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A Linear Algorithm for Convex Drawing of a Planar Graph

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GJCST-F Classification : I.4.7

A LINEAR ALGORITHM FOR CONVEX DRAWING OF A PLANAR GRAPH

A Linear Algorithm for Convex Drawing of a Planar Graph

Thanvir Ahmad [α] & Md. Shahidul Islam [σ]

Abstract - A straight line drawing of a planar graph is called a convex drawing if the boundaries of all faces of that graph are drawn as convex polygon. A graph is planar if it has at least one embedding in the plane such that no two edges intersect at any point except at their common end vertex. Not all planar graphs have convex drawing. In this thesis, we study the characteristics of convex drawing of a planar graph. We develop a method for examining whether a face is drawn as a convex polygon or not.

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I. Introduction

Some planar graphs can be drawn in such a way that each edge is drawn as a straight line segment and each face is drawn as a convex polygon, as illustrated in Figure 3.1. Such a drawing is called a *convex drawing*. The drawings in Figs. 3.2 are not convex drawings.

Although not every planar graph has a convex drawing, Tutte showed that every 3-connected planar graph has a convex drawing, and obtained a necessary and sufficient condition for a plane graph to have a convex drawing [5]. Furthermore, he gave a "barycentric mapping" method for finding a convex drawing of a plane graph, which requires solving a system of $O(n)$ linear equations [6]. The system of equations can be solved either in $O(n3)$ time and $O(n2)$ space using the ordinary Gaussian elimination method, or in $O(n1.5)$ time and $O(n \log n)$ space using the sparse Gaussian elimination method [LRT79]. Thus the barycentric mapping method leads to an $O(n1.5)$ time convex drawing algorithm for planar graphs. In this chapter we first give a lemma for a face is drawn as convex polygon or not. Then using that lemma finally we device a linear time algorithm to examine whether a planar graph has convex drawing or not.

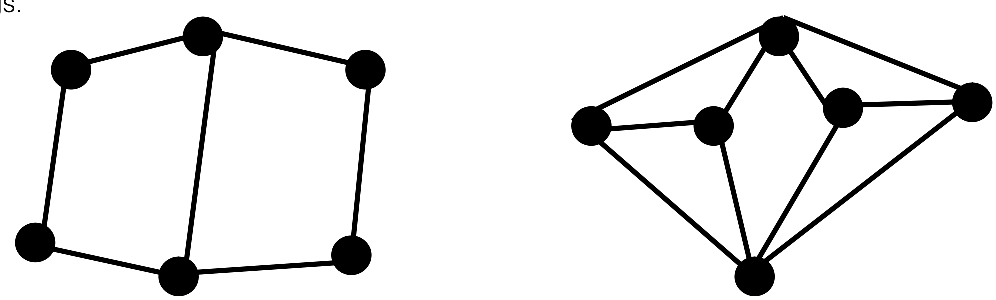

Figure 1.1 : Convex drawing of planar graph

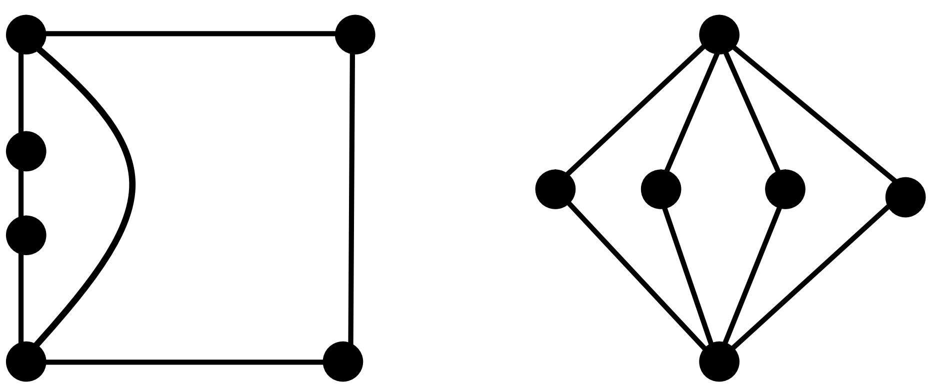

Figure 1.2 : Non convex drawing of planar graph

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II. Definition

By extensively examining the characteristics of convex drawing of a planar graph we derive a lemma for examining whether a planar graph has convex drawing or not. Before introducing the lemma we need to define some terms.

a) Convex Drawing of Planar Graph

A straight line drawing of a planar graph G is called a convex drawing if the boundaries of all faces of G are drawn as convex polygons [8]. Figure 3.3 depicts a convex drawing of a planar graph.

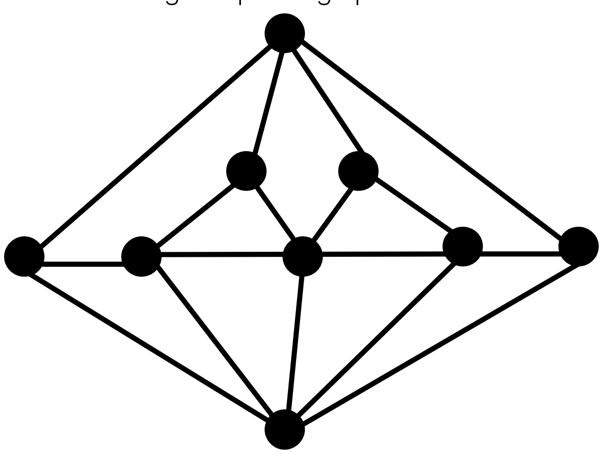


Figure 2.1 : Convex drawing of a planar graph

b) Face

If G is a planar graph, then any plane drawing of G divides the plane into regions, called faces [9]. That is, a face is an area bounded by the edges .One of these faces is unbounded, and is called the infinite face. If f is any face, then the degree of f (denoted by deg f) is the number of edges encountered in a walk around the boundary of the face f. If all faces have the same degree (g, say), the G is face-regular of degree g. For example, the following graph G depicts in Figure 3.4 has six faces, f6 being the infinite face.

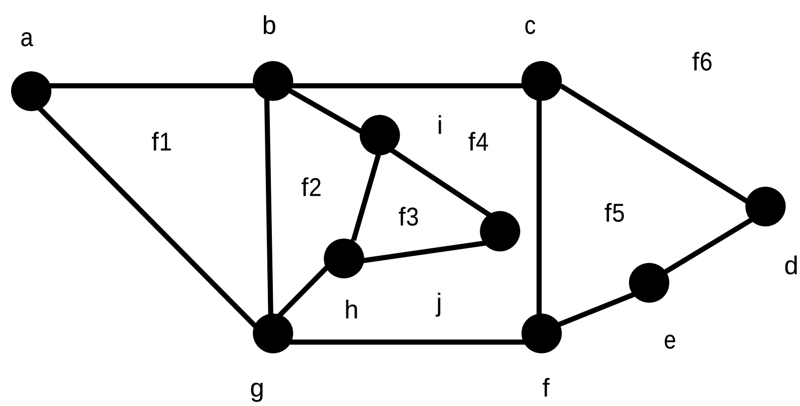


Figure 2.2 : A planar graph with six faces

It is easy to see from above graph that deg $f_1=3$, deg $f_2=4$, deg $f_3=3$, deg $f_4=7$, $f_5=4$. Note that the sum of all the degrees of the faces is equal to twice the number of edges in the graph, since each edge either borders two different faces (such as bg , cd, and cf) or occurs twice when walk around a single face (such as ab and gh). The Euler's formula relates the number of vertices, edges and faces of a planar graph. If n, m, and f denote the number of vertices, edges, and faces respectively of a connected planar graph, then we get $n\text{-}m+f = 2$.The Euler formula tells us that all plane drawings of a connected planar graph have the same number of faces namely, $2+m\text{-}n$.

III. Theorem

(Euler's Formula) Let G be a connected planar graph, and let n, m and f denote, respectively, the numbers of vertices, edges, and faces in a plane drawing of G. Then $n\text{-}m+f=2$.

Proof We employ mathematical induction on edges, m. The induction is obvious for m=0 since in this case n=1 and f=1. Assume that the result is true for all connected plane graphs with fewer than m edges, where m is greater than or equal to 1, and suppose that G has m edges. If G is a tree, then n=m+1 and f=1 so the desired formula follows. On the other hand, if *G* is not a tree, let e be a cycle edge of G and consider G-e.

The connected plane graph *G-e* has n vertices, m-1 edges, and f-1 faces so that by the inductive hypothesis,

$$n-(m-1)+(f-1)=2$$

Which implies that?

$$n-m+f=2$$

(Proved)

(a) Convex polygon

A *convex polygon* is a simple polygons whose interior is a convex set [8]. The following properties of a simple polygon are all equivalent to convexity:

- Every internal angle is less than 180 degrees.
- Every line segment between two vertices remains inside or on the boundary of the polygon.

A simple polygon is *strictly convex* if every internal angle is strictly less than 180 degrees. Figure 3.5(a) and (b) depicts a convex and non-convex polygon respectively.

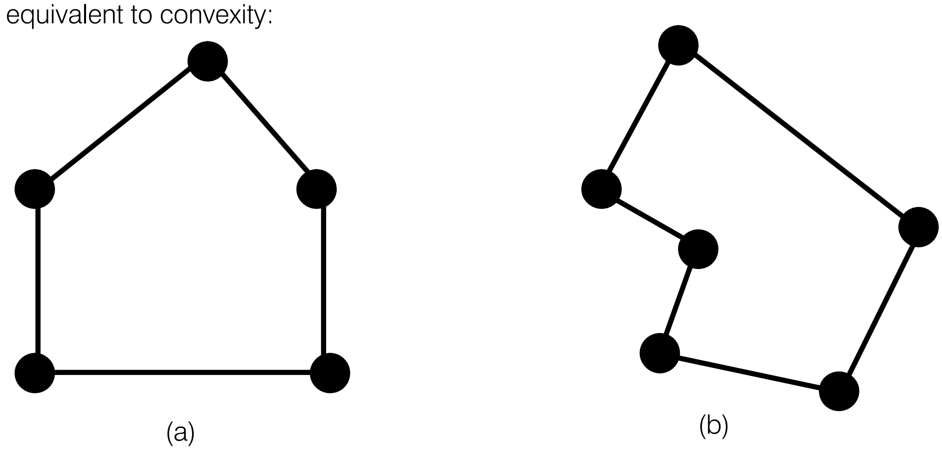


Figure 3.1 : (a) A convex polygon and (b) non-convex polygon

(b) Lemma

A face is drawn as convex polygon if and only if the cross products of adjacent edges of each vertex of that face are same sign.

Proof

Let, a face is assumed to be described by N vertices ordered by,

$$v_0(x_0, y_0), v_1(x_1, y_1), v_2(x_2, y_2), \ldots v_{n-1}(x_{n-1}, y_{n-1})$$

Figure 3.6 (a) and (b) depicts a face in clockwise and anti-clockwise vertex ordering respectively. A simple test of vertex ordering for examining a face is drawn as convex polygon is based on considerations of the cross product between adjacent edges of each vertex of that face. If the cross product is positive then it rises above the plane (z axis up out of the plane) and if negative then the cross product is into the plane.

$$\text{cross product} = ((x_i - x_{i-1}), (y_i - y_{i-1})) \times ((x_{i+1} - x_i), (y_{i+1} - y_i))$$
$$= (x_i - x_{i-1}) * (y_{i+1} - y_i) - (y_i - y_{i-1}) * (x_{i+1} - x_i)$$

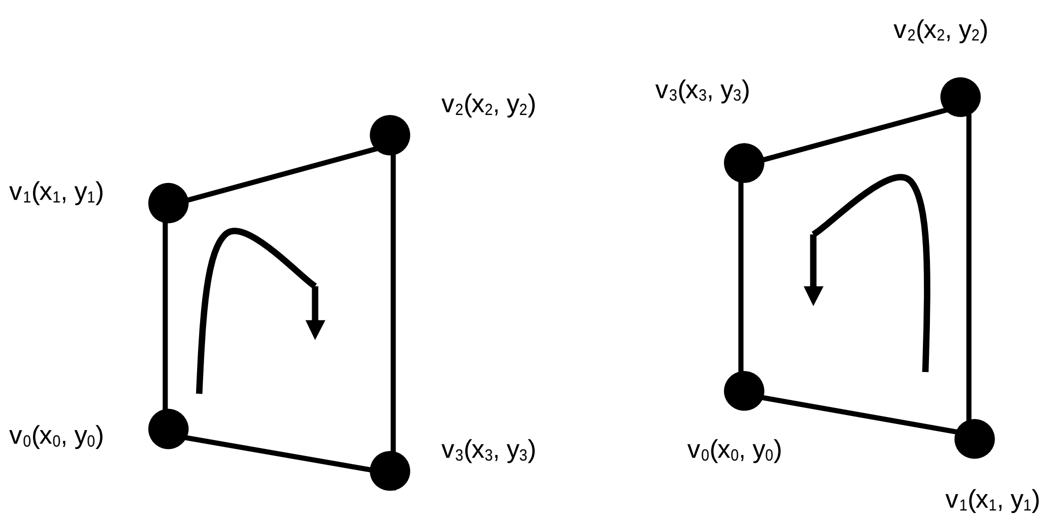
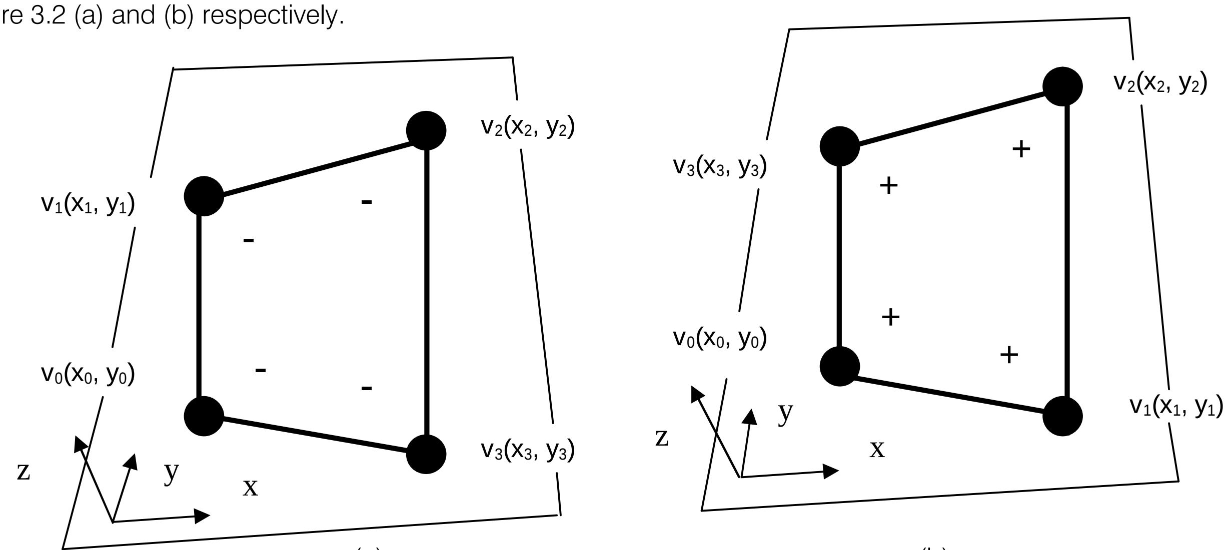


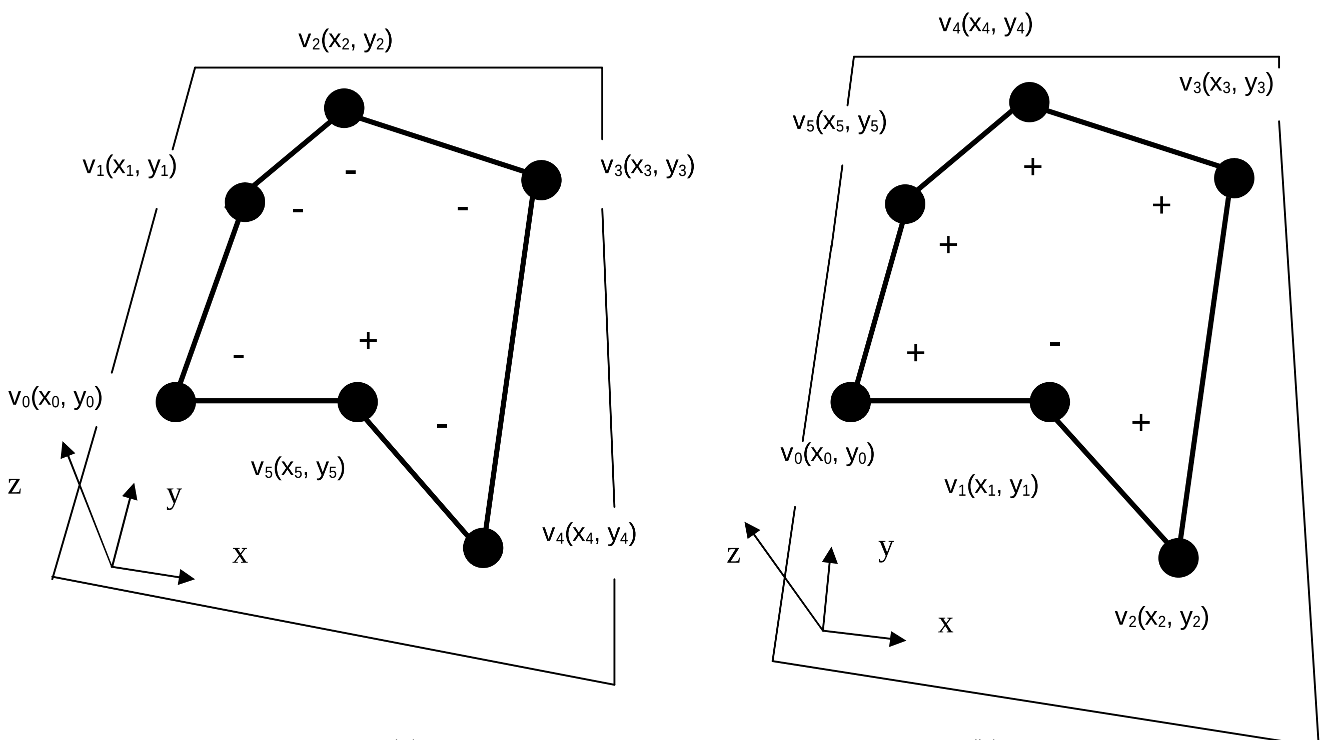
Figure 3.2 : A face in (a) clockwise and (b) anti-clockwise vertex ordering

Figure 3.2 (a) and (b) depicts the cross product sign of adjacent edges of each vertex face depicts in figure 3.2 (a) and (b) respectively.



(a) (b)

Figure 3.3 : (a) Cross products sign of adjacent edges in clockwise direction of convex face and (b) Cross products sign of adjacent edges in anti- clockwise direction of convex in the case of non convex face the cross product sign of adjacent edges of each vertex of that face depicts in figure 3.4(a) and (b)



(a) (b)

Figure 3.4 : (a) Cross products sign of adjacent edges in clockwise direction of non-convex face and (b) Cross products sign of adjacent edges in anti-clockwise direction of non-convex face

A non-convex face has mixture of cross products sign of adjacent edges of each vertex of that face. Hence, a face is drawn as convex polygon if and only if the cross products of adjacent edges of each vertex of that face are same sign.

[Proved]

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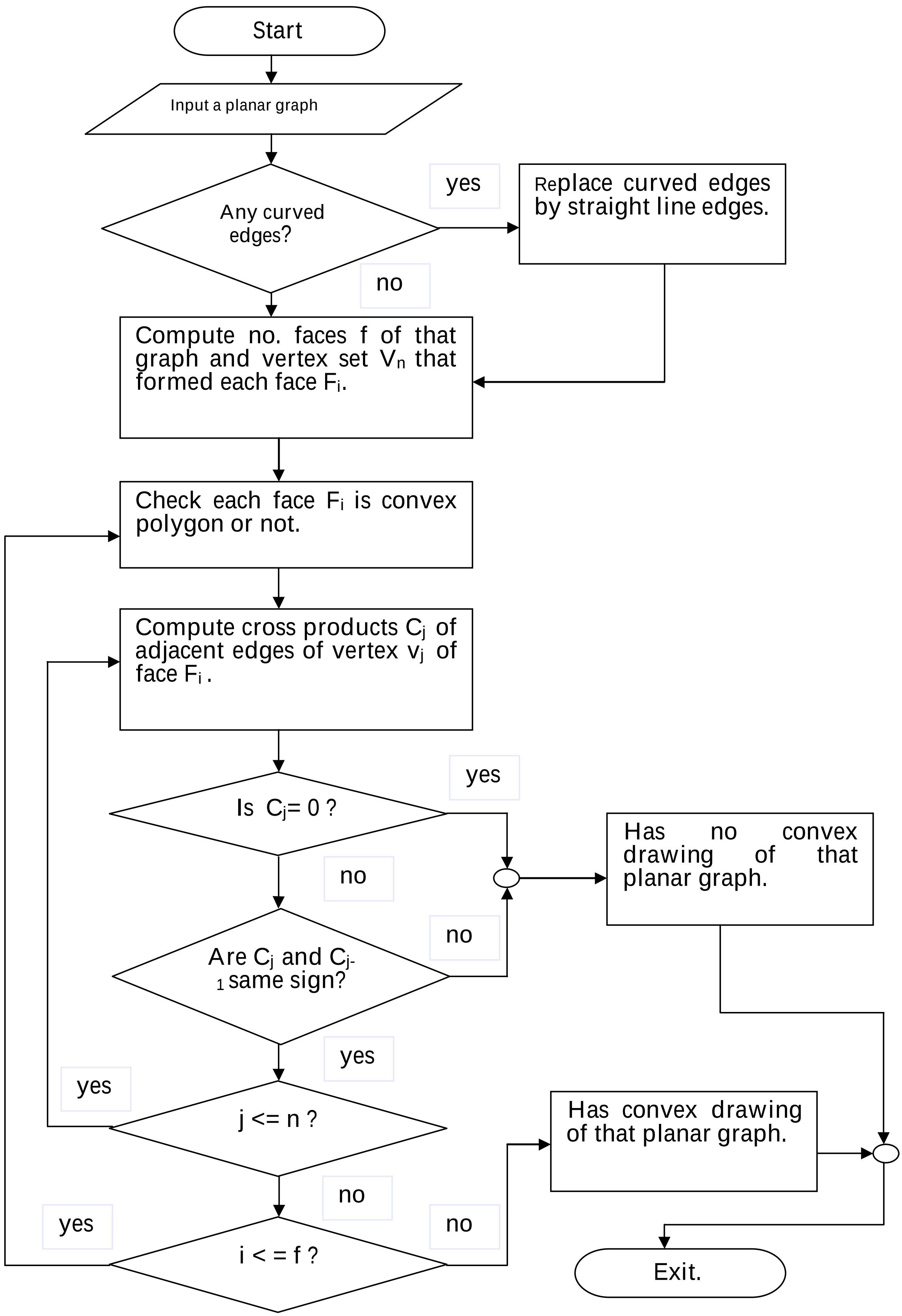
(c) Flowchart : Convex_ Drawing (G)



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(d) Algorithm: Convex_ Drawing (G)

```
Begin

Step 1:  Check input planar graph has curved edges.
                  if (curved edges)
                  then replace curved edges by straightline  edges.
                  else
                  go to step 2.
         end if
Step 2:  Compute no. faces f of that graph and vertex set Vn that formed each face Fi.
                       i.e. , Fi = {v0,v1,v3..................vm} where i=1 to f and m=0 to n.
Step 3.     3(a):    Check each face Fi is convex polygon or not.
                       for each face Fi where  i=1 to no. of face f
                       do
                         j=0 to no. of vertex n to form that face Fi
                            compute cross product Cj of adjacent edges of  vertex vj.
                               cross product of adjacent edges of vertex vj
                                     Cj =(Xj-Xj-1)*(Yj+1-Yj)-(Yj-Yj-1)*(Xj+1-Xj).
                     if ( cross product Cj = 0)
                        then go to step 5.
                     else
                        go to step 3(b).
               end if.
            3(b):        if (j=0)
                       then increment j by 1.
                    else
                       check cross product Cj and Cj-1 are same sign.
                     if (cross product Cj and Cj-1are same sign)
                       then increment j by 1.
                    else
                       go to step 5.
                end if
              end if
            end  do
         end for.
       go to step 4.
Step 4:  Has Convex drawing of that planar graph.
             go to step 6.
Step 5: Has no convex drawing of that planar graph.
             go to step 6.
Step 6:  Exit.

End
```

IV. Conclusions

In this thesis we have studied the convex drawing of a planar graph. Not every planar graph has convex drawing. The results of this thesis are summarized as follows:

- We have derived a method for determining whether a face is drawn as convex polygon or not.
- Finally, using that method we develop a linear time algorithm for examining whether a planar graph has a convex drawing or not.

Some interesting directions in which the future research works can be done are as follows:

- We develop a linear time algorithm for examining whether a planar graph has a convex drawing or not. One can develop an algorithm for converting non-convex drawing of a planar graph to convex drawing.
- One can develop a convex grid drawing of a planar graph on an (n-2) x (n-2) grid.

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