Seismic Data Compression using Wave Atom Transform

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Abstract - Seismic data compression (SDC) is crucially, confronted in the oil Industry with large data volumes and Incomplete data measurements. In this research, we present a comprehensive method of exploiting wave packets to perform seismic data compression. Wave atoms are the modern addition to the collection of mathematical transforms for harmonic computational analysis. Wave atoms are variant of 2D wavelet packets that keep an isotropic aspect ratio. Wave atoms have a spiky frequency localization that cannot be attained using a filter bank based on wavelet packets and offer a significantly sparser expansion for oscillatory functions than wavelets, curvelets and Gabor atoms.

Keywords: seismic data compression (SDC), curvelets, wavelets, wave atom.

GJCST-F Classification: I.5.4

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Seismic Data Compression using Wave Atom Transform

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Abstract- Seismic data compression (SDC) is crucially, confronted in the oil industry with large data volumes and incomplete data measurements. In this research, we present a comprehensive method of exploiting wave packets to perform seismic data compression. Wave atoms are the modern addition to the collection of mathematical transforms for harmonic computational analysis. Wave atoms are variant of 2D wavelet packets that keep an isotropic aspect ratio. Wave atoms have a spiky frequency localization that cannot be attained using a filter bank based on wavelet packets and offer a significantly sparser expansion for oscillatory functions than wavelets. Curvelets and Gabor atoms. Wave atoms capture the coherence of patterns across and along oscillations where as curvelets capture coherence along oscillations only. Wave atoms precisely interpolate between Gabor atoms and (constant support) and directional wavelets (wavelength ~ diameter) in the sense that the period of oscillations of each wave packet (wavelength) is related to the size of essential support by parabolic scaling law. Wave atom transform achieves the best result by some numerical examples.

Keywords: seismic data compression (SDC), curvelets, wavelets, wave atom.

I. Introduction

Modern seismic surveys with higher accuracy memorization that led to ever increasing amounts of seismic data [1 and 2]. Management of these large datasets becomes important for transmission, storage processing and Interpretation. To make the storage more efficient and to reduce the broadcast and cost, many seismic data compression (SDC) algorithms have been developed. During the oil and gas exploration process, the main strategy used by the companies is the construction of sub surface images, which are used both to identify the reservoirs and also to plan the hydrocarbons distillation. The construction of those images begins with seismic survey that produces a huge amount of seismic data. Then, obtained data is transmitted to the processing center generate the subsurface image.

A typical seismic survey can produce hundreds of terabytes of data. Compression algorithms are subsequently desirable to make the storage more effective, and to reduce time and costs related to network and satellite broadcast. Multi-resolution methods are genuinely associated to image processing, biological, computer vision and systematic computing. The curvelet transform is a multiscale directional transform that permits almost best non-adaptive sparse representation of objects with edges. It has generated enhancing importance in the community of applied mathematics and signal processing over the years. A review on the curvelet transform includes its history beginning from wavelets, its logical relationship to other multi resolution multidirectional methods like contourlets and shearlets, its basic theory and discrete algorithm. Further, we agree recent applications in video/image processing, seismic exploration, fluid mechanics, imitation of partial different equations, and compressed sensing [3].

For seismic data compression (SDC), the most important consideration is how to represent seismic signals efficiently, that is to say, using few coefficients to faithfully represent the signals, and therefore preserve the useful information after maximally possible compression. It is easy to comprehend that compression effectiveness is used for different expansion bases. Many orthogonal transforms have been used for data compression. Discrete Fourier Transform (DCT) was the first generation orthogonal transform used in Data compression. Haar Transform use of rectangular basis functions. Slant Transform is an attempt to match basis vectors to the areas stable luminance slope. It has better decor relation efficiency. Discrete cosine Transform is one of the extensive families of sinusoidal transforms. The mainly efficient transform for decor relating input data is the Karhunen loeve Transform also known as Hotelling transform and Eigenvector transform [4].

Curvelets as a multi-scale, anisotropic multi-dimensional transform were introduced, very quickly to be used for seismic data processing and migration using a mapping migration method. Curvelets can build the local slopes information into the representation of the seismic data, and which was proved to be effective in the sparse decomposition of seismic data.

For example, wavelet [5 and 6] based compression algorithm can represent seismic data using only a fraction of the original data size. In this paper, Wave atom transform presents its advantage...
over wavelets, curvelets[7] for conventional image compression. Their features are well suited to seismic data properties and have led to better results in terms of signal-to-noise ratio. Wave atoms come from the property that they also provide an optimally sparse representation of wave propagators, a mathematical effect of autonomous interest, with applications to fast numerical solvers for wave equations.

II. IMAGE COMPRESSION – TRANSFORMS

a) Wavelets

During the last decade the appearance of many transforms called Geometric wavelets have paying attention of researchers working on image analysis. These novel transforms propose a new representation comfortable than the traditional wavelets multi-scale representation. We are responsive that for a particular type of images, we can do better by choosing for this kind of specific images, a more suitable tool than classical wavelets[8,9 and 10].

The orthogonal transforms have been broadly studied and used in image analysis and processing. To defeat the limitations of Fourier analysis many extra orthogonal transforms have been developed. The most important criteria to be fulfilled by the basis functions are localization in equally space and spatially frequency and orthogonality. Various efficient and sophisticated wavelet-based schemes have been developed. In Image compression, the use of orthogonal transform is dual. Primary, it decorrelates the image components and allows to identify the redundancy. Subsequent, it offers a high level of compression of the energy in the spatial frequency domain. These two properties permit to select the most related components of the signal in order to accomplish competent compression. Many orthogonal transforms possess these three characteristics and have been used for data compression.

Wavelets are much modified to isotropic structure; they are not modified for anisotropic structure. This transform cannot effectively represent textures and exceptional details in images for lacking of directionality. 2D wavelet transforms produce high energy coefficients along the contours[11 and 12]. To overcome this limitation, a few solutions have been proposed. A first solution consists in using directional filter banks tuned at fixed scales, orientations and positions. Another solution is exploit an adaptive directional filtering based on a numerical model. So, two important approaches fixed and adaptive have been developed. Figure 1. shows difficulties of wavelet transform to represent regularity of a contour compared to new multi-scale transformed where geometric anisotropy and rotations are taken into description.

b) Ridgelets and Curvelets

Ridgelet transform [13 and 14] have been developed to analyze objects whose significant information is concentrated approximately linear discontinuities such as lines. Ridgelet coefficients are obtained by a One Dimensional wavelet transform of all projections of the image resulting from Radon Transform. Ridgelet transform is that wavelet analysis on One Dimensional slices of the Radon Transform, where the angle is fixed.

Continuous Ridgelet Transform is defined as

$$Rf(a, \theta, b) = \int \int f(x_1, x_2) \psi_{a,\theta,b}(x_1, x_2) dx_1 dx_2$$

Where $\psi_{a,\theta,b} = a^{-1/2} \psi((x_1 \cos(\theta) + x_2 \sin(\theta) - b) / 2)$ is a One Dimensional Wavelet.

Ridgelets are expressed through Radon Transform as:

$$Rf(a, \theta, b) = \int Rf(r, \theta) a^{-1} \psi(t - b) / a dt$$

Where $Rf$ is Radon transform defined by

$$Rf(t, \theta) = \int f(x_1, x_2) \delta(-x_1 \sin \theta + x_2 \cos \theta - t) dx_1 dx_2$$

A curvelet is defined as function $x = f(x_1, x_2)$ at the scale $2^{-j}$, orientation $\theta_j$ and position $x_{j,\theta_j} = R_j^0(k_1 2^{-j}, k_2 2^{-j/2})$ by:

$$c(j, l, k) = \int_{R^2} f(x) \psi_{j,l,k}(x) dx$$

Curve let computation steps:

Step 1: Decomposition into sub bands
Step 2: Partitioning
Step: Ridgelet analysis(Radon Transform + Wavelet transform 1D)
Block size can change from a sub band to another one; the following algorithm will be applied
Step 1: Apply a wavelet transform (J sub bands).
Step 2: Initialize the block size: \( B_{\text{min}} = B_j \).

Step 3: For \( j = 1, \ldots, J \) do

Step 4: Partition the sub bands \( W_j \) in blocks \( B_j \).

Step 5: if \((J \mod 2) = 1\) then \( B_{j+1} = 2B_j \) otherwise \( B_{j+1} = B_j \).

Step 6: Apply Ridgelet transform to each block.

Figure 2. Shows the curvelet tiling in space and frequency domains.

**c) Wave atoms**

In the standard wavelet transform, only the estimate is decomposed, when, we pass from phase to another. While in the wavelet packets, the decomposition could be pursued into the other sets, which is not optimal. The optimality is linked to the greatest energy of decomposition. The notion is then to fetch for the way yielding to the maximum energy through the different sub bands.

Wave atom [15] is a novel member in the family of oriented, multiscale transforms for image processing and also numerical analysis. For the sake of completeness, we remember here some fundamentals notations following

\[
\hat{f}(\omega) = \int e^{-ix\omega}f(x)\,dx \quad (5)
\]

\[
f(x) = \frac{1}{(2\pi)^{1/2}} \int e^{ix\omega}\hat{f}(\omega)\,d\omega \quad (6)
\]

Wave atoms are noted as, with subscript. The indexes are integer-valued related to a point in the phase-space defined as follows. \( x_j = 2^{-j}n,\omega_m = \pi2^j m \) \( C_12^j \leq \max_{m=1,2} |m| \leq C_22^j \), they suggest two parameters are enough to index a lot of known wave packet architectures. The index indicates whether the decomposition is multi scale \((\alpha = 1)\) or not \((\alpha = 0)\); and \( \beta \) indicates whether basis elements are localized and poorly directional \((\beta = 1)\) or, on the opposite side extended and fully directional \((\beta = 0)\)[16,17 and18].

We think that the description in terms of \( \alpha \) and \( \beta \) will clarify the connections between various transforms of modern harmonic analysis. Wavelets correspond to \( \alpha = \beta = 1 \), for ridgelets \( \alpha = 1, \beta = 0[19 \text{ and } 20] \), Gabor transform \( \alpha = \beta = 0 \) and curvelets correspond to \( \alpha = 1, \beta = 1/2 \). Wave atoms are defined for \( \alpha = \beta = 1/2 \). Figure. 3. Illustrates this classification.

In order to introduce the wave atom, let us first consider the 1D case. In practice, wave atoms are constructed from tensor products of adequately chosen 1D wavelet packets. A one-dimensional family of real-valued wave packets \( \psi_m^n(x), j \geq 0, m \geq 0, n \in \mathbb{Z} \), centered in frequency around \( \pm \omega_{j,m} = \pm \pi2^j m \), with \( C_12^j \leq \max_{m=1,2} |m| \leq C_22^j \) and centered in space around \( x_{j,m} = 2^{-j} n \), is constructed. The one-dimensional version of the parabolic scaling inform that the support of \( \psi_m^n(\omega) \) be of length \( O(2^j) \), while \( \omega_{j,m} = O(2^j) \). The desired corresponding tiling of frequency is illustrated at Figure. 4. Filter bank-based wavelet packets is considered as a potential definition of an orthonormal basis satisfying these localization properties. The wavelet packet tree, defining the partitioning of the frequency axis in 1D, depth \( j \) when the frequency is \( 2^j \), as shown in Figure. 4.

Figure 3. Illustrates this classification.
In 2D domain the construction presented above can be modified to certain applications in image processing or numerical analysis: The orthobasis variant. [22,23 and 24]. A two-dimensional orthonormal basis function in frequency plane with four bumps is formed by individually taking products of 1D wave packets. Mathematical formulation and implementations for 1D case are detailed in the earlier section. 2D wave packets. Mathematical formulation and implementations are indexed by \( \mu=(j,m,n) \), where \( m=(m_1,m_2) \) and \( n=(n_1,n_2) \). creation is not a simple tensor product since there is only one scale subscript \( j \). This is similar to the 1D wrapping strategy to two-dimensions except for slight complication. The admissible tiling’s of the frequency plane at scale \( j \) are restricted by

\[
\max_{i=1,2} |m_i| = 4n_j + 1
\]

For some integer \( n_j \) depends on \( j \). we check that this property holds with \( n_0 = 0, n_1 = 1 \) and \( n_2 = 2 \). The rationale for this restriction is that a window needs to be right-handed in both directions near a scale doubling, and that this parity needs to match with the rest of the lattice. The rule is that \( \Psi_{m+} \) is right-handed for \( m \) odd and left-handed for \( m \) even, so for instance \( \Psi_{1}^{m_1}(\omega_1) \Psi_{2}^{m_2}(\omega_2) \) would not be admissible window near a scale doubling, where as \( \Psi_{3}^{m_1}(x_1) \Psi_{3}^{m_2}(x_2) \) is admissible (by a dot in Figure. 5.).

### III. Results and Discussion

This section demonstrates some numerical examples to explain the properties and potential of the wave atom frame and its orthobasis variation.

Now we illustrate the potential of the wave atoms with example. In the example, we consider the compression properties, i.e. the decay rate of the coefficients of images under the wave atom bases. Besides the wave atom orthobasis and the wave atom frame, we include other two bases for comparison: the daubechies db5 wavelet, and a wavelet packet that uses db5 filter and shares the same wavelet packet tree with our wave atom or thobasis.

The quality of reconstructed image is usually specified in terms of peak signal to noise ratio (PSNR).
The PSNR values were calculated using the following expression:

$$psnr = 20 \log_{10} \left( \frac{M_1 \times M_2 \times \max(f(i, j))}{\sum_{i=1}^{M_1} \sum_{j=1}^{M_2} [f(i, j) - f'(i, j)]^2} \right) \text{dB} \quad (12)$$

Here $M_1$ and $M_2$ are the size of the image. $f(i,j)$ is the Original image, $f'(i,j)$ is the decompressed image.

<table>
<thead>
<tr>
<th>S.no.</th>
<th>No. of coefficients used for decompression</th>
<th>PSNR of decompressed image in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>wavelet</td>
</tr>
<tr>
<td>1</td>
<td>5536</td>
<td>38.6992</td>
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<tr>
<td>2</td>
<td>6536</td>
<td>39.2739</td>
</tr>
<tr>
<td>3</td>
<td>7536</td>
<td>39.8192</td>
</tr>
<tr>
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<td>40.3407</td>
</tr>
<tr>
<td>5</td>
<td>9536</td>
<td>40.8406</td>
</tr>
</tbody>
</table>

Table 2: Compression Ratio comparison of wavelet, curvelet and wave atom

<table>
<thead>
<tr>
<th>S.no.</th>
<th>No. of coefficients used for decompression</th>
<th>Compression ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
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<tr>
<td>2</td>
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<td>8536</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>9536</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 3: Execution time comparison of wavelet, curvelet and wave atom

<table>
<thead>
<tr>
<th>S.no.</th>
<th>No. of coefficients used for decompression</th>
<th>Execution time in seconds</th>
</tr>
</thead>
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</tr>
<tr>
<td>5</td>
<td>9536</td>
<td>0.272</td>
</tr>
</tbody>
</table>

From Table 1, we note that PSNR of waveatom decompressed image is high for any no of coefficients used for reconstruction. From Table 2, it is observed that, curvelet representation has more redundant data compared to waveatoms and wavelets. Table 3 shows that, execution time required is less in case of wavelets compared to waveatoms and curvelets. Hence waveatom is the best alternative of the other two techniques.

Figure 6, 7, 8 and 9 show input image, wavelet reconstruction, curvelet reconstruction and wave atom reconstruction respectively and Figure 10 and 11 show graphical representation of PSNR vs. No. of coefficients used for reconstruction and PSNR vs. compression ratio respectively for the three considered compression techniques. It is observed from the below figures, that waveatom compression technique outperforms than wavelet and curvelet techniques.
IV. Conclusions

We have shown that for a seismic data images, we can find a transform that is more appropriate than Curvelets and wavelets. Using Wave atom transform we obtained better PSNR and Compression Ratio than other transforms.

REFERENCES RÉFÉRENCES REFERENCIAS


Figure 7: Wavelet reconstruction

Figure 8: Curvelet reconstruction

Figure 9: Wave atom reconstruction

Figure 10: PSNR vs. No. of coefficients used for reconstruction

Figure 11: PSNR vs. Compression Ratio


