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Abstract- In this article, we implement the exp($-\Phi(\eta)$)-expansion method for seeking the exact solutions of NLEEs via the Benjamin-Ono equation and achieve exact solutions involving parameters. Abundant traveling wave solutions with arbitrary parameters are successfully obtained by this method and the wave solutions are expressed in terms of the hyperbolic, trigonometric, and rational functions. It is established that the exp($-\Phi(\eta)$)-expansion method offers a further influential mathematical tool for constructing the exact solutions of NLEEs in mathematical physics. The obtained results show that exp($-\Phi(\eta)$)-expansion method is very powerful and concise mathematical tool for nonlinear evolution equations in science and engineering.

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GJSFR-F Classification : *MSC 2010: 35C07, 35C08, 35P99.*

TRAVELING WAVE SOLUTIONS OF NONLINEAR EVOLUTION EQUATIONS VIA EXPEXPANSION METHOD

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Rafiqul Islam [α], Md. Nur Alam [σ], A.K.M. Kazi Sazzad Hossain [ρ], Harun-Or-Roshid [ω] & M. Ali Akbar [¥]

Abstract- In this article, we implement the $\exp(-\Phi(\eta))$-expansion method for seeking the exact solutions of NLEEs via the Benjamin-Ono equation and achieve exact solutions involving parameters. Abundant traveling wave solutions with arbitrary parameters are successfully obtained by this method and the wave solutions are expressed in terms of the hyperbolic, trigonometric, and rational functions. It is established that the $\exp(-\Phi(\eta))$-expansion method offers a further influential mathematical tool for constructing the exact solutions of NLEEs in mathematical physics. The obtained results show that $\exp(-\Phi(\eta))$-expansion method is very powerful and concise mathematical tool for nonlinear evolution equations in science and engineering.

Keywords: the $\exp(-\Phi(\eta))$-expansion method; complixiton soliton solutions; the benjamin-ono equation; traveling wave solutions; nonlinear evolution equation.

I. Introduction

Nonlinear wave phenomena appears in various scientific and engineering fields such as fluid mechanics, plasma physics, optical fibers, biophysics, geochemistry, electricity, propagation of shallow water waves, high-energy physics, condensed matter physics, quantum mechanics, elastic media, biology, solid state physics, chemical kinematics, chemical physics and so on. This is also noticed to arise in engineering, chemical and biological applications. The application of nonlinear traveling waves has been brought prosperity in the field of applied science. In order to understand better the nonlinear phenomena as well as further application in the practical life, it is important to seek their more exact travelling wave solutions. Essentially all the fundamental equations in physical sciences are nonlinear and, in general, such NLEEs are often very complicated to solve explicitly. The exact solutions of NLEEs play an important role in the study of nonlinear physical phenomena. Therefore, the powerful and efficient methods to find exact solutions of nonlinear equations still have drawn a lot of interest by diverse group of scientists. In the past three decades, there has been significant progress in the development of finding effective methods for obtaining exact solutions of NLEEs. These methods are the homogeneous balance method [1], the tanh-function method [2], the extended tanh-function method [3, 4], the Exp-function method [5, 6], the sine-cosine method [7], the modified Exp-function method [8], the generalized Riccati equation [9],

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Ref

2. A.M. Wazwaz, The tanh method for generalized forms of nonlinear heat conduction and Burgers–Fisher equations, Appl. Math.Comput. 169 (2005) 321–338.

the Jacobi elliptic function expansion method [10, 11], the Hirota's bilinear method [12], the Miura transformation [13], the (G'/G)-expansion method [14-18], the novel (G'/G)-expansion method [19, 20], the modified simple equation method [21, 22], the improved (G'/G)-expansion method [23], the inverse scattering transform [24], the Jacobi elliptic function expansion method [25, 26], the new generalized (G'/G)-expansion method [27-31], the $\exp(-\Phi(\eta))$-expansion method [32, 33] and so on.

The objective of this article is to apply the $\exp(-\Phi(\eta))$-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the Benjamin-Ono equation.

The outline of this paper is organized as follows: In Section 2, we give the description of the $\exp(-\Phi(\eta))$-expansion method. In Section 3, we apply this method to the Benjamin-Ono equation, graphical representation of solutions. Conclusions are given in the last section.

II. Description of the $\exp(-\Phi(\eta))$-Expansion Method

Let us consider a general nonlinear PDE in the form

$$F(v, v_t, v_x, v_{xx}, v_{tt}, v_{tx}, \ldots), \tag{1}$$

where $v = v(x,t)$ is an unknown function, F is a polynomial in $v(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following, we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable η

$$v(x,t) = v(\eta), \ \eta = x \pm V t, \tag{2}$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $v = v(\eta)$:

$$\Re(v, v', v'', v''', \cdots), \tag{3}$$

where $\Re$ is a polynomial of v and its derivatives and the superscripts indicate the ordinary derivatives with respect to η.

Step 2. Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$v(\eta) = \sum_{i=0}^{N} A_i (\exp(-\Phi(\eta)))^i, \tag{4}$$

where A_i $(0 \le i \le N)$ are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\eta)$ satisfies the following ordinary differential equation:

$$\Phi'(\eta) = \exp(-\Phi(\eta)) + \mu \exp(\Phi(\eta)) + \lambda, \tag{5}$$

Eq. (5) gives the following solutions:

Ref

13. T.L. Bock and M.D. Kruskal, A two-parameter Miura transformation of the Benjamin-One equation, Phys. Lett. A 74 (1979) 173-176.

Family 1: When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$\Phi(\eta) = \text{In}\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\ \tan\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{6}$$

Ref

34. A. Neyrame, A. Roozi, S. S .Hosseini, S. M. Shafiof, Exact travelling wave solutions for some nonlinear partial differential equations, journal of King Saud University (Science), (2010), 22, 275-278.

Family 2: When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$\Phi(\eta) = \text{In}\left(\frac{\sqrt{(4\mu - \lambda^2)}\ \tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{7}$$

Family 3: When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$\Phi(\eta) = -\text{In}\left(\frac{\lambda}{\exp(\lambda(\eta + E)) - 1}\right) \tag{8}$$

Family 4: When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(\eta) = \text{In}\left(-\frac{2(\lambda(\eta + E) + 2)}{\lambda^2(\eta + E)}\right) \tag{9}$$

Family 5: When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(\eta) = \text{In}(\eta + E) \tag{10}$$

$A_N, \cdots\cdots, V, \lambda, \mu$ are constants to be determined latter, $A_N \neq 0,$ the positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\Phi(\eta))$. As a result of this substitution, we get a polynomial of $\exp(-\Phi(\eta))$. We equate all the coefficients of same power of $\exp(-\Phi(\eta))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_N, \cdots\cdots, V, \lambda, \mu$. Substituting the values of $A_N, \cdots\cdots, V, \lambda, \mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

III. Application of the Method

In this section, we will present the $\exp(-\Phi(\eta))$-expansion method to construct the exact solutions and then the solitary wave solutions of the Benjamin-Ono equation [34]. Let us consider the Benjamin-Ono equation,

$$v_t + hv_{xx} + vv_x = 0\,. \tag{11}$$

We utilize the traveling wave variable $v(\eta)=v(x,t)$, $\eta=x-\omega t$, Eq. (11) is carried to an ODE

$$-Vv'+hv''+\frac{1}{2}(v^2)'=0. \tag{12}$$

Eq. (12) is integrable, therefore, integrating with respect to η once yields:

$$P-\omega v+hv'+\frac{1}{2}v^2=0, \tag{13}$$

where P is an integration constant which is to be determined.

Taking the homogeneous balance between highest order nonlinear term w^2 and linear term of the highest order w' in Eq. (13), we obtain $N=1$. Therefore, the solution of Eq. (13) is of the form:

$$v(\eta)=A_0+A_1(\exp(-\Phi(\eta))), \tag{14}$$

where A_0, A_1 are constants to be determined such that $A_N\neq 0$, while λ,μ are arbitrary constants. It is easy to see that

$$v'(\eta)=-A_1(\exp(-2\Phi(\eta))+\mu+\lambda\exp(-\Phi(\eta))). \tag{15}$$

$$v^2(\eta)=A_0{}^2+2A_0A_1\exp(-\Phi(\eta))+A_1{}^2\exp(-2\Phi(\eta))). \tag{16}$$

Substituting v, v', v^2 into Eq. (13) and then equating the coefficients of $\exp(-\Phi(\eta))$ to zero, we get

$$\frac{1}{2}A_1{}^2-hA_1=0, \tag{17}$$

$$A_0A_1-hA_1\lambda-\omega A_1=0, \tag{18}$$

$$P-\omega A_0+\frac{1}{2}A_0{}^2-hA_1\mu=0, \tag{19}$$

Solving the Eq. (17)-Eq. (19) yields

$$P=\frac{1}{2}A_0{}^2-A_0h\lambda+2h^2\mu,\ \ \omega=A_0-h\lambda,\ A_0=A_0,\ \ A_1=2h.$$

where λ,μ are arbitrary constants.

Now substituting the values of ω, A_o, A_1 into Eq. (14) yields

$$v(\eta)=A_0+2h(\exp(-\Phi(\eta))), \tag{20}$$

where $\eta=x-(A_0-h\lambda)t$.

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Now substituting Eq. (6)- Eq. (10) into Eq. (20) respectively, we get the following five traveling wave solutions of the Benjamin-Ono equation.

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0$,

$$v_1(\eta) = A_0 - \frac{4h\mu}{\sqrt{\lambda^2 - 4\mu}\ \tan(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + E)) + \lambda}.$$

where $\eta = x - (A_0 - h\lambda)t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda^2 - 4\mu < 0$,

$$v_2(\eta) = A_0 + \frac{4h\mu}{\sqrt{4\mu - \lambda^2}\ \tan(\frac{\sqrt{4\mu - \lambda^2}}{2}(\eta + E)) - \lambda}.$$

where $\eta = x - (A_0 - h\lambda)t$. E is an arbitrary constant.

When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0$,

$$v_3(\eta) = A_0 + \frac{2h\lambda}{\exp(\lambda(\eta + E)) - 1}.$$

where $\eta = x - (A_0 - h\lambda)t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0$,

$$v_4(\eta) = A_0 - \frac{h\lambda^2(\eta + E)}{(\lambda(\eta + E)) + 2)}.$$

where $\eta = x - (A_0 - h\lambda)t$. E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0$,

$$v_5(\eta) = A_0 - \frac{2h}{(\eta + E)}.$$

where $\eta = x - (A_0 - h\lambda)t$. E is an arbitrary constant.

a) Graphical representation of the solutions

The graphical illustrations of the solutions are given below in the figures with the aid of Maple.

Notes

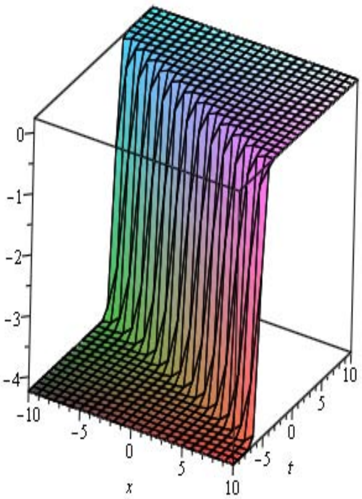

Fig. 1 : The modulus of Kink wave solution $v_1(\eta)$.

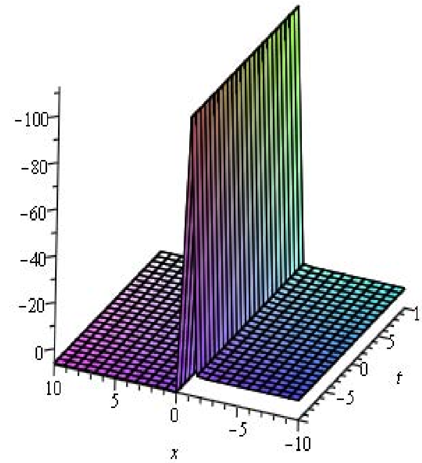

Fig. 2 : The modulus of solitary wave solution $v_2(\eta)$.

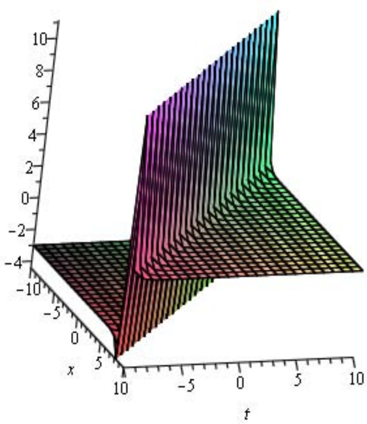

Fig. 3 : The modulus of solitary wave solution $v_3(\eta)$

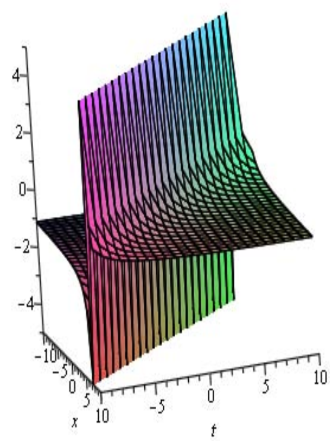

Fig. 4 : The modulus of solitary wave solution $v_4(\eta)$

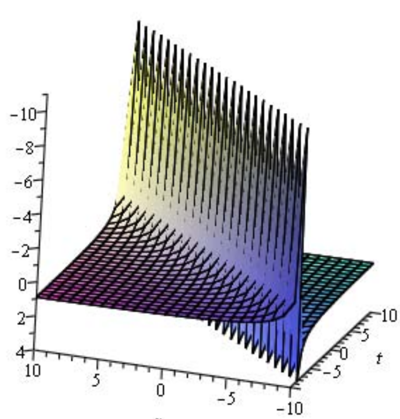

Fig. 5 : The modulus of solitary wave solution $v_5(\eta)$.

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IV. Conclusion

In this study, we considered the Benjamin-Ono equation. We apply the $\exp(-\Phi(\eta))$-expansion method for the exact solution of this equation and constructed some new solutions which are not found in the previous literature. The method offers solutions with free parameters that might be imperative to explain some intricate physical phenomena. This study shows that the $\exp(-\Phi(\eta))$-expansion method is quite efficient and practically well suited to be used in finding exact solutions of NLEEs. Also, we observe that the $\exp(-\Phi(\eta))$-expansion method is straightforward and can be applied to many other nonlinear evolution equations.

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