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## Light Deflection in Massive Dyonic Black Holes

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# Light Deflection in Massive Dyonic Black Holes

H. R. Fazlollahi

**Abstract-** Following Rindler-Ishak method [1], we study the bending of light around general form of dyonic black holes in massive gravity [2]. We show that when the Schwarzschild-de Sitter geometry is taken into account,  $\Lambda$  does indeed contribute to the bending of light.

## INTRODUCTION

Discovering dark energy as source of accelerating expansion of our universe [3], many efforts have gone into understanding its nature. One of the prime candidates is the cosmological constant  $\Lambda$  [4] which its effects on local phenomena such as null geodesics, time delay of light [5], and the perihelion precession [6] are studied. In these circumstance, local cases, many authors have investigated the effects of cosmological constant on the bending of light.

The argument for the non-influence of  $\Lambda$  was first discussed by Islam through investigating the null geodesic equation in a spherically symmetric space-time [7] and has been re-affirmed by other authors, see e.g. [8]. However, in the last decade, Rindler and Ishak [1], by considering the intrinsic properties of the Schwarzschild-de Sitter space-time proposed a new method for calculating the deflection angle of light. Also, different aspects of their method such as integration of the gravitational potentials and Fermats principle have been studied [9]. Sultana in [10] and Heydari-Fard et al [11] have investigated light bending in Kerr-de Sitter and Reissner-Nordstrom-de Sitter space-time through Rindler-Ishak method. Also, this method has been applied to investigate Mannheim-Kazanas solution of conformal Weyl gravity [12].

Dyonic black holes enjoy the duality of electric/magnetic

charges and possibly mass/dual mass [13]. In [14] is shown that two constants of a Taub-NUT system can be interpreted as a gravitating dyon with both ordinary mass and its dual where role of Nut charge is the mass duality such as the duality between electric and magnetic charges in the U(1) Maxwell theory [15]. Dyonic black hole and its properties have been investigated in literature (see [16]). In this letter, we take into account bending of light around dyonic black holes in massive gravity theory to examine massive gravity effects on light deflection.

For static spherically symmetric space-time, the dyonic black hole in massive gravity, massive dyonic black hole, is given by [2].

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where

$$f = 1 + \frac{r}{l^2} - \frac{2m_0}{r} + \frac{q_E + q_M}{r^2} + m^2 \frac{cc_1}{2}r + c^2c_2, \quad (2)$$

where  $m_0$  is the total mass of dyonic black holes,  $m$  denotes the massive parameter,  $c$ ,  $c_1$  and  $c_2$  are constants of model,  $l^2 = -3/\Lambda$  and  $q_E$  and  $q_M$  identified as electric and magnetic charges, respectively.

The standard approach for calculating the bending angle is [17]

$$\Delta\phi = 2|\phi(\infty) - \phi(r_0)| - \pi \quad (3)$$

where  $r_0$  denotes the closest distance to the black hole. However, the space-time here is not asymptotically flat, and so we cannot use usual way to calculate the deflection angle of light around massive dyonic black hole. Surprisingly, the Rindler-Ishak method proposed in [1] gives new approach to calculate the deflection angle in an asymptotically non-flat space-time. Rindler and Ishak have shown that by considering the effects of cosmological constant on the geometry of space-time, one can obtain the contribution of  $\Lambda$  to the bending angle near massive celestial objects (for example see [20]). Using the Euler-Lagrange equations for null geodesics in equatorial plan,  $\theta = \pi/2$ , we obtain the following equation

$$\frac{d^2u}{d\phi^2} + u = -\frac{cc_1m^2}{4} - c_2c^2m^2u + 3m_0u^2 - 2(q_E^2 + q_M^2)u^3, \quad (4)$$

where  $u \equiv 1/r$ . For  $c = 0$ , and in the absence of electric and magnetic charges, we find the standard orbital equation for light bending in Schwarzschild-de Sitter space-time (see e.g. [18]).

The orbit that is usually considered as small perturbation of the undeflected straight line in flat space

$$r \sin \phi = R \quad (5)$$

So according to the standard orbital equation of deflection of light, we have two different approaches to consider differential equation (4): first approximation case (see [1], [18]), where the small perturbation of the undeflected straight line (5) substituted into the right-hand terms of (4) or solving it by using a perturbation method up to the third order and consider a solution as

$$u = u_0 + \delta u_1 + \delta u_2 + \delta u_3 + \dots \quad (6)$$

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where  $u_0 = \frac{\sin \phi}{R}$  and corrections  $\delta u_1$ ,  $\delta u_2$  and  $\delta u_3$  satisfy the following equations

$$\frac{d^2(\delta u_1)}{d\phi^2} + \delta u_1 = -\frac{cc_1 m^2}{4} + 3m_0 u_0^2 \tag{7}$$

$$\frac{d^2(\delta u_2)}{d\phi^2} + \delta u_2 = 6m_0 u_0 \delta u_1 \tag{8}$$

$$\frac{d^2(\delta u_3)}{d\phi^2} + \delta u_3 = 6m_0 u_0 \delta u_2 + 3m_0 \delta u_1^3 - c_2 c^2 m^2 u_0 + -2(q_E^2 + q_M^2) u_0^3 \tag{9}$$

here we use perturbation method for small effects of electric-magnetic charge and massive parameter m on the deflection of light with respect to standard one.

Applying these approaches on equation (4) gives:

$$u_1 = \frac{1}{r} = -\frac{m^2 cc_1}{4} + \frac{\sin \phi}{R} + \frac{m^2 c^2 c_2}{2R} (\phi \cos \phi - \sin \phi) +$$

$$\frac{m_0}{R^2} (1 + \cos^2 \phi) - \frac{q_E^2 + q_M^2}{4R^3} (\sin \phi \cos^2 \phi - 3\phi \cos \phi + 2 \sin \phi), \tag{10}$$

$$u_2 = \frac{1}{r} = \frac{m_0}{R^2} \left(1 + \frac{3m_0^2}{R^2}\right) + \frac{\sin \phi}{R} \left(1 - \frac{m^2 c^2 c_2}{4} - \frac{5(q_E^2 + q_M^2)}{16R^2}\right) -$$

$$\frac{m^2 cc_1}{2} \left(1 - \frac{3}{8} m^2 cc_1 m_0 + \frac{3m_0^2}{R^2}\right) + \frac{\phi \cos \phi}{2R} (m^2 c^2 c_2 + \frac{3}{2} m^2 cc_1 m_0 + \frac{3(q_E^2 + q_M^2)}{2R^2} - \frac{15m_0^2}{2R^2}) + \frac{m_0 \cos^2 \phi}{R^2} \left(1 - \frac{3m^2 cc_1 m_0}{2} + \frac{15m_0^2}{2R^2}\right) - \frac{3\phi \sin \phi \cos \phi m_0^2}{2R^2} (m^2 cc_1 - \frac{5m_0}{R^2}) - \frac{\cos^2 \phi \sin \phi}{4R^3} (q_E^2 + q_M^2 + 3m_0^2) - \frac{m_0^3}{2R^4} \cos^4 \phi \tag{11}$$

where  $u_1$  and  $u_2$  are solutions of equation (4) for first approximation and perturbation method, respectively.

To obtain the one sided deflection angle at the point where

$\phi \ll 1$ , we obtain

$$u_1 \approx \frac{\phi}{R} + \frac{2m_0}{R^2} - \frac{m cc_1}{4}, \tag{12}$$

$$u_2 \approx \frac{2m_0}{R^2} \left(1 + \frac{5m_0^2}{R^2}\right) + \frac{\phi}{R} \left(1 - \frac{9m^2 c^2 c_2}{2R^2} + \frac{3(q_E^2 + q_M^2)}{16R^2}\right) - \frac{m^2 cc_1}{4} \left(1 - \frac{3}{4} m^2 cc_1 m_0\right) + \frac{m^2 c}{4R} (3\phi m_0 c_1 + \phi cc_2 - \frac{12c_1 m_0^2}{R}). \tag{13}$$

According to the Rindler-Ishak method, one able to compute angle  $\psi$  between photon orbit direction  $d$  and direction of  $\phi = \text{const.}$  line,  $\delta$ , by the invariant formula (see Figure 1)

$$\cos \psi = \frac{g_{ij} d^i \delta^j}{\sqrt{g_{ij} d^i d^j} \sqrt{g_{ij} \delta^i \delta^j}} \tag{14}$$

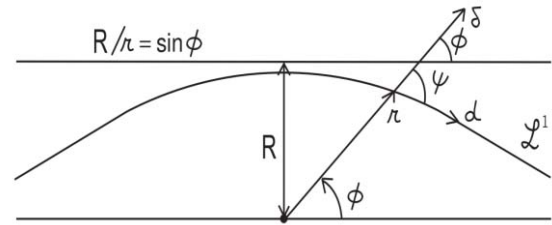


Fig. 1: The orbital map, light bending in the space-time of a black hole. The one-sided deflection angle is  $\psi - \phi \equiv \epsilon$ .

where  $g_{ij}$  are the coefficients of the 2-metric on  $\theta = \pi/2$  and  $t = \text{const.}$  surface. Substituting  $d = (dr, d\phi)$  and  $\delta = (\delta r, 0)$  in Eq. (9), gives

$$\cos \psi = \frac{|dr/d\phi|}{\sqrt{(dr/d\phi)^2 + f(r)r^2}} \tag{15}$$

or equivalently

$$\tan \psi = \frac{r\sqrt{f(r)}}{|dr/d\phi|} \tag{16}$$

using equations (12) or (13) for  $m \ll 1$

$$\frac{dr}{d\phi} \approx -\frac{r^2}{R} \tag{17}$$

finally, by substituting in equations (12) and (13), we find their corresponding expressions for the total deflection angle

$$2\epsilon_1 \approx \frac{4m_0}{R} \left(1 - \frac{2m_0^2}{R^2} + \frac{2m_0^2}{R^2} (q_E^2 + q_M^2)\right) - \frac{2R^3}{(m^2 R^2 cc_1 - 8m_0)l^2} + \frac{cm_0 m^2}{2R^3} (2R^2 (cc_2 + c_1 m_0) - 3c_1 m_0 (q_E^2 + q_M^2)) - \frac{m^4 c^2 c_1}{128R} (c_1 (q_E^2 + q_M^2) (m^2 R^2 cc_1 - 24m_0) + 8R^2 (2cc_2 + c_1 m_0)) \tag{18}$$

$$2\epsilon_2 \approx \frac{4m_0}{R} \left(1 - \frac{2m_0^2}{R^2} + \frac{2m_0^2}{R^4} (q_E^2 + q_M^2)\right) + \frac{m^2 c^2 c_2 m_0}{R} + \frac{R^3}{4m_0 l^2} + \frac{Rm^2 cc_1}{4} \tag{19}$$

The deflection angle is modified by new terms containing the massive parameter and cosmological constant in both equations (18) and (19).

Canceling out massive parameter and cosmological constant effects gives the same results for both approaches in equations (18) and (19) as

$$2\epsilon \approx \frac{4m_0}{R} \left(1 - \frac{2m_0^2}{R^2} + \frac{2m_0^2}{R^4} (q_E^2 + q_M^2)\right) \tag{20}$$

which equals to deflection light equation for charged Schwarzschild black holes.

The effect of the cosmological constant, electric charge and magnetic charge on deflection angle at small scales such as the solar system is expected to be negligible. So by using  $l \rightarrow \infty$  or  $\Lambda \approx 0$  and canceling out  $q_E$  and  $q_M$  from equations (18) and (19), we have

$$2\epsilon_1 \approx \frac{4m_0}{R} \left(1 + \frac{cm^2}{4}(cc_2 + c_1m_0)\right) \quad (21)$$

$$2\epsilon_2 \approx \frac{4m_0}{R} \left(1 + \frac{c^2c_2m^2}{4}\right) \quad (22)$$

To find constraint on constant m, we use the observational data on light deflection by the sun, from long baseline radio interferometry [19]. According to this observational data,  $\delta\phi_{LD} = \delta\phi_{LD}^{(GR)}(1 + \Delta_{LD})$  with  $\Delta_{LD} \leq 0.0002 \pm 0.0008$ , where  $\delta\phi_{LD}^{(GR)} = 1.7510$  arc sec. Assuming  $\Delta_{LD}$  as the geometric effects of the conformal terms, the observational results constrain the two last equations as follows

$$m^2 \leq \frac{4\Delta_{LD}}{(c_2+c_1m_0)} \quad (23)$$

Assuming

$$m^2 \leq \frac{4\Delta_{LD}}{c_2} \quad (24)$$

where we set  $c = 1$ . This selection leads us to find massive parameter m as function of constants  $c_1$  and  $c_2$ . Taking for R and  $m_0$  values of the radius and mass of the sun,  $R_\odot \approx 6.95 \times 10^8$  m and  $M_\odot \approx 1.99 \times 10^{30}$  kg, we find following constraints on m according to our approaches, first approximation and perturbative method

$$|m| \leq \frac{0.0283}{\sqrt{1.99 \times 10^{30} c_1 + c_2}} \quad \text{or} \quad |m| \leq \frac{0.0282}{\sqrt{c_2}} \quad (25)$$

In conclusion, in this letter, we have investigated the bending of light in the dyonic black holes in massive gravity.

Following Rindler-Ishak method, we have shown that when the geometry of the Schwarzschild-de Sitter space-time is taken into account,  $\Lambda$  indeed contributes to the light-bending in massive dyonic black holes. Also, using the observational data on bending of light by the sun and constant  $c = 1$  leads us to find strong constraint on massive constant m.

Generally, if we assume that both approaches give same result, one need to set  $c_1 = 0$ .

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