set. The aim of this paper is to draw a new technique to obtain a new class of sets, called m-closed sets. This class is obtained by generalizing semi-closed sets via semi-generalized open sets. It is shown that the class of m-closed sets properly contains the class of semi-closed sets and is properly

contained in the class of semi-preclosed sets. Also it is shown that the class of m-closed sets is independent from the class of preclosed sets/the class of generalized closed sets/the class of gα-closed sets/the class of αg-closed sets. P. Bhattacharya and B.K. Lahiri[5], D.S. Jancovic & K.L. Reilly[7] and H. Maki et al[8] introduced semi-$T_{½}$ spaces, semi-T_D and $\alpha T_{½}$, semi-T_D and semi-$T_{½}$ spaces respectively. J. Dontchev[9&10] shown that $\alpha T_{½}$ separation axioms are equivalent. R-Devi *et al*[11] introduced αT_b spaces and T_b spaces respectively. As an application of m-closed sets, we introduced a new class of spaces, namely m-$T_{1/3}$ spaces. Also characterize m-$T_{1/3}$ spaces and show that the class of m-$T_{1/3}$ spaces properly contains contains the class of semi-$T_{½}$ spaces, the class of αT_b spaces and the class of semi-$T_{1/3}$ spaces.

II. Definitions And Notations

Definitions 2.1: Let A be a subset of topological space X then A is called.

(a). a *generalized closed (g-closed)* set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(b). A *semi-generalized closed (sg-closed)* set if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of a sg-closed set is called a sg-open set.

(c). A *generalized semi-closed (gs-closed)* set if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(d). An *α-generalized closed (αg-closed)* set if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ).

(e). A *generalized α-closed (gα-closed)* set if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X,τ).

(f). A *gα**-closed* set if cl(A) $\subseteq$ int(cl(U)), whenever A $\subseteq$ U and U is α-open in (X, τ).

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A *δ-generalized closed (δg-closed)* set if $cl\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(g). A *generalized semi-preclosed (gsp-closed)* set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(h). A *δ-closed* set if $A = cl\delta(A)$ whenever $cl\delta(A) = \{x \in X | int(cl(U)) \cap A \neq \phi, x \in U \text{ and } U \in \tau\}$.

(i). A *regular-open* set if $A = int(cl(A))$ and a regular-closed set if $cl(int(A)) = A$.

(j). A *semi-regular* set if it is both *semi-open* and *semi-closed* in (X,τ).

(k). A *preopen* set if $A \subseteq int(cl(A))$ and *preclosed* set if $cl(int(A)) \subseteq A$.

(l). A *semi-open set* if $A \subseteq cl(int(A))$ and a *semi-closed set* if $int(cl(A)) \subseteq A$.

(m). A *semi-preopen* set or *β-open* set if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set or β-closed set if $int(cl(int(A))) \subseteq A$.

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Definitions 2.2: A mapping $f: (X,\tau) \to (Y, \sigma)$ is said to be

(a). *Pre-semi-open* if f (U) is semi-open in (Y, σ) for every semi-open set U of (X,τ).

(b). *Pre-semi-closed* if f (U) is semi-closed in (Y,σ) for every semi-closed set U of (X,τ).

(c). *Semi-continuous* if f^{-1} (V) is semi-open in (X,τ) for every open set V of (Y, σ).

(d). *Pre-continuous* if f^{-1} (V) is pre-closed in (X,τ) for every closed set V of (Y, σ).

(e). *g-continuous* if f^{-1} (V) is g-closed in (X,τ) for every closed set V of (Y, σ).

(f). *αg-continuous* if f^{-1} (V) is αg-closed in (X,τ) for every closed set V of (Y, σ).

(g). *gα-continuous* if f^{-1-} (V) is gα-closed in (X,τ) for every closed set V of (Y, σ).

(h). *α-continuous* if f^{-1} (V) is α-closed in (X,τ) for every closed set V of (Y, σ).

(i). *β-continuous* if f^{-1} (V) is semi-preopen in (X,τ) for every open set V of (Y, σ).

(j). *sg-continuous* if f^{-1} (V) is sg-closed in (X,τ) for every closed set V of (Y, σ).

(k). *gs-continuous* if f^{-1} (V) is gs-closed in (X,τ) for every closed set V of (Y, σ).

(l). *gsp-continuous* if f^{-1} (V) is gsp-closed in (X,τ) for every closed set V of (Y, σ).

(m). *Irresolute* if f^{-1} (V) is semi-open in (X,τ) for every semi open set V of (Y, σ).

(n). *sg-irresolute* if f^{-1} (V) is sg-closed in (X,τ) for every sg-closed set V of (Y, σ).

Definitions 2.3: A topological space (X,τ) is called a

(a). $T_{½}$ space if every g-closed set is closed.

(b). *Semi-*$T_{½}$ space if every sg-closed set is semi-closed.

(c). *Semi-*T_D space if every singleton is either open or nowhere dense.

(d). α $T_{½}$ space if every gα**-closed set is α-closed.

(e). $\propto$ T*i space if a (X,τ^{α}) is T_i where i=1, ½.

(f). αT_b space if every αg-closed set is closed.

(g). T_b space if every gs-closed set is closed.

(h). αT_m space if every gα**-closed set is closed.

(i). *Feebly-*T_1 space if every singleton is either nowhere dense or clopen.

(j). $T_{¾}$ space if every δ-g-closed set is δ-closed.

(k). *Semi-*T_1 space if for any $x,y \in X$ such that $x \neq y$, there exists two semi-open sets G and H such that $x \in G$, $y \in H$ but $x \notin H$ and $y \notin G$.

III. M-CLOSED SET AND ITS PROPERTIES

Definition 3.1: A subset A of a topological space (X,τ) is called a *m-closed set* if $scl(A) \subseteq U$ wherever $A \subseteq U$ and U is a sg-open set of (X, τ).

Theorem 3.1: (i). m-closedness and g-clsedness are independent notions.
(ii). m-closedness is independent from gα-closedness, αg-closedness and preclosedness.
Proof: It can be seen by the following examples,

Example 3.1.1: Let X = {a, b, c} τ = {ϕ, X, {a}, {a, c}} and C = {c} and D = {a, b}, C is a m-closed set but not even a g-closed set of (X,τ). D is g-closed set but not a m-closed set of (X,τ).
The following two examples show that m-closedness is independent from gα- closedness, αg-closedness and preclosedness.
Example 3.1.2: Let X = {a, b, c,} τ = {ϕ, X, {a}, {b}, {a, b}} and K = {a}. K is m- closed but it is neither a gα-closed nor a αg-closed set. Also K is not preclosed set.

Example 3.1.3: Let $X = \{a, b, c,\}$ $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $L = \{b\}$. Here L is not a m-closed set of (X, τ). However L is a $g\alpha$-closed set. Hence it is a αg-closed set. Also L is a preclosed set of (X,τ).

Lemma 3.1: For a subset A of a space (X,τ), the following conditions are equivalent.
(i). A is pre-open, sg-open and m-closed.
(ii). A is pre-open, sg-open and semi-closed.
(iii). A is regular open.
The following example shows that a subset B of a space (X, τ) need not be a closed set even though B is pre-open, sg-open and a Q-set.

Example: Let $X= \{a, b, c\}$, $\tau= \{\phi, X, \{a\}\}$ and $B= \{a\}$. Clearly B is pre-open, sg-open and a Q-set but not a closed set.

Theorem 3.2: For a subset A of a space (X,τ), the following conditions are equivalent.
(i) A is clopen.
(ii) A is preopen, sg-open, Q-set, and m-closed.

Proof: (i) $\Rightarrow$ (ii): It is obvious.

Proof: (ii) $\Rightarrow$ (i): Since A is preopen, sg-open and a m-closed set of (X,τ), then sg Lemma 1, A is a regular open set. This implies A is open. On the other hand, $A = int(cl(A)) = cl(int (A)) \subseteq cl(A)$. Since A is a Q-set, so A is closed. Therefore A is a clopen set of (X,τ). □

Corollary 3.1: The union of two m-closed sets need not to be m-closed.
Proof: Let $X = \{a. b, c\}$, $\tau = \{\phi, X, \{a\} \{b\}, \{a, b\}\}$ $A = \{a\}$ and $B = \{b\}$. Then both A and B are m-closed but $A\cup B$ is not a m-closed set of (X,τ).

IV. M-$T_{1/3}$ SPACE AND ITS PROPERTIES

Definition 4.1: A space (X,τ) is said to be an m-$T_{1/3}$ space if every m-closed set in it is semi-closed.

Theorem 4.1: For a space (X,τ), the following conditions are equivalent.
(i). (X, τ) is a m-$T_{1/3}$ space.
(ii). Every singleton of X is either sg-closed or semi-open.
(iii). Every singleton of X is either sg-closed or open.

Proof: (i) $\Rightarrow$ (ii). Let $x \in X$ and assume that $\{x\}$ is not a sg-closed set of (X,τ). Then X-$\{x\}$ is a sg-open of (X, τ). So x is the only sg-open set containing X-$\{x\}$. Therefore X-$\{x\}$ is a m-closed set of (X,τ). Since (X,τ) is a m-$T_{1/3}$ space, then X-$\{x\}$ is a semi-closed set of (X,τ). □

Proof: (iii) $\Rightarrow$ (i) Let A be a m-closed set of (X, τ). Clearly $A \subseteq scl(A)$. Let $x \in X$. By supposition, $\{x\}$ is either sg-closed or semi-open. □
Case I: Let us suppose that $\{x\}$ is sg-closed. By Theorem 1.3.2 scl (A)-A does not contain any non-empty sg-clsed set. Since $x \in scl(A)$, then $x\in A$.
Case II: Let us suppose that $\{x\}$ is a semi-open set. Since $x \in scl(A)$, then $\{x\}\cap A \neq \phi$. So, $x \in A$. Hence in any case $scl(A) \subseteq A$. Therefore $A = scl(A)$ or equivalently A is a semi-closed set of (X, τ). Hence (X, τ) is a m-$T_{1/3}$ space.
Proof: (ii) $\Leftrightarrow$ (iii). It follows from the fact that a singleton is semi-open if it is open. .

V. APPLICATIONS OF GENERAL TOPOLOGY

Here we would like to discuss in brief the use of general topology in industries through by some areas of sciences.

a) *Application in BIOLOGY*

In recent years, topologists have developed the discrete geometric language of knots to a fine mathematical art one of the most interesting new scientific application of topology is the use of knot theory in analysis of DNA experiments. One of the important issues in molecular

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biology in the 3-dimensional structure of proteins and DNA in solution in the Cell and the relationship between structure and functions. Generally, protein and DNA structures are determined by X-rays crystallization and the manipulation required preparing a specimen for electron microscope. The DNA molecules are long and thin and the packing of DNA molecules in the cell nucleus is very complex. The biological solution to this entanglement problem is the existence of enzymes, which convert DNA from one topological form to another and appear to have a preformed role in the central genetic events of DNA replication, recombination and transcription. The topological approach to enzymology aims to exploit knot theory directly to reveal the secrets of enzyme action. How recent results in 3-dimensional topology have proved to be of use in the description and quantization of the action of cellular enzymes on DNA is best described by D.W.Sumners in his research paper published in 1995.

b) Application in CHEMISTRY

As a natural continuation of classical knot theory, chemists have been trying to synthesize and measure molecules with topologically interesting structures. The idea of molecules made of linked rings as a realistic possibility, was discussed at least as early as 1912. The most important tools in the topological method of making chemical predictions are known as indices. They derive from algorithms of procedures for converting the topological structure of a molecule into a single characteristic number. For example, an index might involve adding together the total number of rings in a molecule, or a number of atoms that are connected to three or more other atoms. The topological method has found applications beyond the simple prediction of chemical properties. It has the potential to help in modeling the behavior of gases, liquids and solids and of both organic & inorganic species, in developing new anesthetics and psychoactive drugs, in predicting the degree to which various pollutants might spread in the environment and the harm they might do once they spread, in estimating the cancer causing potential of certain chemicals and even in developing in beer with a well balanced taste.

c) Application in Physics

According to Normal Howes – uniform structures are the most important constructs from the physicist's point of view. The importance of uniform spaces from the physicist's points of view is also well brought out by the proceedings of the Nashville Topological Conference .In fact; topology has intrigued particle physicists for a long time. Recall that Donaldson used the Yand Mills field equations of mathematical physics, themselves generalizations of Maxwell's equations to study in 4 – space, there by reversing tradition by applying methods from physics to the understanding of topology.

d) Application in Computer Science

Recent developments in topology are penetrating other fields is best illustrated by the topics discussed at an extra ordinary research conference which was held at Barkley in 1990 in honor of the great topologist Stephen Samale's 60^{th} birthday. The proceeding were published with title "Form topology to Computation: Unity and diversity in mathematical sciences" edited by Hirsch, Marsden and Shub. There seems to not many examples of the use of topology in computer science, perhaps because it is not clear hot it is related to the fundamental questions. However, in recent years, there have been some interesting results. The problem of the minimal number of conditional statements in an algorithm, to solve a particular problem, seems particularly well suited for the topological approach.

e) Application in Business Economics

Topology has had tremendous effect on developments in economics. The study of conflicts of interest between individuals is what makes economics interesting and mathematically complex. Indeed we now know that the space of all individual preferences, which define the individual optimization problems, is topologically nontrivial and that is topological complexity is responsible for the impossibility of treating several individual preferences as if they were one. Because it is not possible, in general, to define a single optimization problem. Because of the complexity arising from simultaneous optimization problems, economic differs from physics where many of the fundamental relations derive from a single optimization problem. The attempt to find solutions to conflicts among individual interests led to there different theories about how economics are organized and how they behave.

f) Application in Engineering

Topology has also found applications in engineering. a problem of great importance to an electric industry, which had failed of solution by its own engineers, has been solved by using methods of set theoretic topology. In particular, Daniel R. Baker has established that topological techniques are used in several robotics applications. Topology has been applied to production and distribution problems as well.

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