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ECG Signal Denoising by Morphological Top-Hat Transform

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GJCST-C Classification : *H.5.5*

ECG SIGNAL DENOISING BY MORPHOLOGICAL TOP-HAT TRANSFORM

Strictly as per the compliance and regulations of:

ECG Signal Denoising by Morphological Top-Hat Transform

S.A.Taouli [α] & F. Bereksi-Reguig [σ]

Abstract - The electrocardiogram ECG signal plays an important role in the primary diagnosis, prognosis and survival analysis of heart diseases. The ECG signal contains an important amount of information that can be exploited in different manners. However, during its acquisition it is often contaminated with different sources of noise making difficult its interpretation. In this paper, a new approach based on Morphological Top-Hat Transform (MTHT) is developed in order to suppress noises from the ECG signals. The morphological operators (dilation, erosion, opening, closing) constitute the fundamental stage of Top-Hat transform. Method presented in this paper is compared with the Visu Shrink, Sure Shrink, and Bayes Shrink methods. The experimental results indicated that the proposed methods in this work were better than the compared methods in terms of retaining the geometrical characteristics of the ECG signal, SNR. Due to its simplicity and its fast implementation, the method can easily be used in clinical medicine.

Keywords : ECG signal, denoising, mathematical morphology, morphological top-hat transform.

I. Introduction

During its acquisition the ECG signal is corrupted with different types of noises. Noises such as the power line interference (50 Hz), the muscle artifact due to the EMG (electromyogram), the baseline wandering due to the rythmic inhalation and exhalation during respiration are examples of noises which corrupt the ECG signals [1-2]. In order to reduce the noise in ECG signals many techniques are available such as digital filters (FIR or IIR), adaptive method, wavelet transform thresholding and Empirical Mode Decomposition methods [3]. However, digital filters and adaptive methods can be applied to signal whose statistical characteristics are stationary in many cases.

Recently the wavelet transform has been proven to be an useful tool for non-stationary signal analysis [4]. Thresholding is used in wavelet domain to smooth out or to remove some coefficients of wavelet transform subsignals of the measured signal. The noise content of the signal is reduced, effectively, with in the non-stationary environment. The denoising method that applies thresholding in wavelet domain has been proposed by Donoho [5-6]. It has been proved has Been proved that the Donoho's method for noise reduction works well for a wide class of one dimensional and two dimensional signals. Other approaches for threshold value estimators can be found in [7-10]. Visu Shrink [9, 11] utilizes the universal threshold estimator, which is $\sqrt{2\log(N)}$ for a vector d_i of the detail coefficients of length N . Sure Shrink is based on Stein's unbiased risk estimator [12]. Sure Shrink has serious drawbacks in situations of extreme sparsity of the wavelet coefficients [13]. In [11], Bayes Shrink was used for the threshold estimator, which is a data-driven sub and adaptive technique. Others methods, which has also been widely used is the Least Mean Square adaptive algorithm (LMS) [9]. But this algorithm is not able to track the rapidly varying non-stationary signals such as ECG signal within each heart beat; this causes excessive low pass filtering of mean parameters such as QRS complex.

This paper considers as a possible alternative, the application of Morphological Top-Hat Transform, (MTHT) based on morphological signal-processing concepts. Morphological signal processing comprises a broad collection of theoretical concepts and mathematical tools for signal analysis, non-linear signal operators, design methodologies and application systems that are related to Mathematical Morphology (MM).

Morphological operators of opening and closing are simple, and the morphological Top-Hat transform arising from these operators, have been prove to be powerful tools and have been used in different applications, giving excellent results in areas such as noise reduction, edge detection and object recognition [14].

In this work, we are interested in morphological Top-Hat transform. The filter is implemented under MATLAB 7 environment. The filter is evaluated using ECG signals from the MIT- BIH universal data base [15].

The paper is divided into five sections. After this introduction section 1, the Section 2 presents a brief description of basic morphological signal processing. Section 3 describes the morphological Top-Hat transform algorithm for ECG signal. In section 4, some experimental results are presented and discussed. Finally, a conclusion is given in section 5.

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II. Mathematical Morphology

During the last decade, mathematical morphology was established like powerful method for signal processing and became a complete mathematical theory. It was applied successfully in various disciplines, such as mineralogy, medical diagnosis, and histology. It found also increasing applications in the digital signal processing, computer vision and pattern recognition.

a) *Basic mathematical morphology transforms*

The basic concept of morphological signal processing is to modify the shape of a signal, equivalently considered as a set, by transforming it through its interaction with another object, called structuring element. In practice, the structuring element is compact and of a simple shape than the original object. The basic operators of morphology transform include dilation, erosion, opening and closing [16-19].

Let $f(n)$ be the original 1-D signal, which is the discrete function over a domain $f(n)=\{0,1.........., N-1\}$. And let $B(m)$ be the structuring element, which is the discrete function over a domain $B(m)=\{0,1.........., M-1\}$ two basic morphological operators, the erosion and the dilation, can be defined as

$$(f\Theta B)(n) = \underset{m=0,...M-1}{MIN}\{f(n+m)-B(m)\} \qquad (1)$$

$$(f \oplus B)(n) = \underset{m=0,...M-1}{MAX}\{f(n-m)+B(m)\} \qquad (2)$$

Based on de dilation and erosion, two other basic morphological operators, the opening ($\circ$) and the closing $(\bullet)$ can be further defined:

$$(f \circ B)(n) = (f\Theta B \oplus B)(n) \qquad (3)$$

$$(f \bullet B)(n) = (f \oplus B\Theta B)(n) \qquad (4)$$

Table 1 and 2 illustrate successively the basic properties of dilation and erosion, closing and opening operations.

To obtain the eroded function of $f(n)$, we attribute to $f(n)$ its minimal value in the field of the structuring element $B(m)=(0,0,0,0,0)$ which is a line segment, and with each new displacement of $B(m)$, the structuring element $B(m)$ plays the same role as a moving window. The width of such window is chosen empirically $B=5$. This is illustrated in Fig.1, where these operations are applied to an ECG signal.

The illustration shows a reduction in the peaks of ECG signal and a widening of the valleys. Erosion is an operator of shrinking in which the values of $f\Theta B$ are always less than those of f.

Dilataion
Commutative: $A \oplus B = B \oplus A$
Associative: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
Increasing: $(A)_x \oplus B = (A \oplus B)_x$
Duality: $A \oplus B = (A^c \ominus B)^c$
Erosion
Non-commutative : $A \ominus B \neq B \ominus A$
Translation Invariance : $(A)_x \ominus B = (A \ominus B)_x$
Increasing: $B_1 \subseteq B_2 \Rightarrow A_1 \ominus B_2 \subseteq A \oplus B_1$
Duality: $(A \ominus B^c)^c = A^c \oplus B$

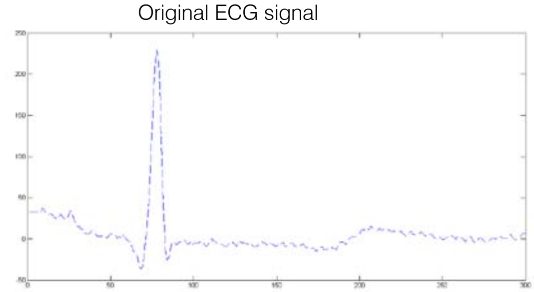
Table 1 : Basic properties of dilation and erosion

Closing
Extensivity: $A \subseteq A \bullet B$
Idempotence: $(A \bullet B) \bullet B = A \bullet B$
Translation Invariance: $:(A)_x \bullet B = (A \bullet B)_x$
Increasing: $A_1 \subseteq A_2 \Rightarrow A_1 \bullet B \subseteq A_2 \bullet B$
Duality: $(A \bullet B)^c = AoB$
Opening
Antiextensivity $AoB \subseteq A$
Idempotence: $(AoB)oB = AoB$
Translation Invariance: $: (A)_x oB = (AoB)_x$
Increasing: $A_1 \subseteq A_2 \Rightarrow A_1 oB \subseteq A_2 oB$
Duality: $(AoB)^c = A \bullet B$

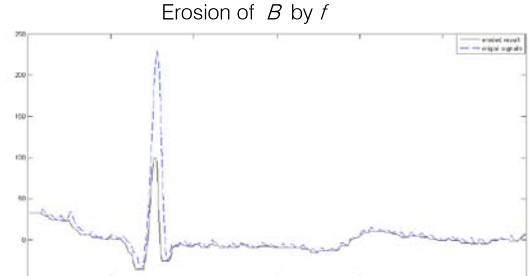
Table 2 : Basic properties of closing and opening

Similarly, the dilation can be performed by taking of set sums. Its complexity is the same as erosion and is related to convolution, where instead of doing summation of products, a maximum of sums is computed.

Original ECG signal



Erosion of B by f



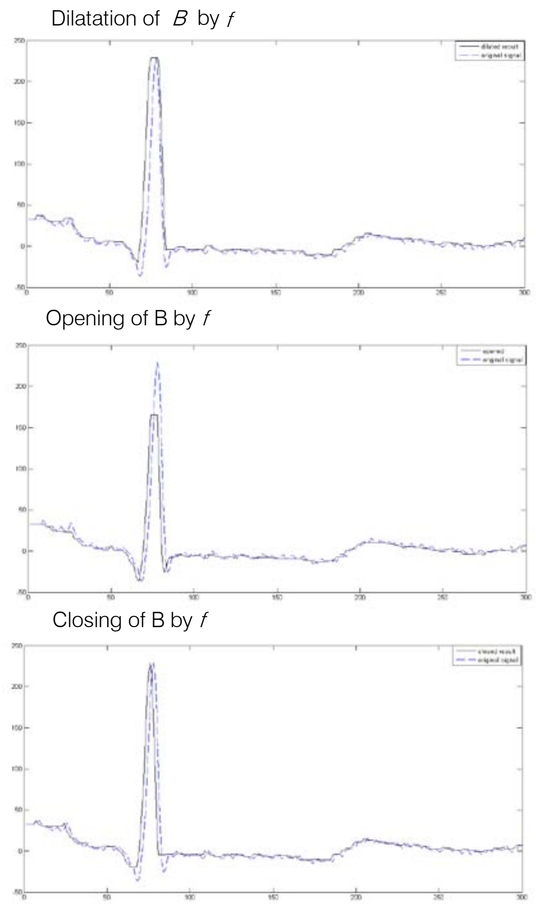


Fig. 1 : Erosion, Dilation, Opening, Closing of ECG signal by structuring element

This transformation fills the valleys and thickens the peaks. Fig. 1 shows that dilation is an operation of expansion in which values of $f \oplus B$ are always greater than those of f .

Morphological opening can be expressed as an erosion operation followed by a dilation of the eroded result, using the same structuring element. The dual operation of opening is the closing operation.

Morphological closing can be expressed as a dilation operation followed by an erosion of the dilated result, using the same structuring element.

Figure1. shows that the opening by B smooths the graph off from below by cutting down its peaks.

The closing smooths the graph of f from above by filling up its valleys (suppress peaks). Subtracting from f its opening or closing by B provides respectively the peaks and valleys of f .

The width of these peaks and valleys depends on the size of B . Therefore, opening and closing by a structuring element B can be used effectively to suppress noise in ECG signals.

b) Morphological structuring element (SE)

After selecting the morphological operator, the SE is the next key component of the morphology analysis to be defined. Generally, only when the shape of the signal is matched to those of SE, the signal can be preserved. Therefore, the shape, length (domain) and height (amplitude) of SE should be selected according to the signal to be analyzed.

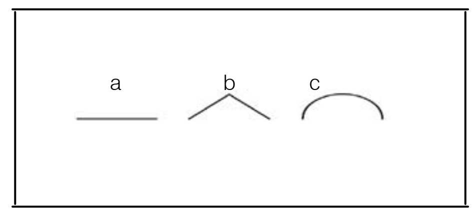


Fig. 2 : Structuring element E of various shapes: (a) flat SE; (b) triangular SE; and (c) semicircular SE

The shapes of SE can vary from regular to irregular curves, such as flat, triangle, semicircle, and so on figure 2. illustrates some of the common SE.

III. Algorithm of the Morphological Top-Hat Transform for ECG Signal

In the morphological Top-Hat transform algorithm, the noise suppression is performed as follows (20):

$$
\begin{aligned}
f &= f_o \bullet B - f_o \circ B \\
&= \left(f_o \bullet B - f_o\right) + \left(f_o - f_o \circ B\right) \\
&= \left(f_o \oplus B_1 \Theta B_2 - f_o\right) + \left(f_o - f_o \Theta B_1 \oplus B_2\right) \qquad (5)
\end{aligned}
$$

$f_o \bullet B - f_o$ and $f_o - f_o \circ B$ are two types of the morphological Top-Hat Transform [21]. The morphological Top-Hat transform is a high-pass filter with good performances. $f_o \bullet B - f_o$ is called the Black Top-Hat transform, which is used to extract negative impulsive features; $f_o \circ B - f_o$ is called the White Top-Hat transform, which is used to extract positive impulsive features. Thus filter can be used to extract the positive and negative features simultaneously. Figure3. illustrates a bloc diagram describing the structure of the morphological Top-Hat transform of the ECG signals. It consists of three blocs: The first is concerned with the acquisition of ECG signals (f_o: original ECG signal). This step is followed by another step which allows the detection of the noise. This detection is achieved using the morphological operators defined in equation (5). B_1 and B_2 are structuring elements for opening and closing. These operations are used simultaneously on the original signal. The following step is the subtraction of the resulted closing and opening operations. This filtered

ECG signal f is the resultant signal after noise suppression.

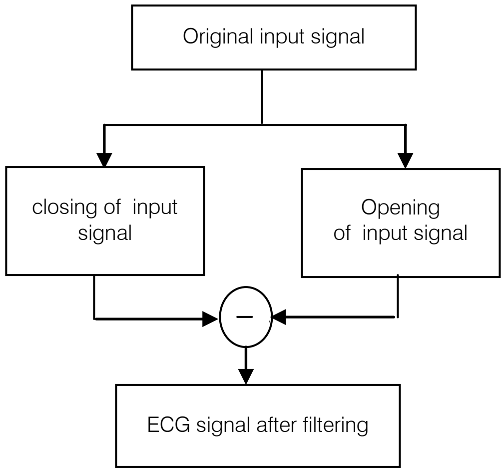


Fig. 3 : Bloc diagram of Top-Hat transform

The $B_{pair}(B_1, B_2)$ is selected according to the purpose of analysis and to the morphological properties of the ECG signal. B_1 is selected to be a triangular shape to retain the peaks and valleys and B_2 is a line segment to remove the noise.

IV. Results and Discussion

We use the MIT-BIH arrhythmia to evaluate the morphological Top-Hat algorithm. All the programs are written in Matlab 7 environment under the platform Pentium 4.

After the acquisition of ECG signal, the following stage is the suppression of the noise. It consists on the application of operators of Morphological Top-Hat transform. In fact, the input signal simultaneously is processed by the operations "closing" and "opening", followed by a subtraction, to generate at the end the filtered signal. Thus, the Top-Hat transform can be used to extract the positive and negative features simultaneously.

It should be noted that the shape of the structuring element in the suppression of noise is different compared to that from the correction of the baseline. Indeed, it can take two different forms of equal lengths: a triangular form B_1 to maintain the peaks and the valleys on a straight form (segment of null amplitude) B_2

In our case the size of the structuring element was fixed at 5 samples units each, with values of $B_1 = (0,1,5,1,0)$, $B_2 = (0,0,0,0,0)$. This value is fixed in an empirical way where the minimum and the maximum are fixed at optimal values in the stage of the suppression of the noise. The algorithm is applied respectively to the records 101, 105, 108 and 209 of the MIT-BIH data base. As shown in Figures. (4-5-6-7), good performance of suppression of the noise can be observed.

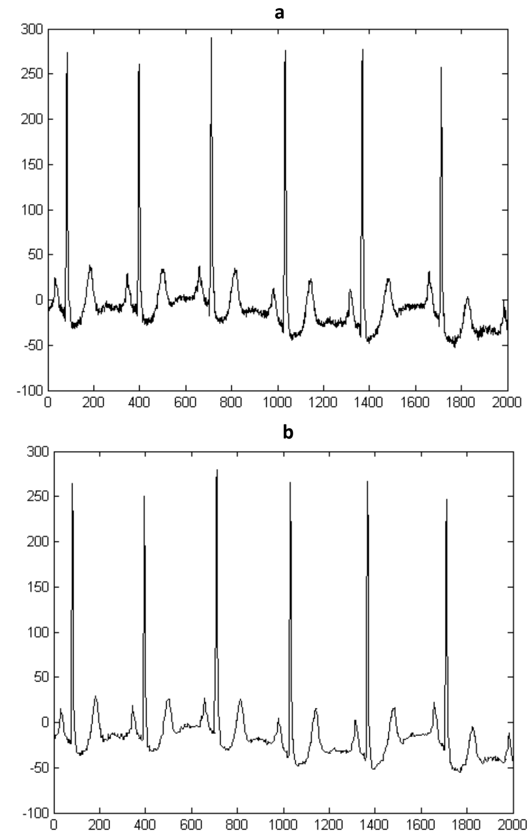
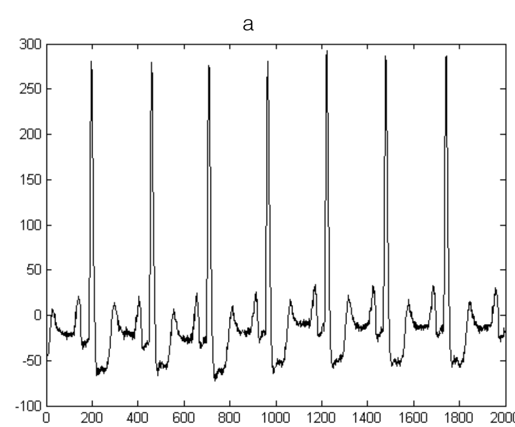


Fig. 4 : Results of MTHT (a) original ECG signal 101; (b) denoised signal



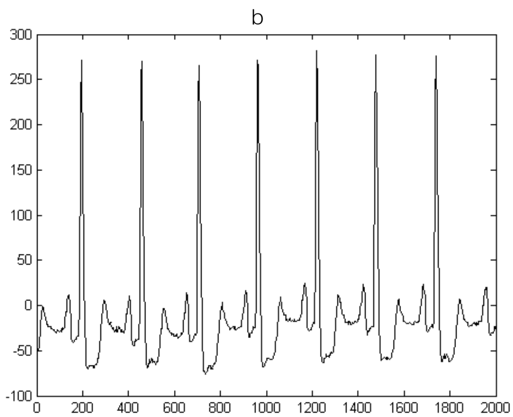


Fig. 5 : Results of MTHT (a) original ECG signal 105; (b) denoised signal

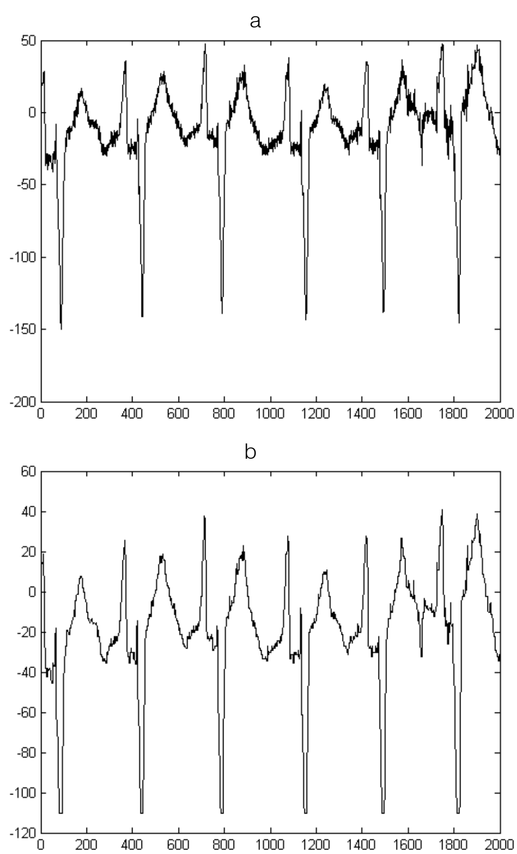


Fig. 6 : Results of MTHT (a) original ECG signal 108; (b) denoised signal

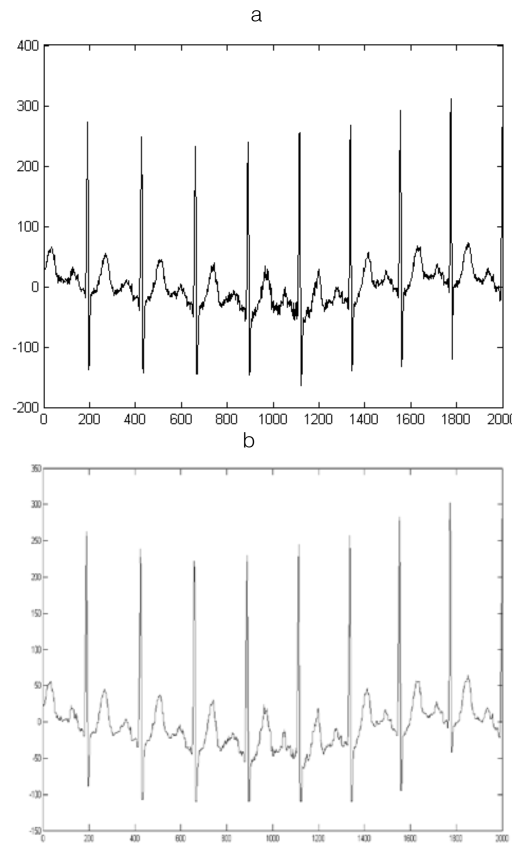


Fig. 7 : Results of MTHT (a) original ECG signal 209; (b) denoised signal

a) Simulation Study

In order to test the performance of the developed Top-Hat transform algorithm in suppressing noise, the algorithm is applied to ECG signal recordings of the data base "MIT-BIH Arrhythmia Database" to which simulated Gaussian noise of a 5 dB level has been added.

The resulting test signal is given by:

$$S(n) + f(n) = f_S(n) \qquad (6)$$

Where $S(n)$ is the simulated Gaussian noise, $f(n)$ is the ECG signal recording and $f_S(n)$ is the noisy ECG signal.

Figures 8. upto 14 illustrats the results obtained after applying the proposed algorithm a noisy ECG signal recordings. As it can be noticed in each figure, the denoised ECG signals (c) are well resolved mainly the different waves, and the added noise is completed suppressed.

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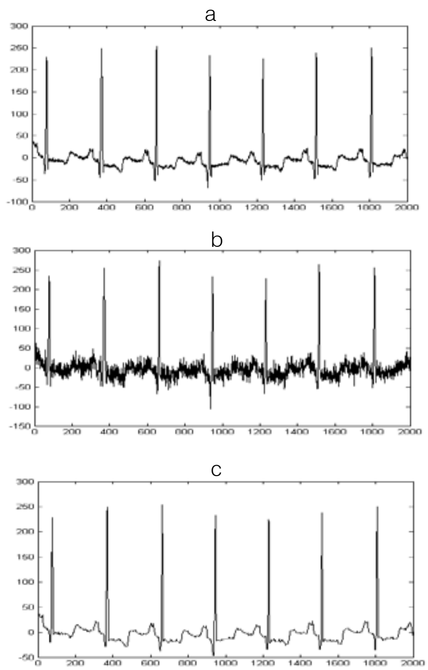

Fig. 8 : Application of the proposed denoising algorithm to a 5 dB noisy normal sinus rhythm ECG signal record 100. a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals.

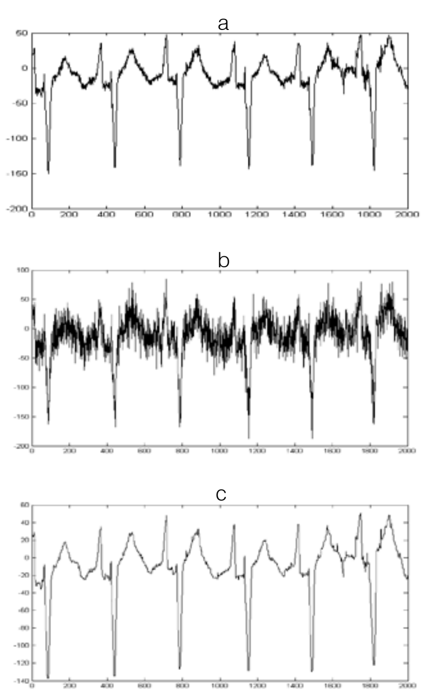

Fig. 9 : Application of our denoising algorithm to a 5 dB noisy ECG signal with a PVC record 108 a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals.

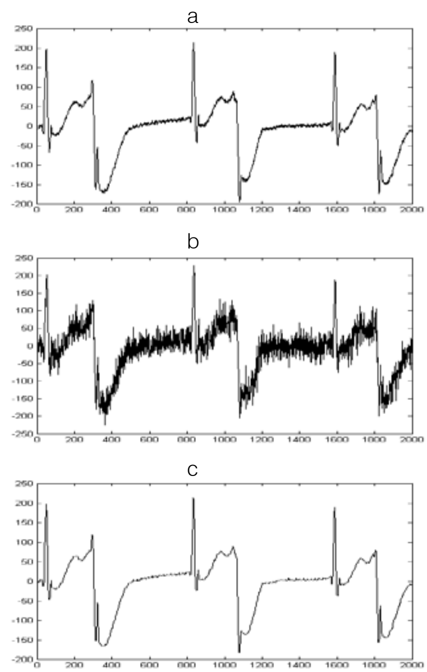

Fig. 10 : Application of our denoi7 a) Input ECG signal; b) with added Gaussian noise EC sing algorithm to a 5 dB noisy ECG signal with a PVC record 20G signal; c) denoised ECG signals.

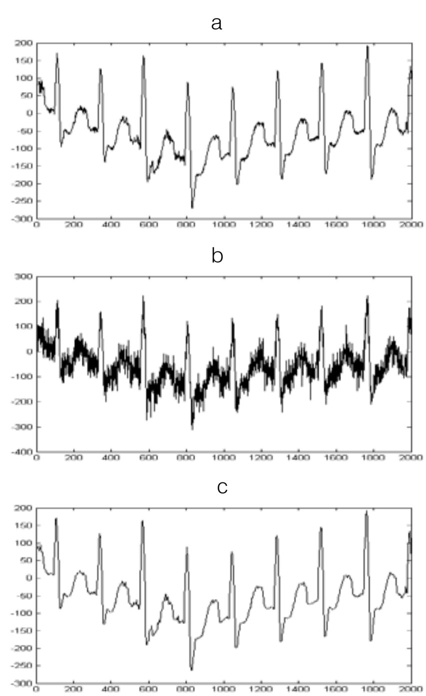

Fig. 11 : Application of our denoising algorithm to a 5 dB noisy ECG signal containing a baseline wandering record 109: a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals.

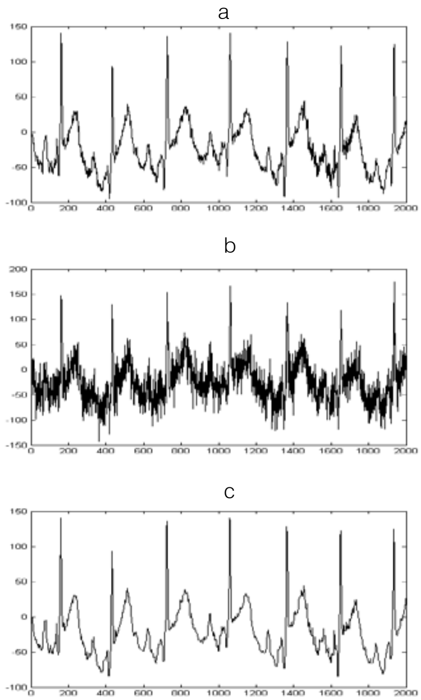

Fig. 12 : Application of our denoising algorithm to a 5 dB noisy ECG signal containing a baseline wandering record 228: a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals.

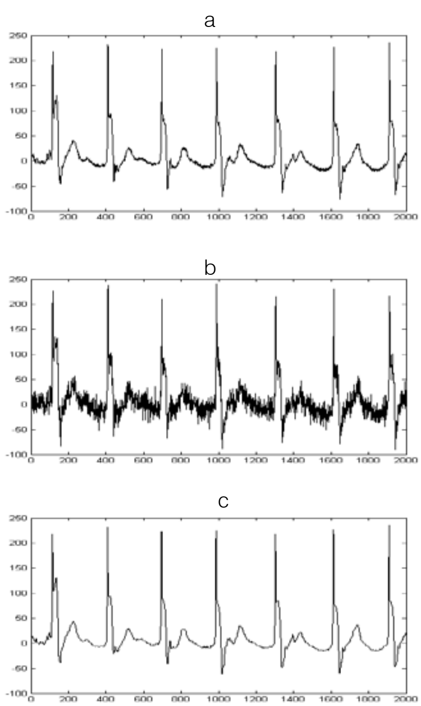

Fig. 13 : Application of our denoising algorithm to a 5 dB noisy ECG signal containing a various waves record.

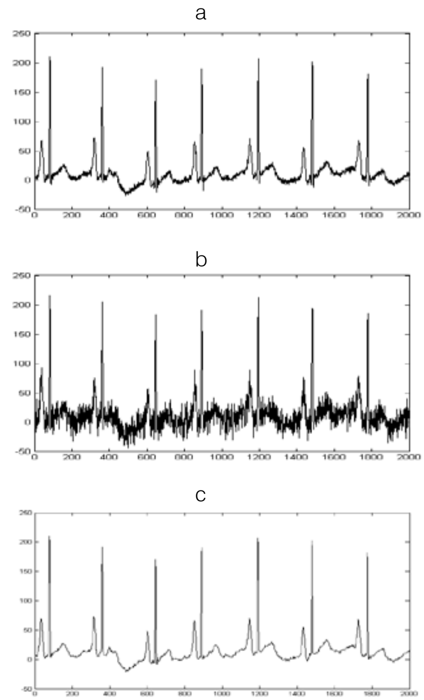

Fig. 14 : Application Application of our denoising algorithm to a 5 dB noisy ECG signal containing a various waves record 222: a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals.

b) Evaluation criteria values

General comparative study was related to three recordings of the database ` MIT-BIH Arrhythmia Database' which are ` 100.dat ', ` 101.dat ', and ` 103.dat '.

Tables 3 and 4 summarize the values of signal to noise ratio SNR (eq.7) and the Mean Squared error MSE (eq.8), respectively, of the application of three approaches of filtering (Top-Hat transform, the wavelet denoising utilize the Visu Shrink, and low pass Butterworth filter [22-23] to each recording.

Table 3: The obtained output SNR (in dBs) values for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat').

Table 4: The obtained MSE (in dBs) values for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat').

One note that the average values of the average quadratic error and the signal report/ratio on noise are, in fact, calculated for various successive segments of 4096 samples.

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$$SNR = 10\log \frac{\sum_{n=1}^{N}(f_S(n))^2}{\sum_{n=1}^{N}(f_S(n) - f_d)^2} \qquad (7)$$

$$MSE = \frac{\sum_{n=1}^{N}(f_S(n) - f_d(n))^2}{N} \times 100 \qquad (8)$$

Where f_S denotes the reference ECG signal and f_d represents the constructed denoised ECG signal. Whereas N is the length of the data segment.

Values SNR and MSE obtained of the general comparative study (of tables 3 and 4) show, the superiority of performance of the algorithm Top-Hat transform.

Table 5: Presents the SNR between the denoised ECG signal and the clean ECG signal, as obtained for each method. The ECG signal was corrupted by adding white Gaussian noise. The noise level was adjusted to specific values in such a way as to obtain the different SNRs. Top-Hat transform was used by the proposed algorithm in Table 5.

Table 6: Lists the SNR of the proposed method and the other methods for the ECG signals (MIT-BIH database record 103) corrupted with electrode motion artifact noise.

Table 7: Presents the SNR of the proposed method and the other methods for the ECG signals (MIT-BIH database record 103) corrupted with muscle artifact noise.

Table 8: Lists the SNR of the proposed method and the others for the ECG signals (MIT-BIH database record 109) corrupted with electrode motion noise.

The results clearly indicate that the proposed method has stronger denoising abilities than the others methods. Although the other methods removed the

Table 3 : The obtained output SNR (in dBs) values for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat').

Method record	Input SNR (dB)	Top-Hat transform	Low pass Butterworth filer	VisuShrink
100 dat	5	$0.0005*10^{-5}$	$0.0015*10^{-5}$	$0.00098*10^{-5}$
101 dat	5	$0.0002*10^{-4}$	$0.0016*10^{-4}$	$0.0011*10^{-4}$
103 dat	5	$0.008*10^{-4}$	$0.0016*10^{-4}$	$0.0012*10^{-4}$

102 : a) Input ECG signal; b) with added Gaussian noise ECG signal; c) denoised ECG signals

Table 4 : The obtained MSE (in dBs) values for each tested filtering method applied to the set of MIT-BIH arrhythmia Databases ECG records ('100.dat', '101.dat' and '103.dat').

Method record	Input SNR (dB)	Top-Hat transform	Low pass Butterworth filer	VisuShrink
100 dat	5	13.2025	5.4718	4.8539
101 dat	5	12.5645	6.7585	6.8030
103 dat	5	12.9845	9.1075	7.9001

Table 5 : The performance (SNR) of the proposed method and others for the ECG signal (MIT-BIH database record 103) corrupted with Gaussian noise.

Input SNR	Output SNR Proposed method	Output SNR for VisuShrink	Output SNR for SureShrink	Output SNR for ayesShrink
6.8	12.08	10.50	10.82	10.54
9.29	13.88	12.73	13.24	12.88
12.81	17.80	15.84	16.57	15.99
15.83	20.73	18.31	19.25	18.33

Table 6 : The performance (SNR) of the proposed method and others for the ECG signal (MIT-BIH database Record 103) corrupted with electrode motion noise.

Input SNR	Output SNR Proposed method	Output SNR for VisuShrink	Output SNR for SureShrink	Output SNR for ayesShrink
6.8	11.30	9.45	10.85	10.72
9.29	13.45	11.65	13.27	13.05
12.81	16.70	15.74	16.60	16.14
15.83	19.60	17.34	19.31	18.48

Table 7 : The performance (SNR) of the proposed method and others for the ECG signal (MIT-BIH database record 103) corrupted with muscle artifact noise.

Input SNR	Output SNR Proposed method	Output SNR for VisuShrink	Output SNR for SureShrink	Output SNR for ayesShrink
6.8	13.20	11.15	12.88	12.68
9.29	15.76	13.24	15.26	14.91
12.81	18.92	16.28	18.48	17.79
15.83	21.49	18.74	21.02	19.85

Table 8 : The performance (SNR) of the proposed method and others for the ECG signal (MIT-BIH database record 109) corrupted with electrode motion noise.

Input SNR	Output SNR Proposed method	Output SNR for VisuShrink	Output SNR for SureShrink	Output SNR for ayesShrink
6.8	10.96	10.57	10.91	10.85
9.29	13.33	12.77	13.26	13.22
12.81	16.77	15.76	16.65	16.62
15.83	19.80	18.25	19.28	19.19

Noise, the signal was distorted, as well. Our method showed good performance with a Top-Hat transform. The values obtained from our proposed method were always among the ones that had the highest SNR.

V. Conclusion

A new algorithm Morphological Top-Hat transform for noise suppression using nonlinear transform was developed and evaluated. It consists morphological operators which are closing and opening. These operators are used as structuring element SE. Such SE was chosen as a triangular shape to maintain the peaks and the valleys on a straight form. The algorithm was implemented and evaluated a same ECG signals for the MIT-BIH data base. The experimental application of this algorithm result is better than Visu Shrink, Sure Shrink, Bayes Shrink and Low passes Butterworth filer, particularly, in ECG signal denoising.

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