Pace Carabelli and Dow, There is No Common Discourse Language Logic in Keynes’s *A Treatise on Probability*

By Michael Emmett Brady  
*California State University*

**Abstract**—For 45 years, both A. Carabelli and S. Dow have been arguing that Keynes’s non-demonstrative logic in the *A Treatise on Probability* is a common discourse logic (rhetoric). They provide no textual evidence anywhere in Keynes’s *A Treatise on Probability* to support this claim. They have never supported, through the citation of specific pages or paragraphs in Keynes’s book, their claims that Keynes’s logic is NOT a formal logic. What is in the *A Treatise on Probability* is a version of Boole’s relational, propositional logic that Keynes combined with a first order (predicate) logic. These are mathematical, formal, symbolic logics. They have nothing to do with common discourse logics using the English language. A simultaneous reading of chapters I and II of Keynes’s book and chapters I, XI, and XII of Boole’s *The Laws of Thought* lead to one, and only one, conclusion—Keynes’s logic is a formal logic derived from G. Boole.

The only conclusion that follows from Keynes’s application of Boolean logics is that Carabelli and Dow have been severely confused for 45 years in what a formal logic is and what a common discourse logic is.

**GJHSS-D Classification: FOR Code:** 2202

---

© 2023. Michael Emmett Brady. This research/review article is distributed under the terms of the Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0). You must give appropriate credit to authors and reference this article if parts of the article are reproduced in any manner. Applicable licensing terms are at https://creativecommons.org/licenses/by-nc-nd/4.0/.
Pace Carabelli and Dow, There is No Common Discourse Language Logic in Keynes’s 
A Treatise on Probability

Michael Emmett Brady

Abstract- For 45 years, both A. Carabelli and S. Dow have been arguing that Keynes’s non demonstrative logic in the A Treatise on Probability is a common discourse logic (rhetoric). They provide no textual evidence anywhere in Keynes’s A Treatise on Probability to support this claim. They have never supported, through the citation of specific pages or paragraphs in Keynes’s book, their claims that Keynes’s logic is NOT a formal logic. What is in the A Treatise on Probability is a version of Boole’s relational, propositional logic that Keynes combined with a first order (predicate) logic. These are mathematical, formal, symbolic logics. They have nothing to do with common discourse logics using the English language. A simultaneous reading of chapters I and II of Keynes’s book and chapters I, XI, and XII of Boole’s The Laws of Thought lead to one, and only one, conclusion- Keynes’s logic is a formal logic derived from G. Boole.

The only conclusion that follows from Keynes’s application of Boolean logics is that Carabelli and Dow have been severely confused for 45 years in what a formal logic is and what a common discourse logic is. Given that Keynes’s non-demonstrative logic is a formal, mathematical, symbolic logic, the only conclusion possible is that Keynes is a formalist and a logicist, who was, and is, vastly superior to any economist, either orthodox or heterodox, in the 20th and 21st centuries.

Dow and Carabelli are not alone in their confusions. For another example, R. O’Donnell (See O’Donnell, 1989, 1990 a, b) is another economist who completely overlooked that all of the foundations of Keynes’s logical approach to probability are to be found in Boole’s The Laws of Thought (1854). Contrary to O’Donnell, it is Boole, not Keynes, who developed the first technically advanced, logical theory of probability in history.

I. Introduction

The paper will be organized in the following manner. Section Two will cover Keynes’s relational, propositional logic as introduced by him in chapters I and II of the A Treatise on Probability, while simultaneously demonstrating that Keynes is using Boole’s basic approach. Section Three will examine Carabelli’s claims that Keynes’s logic is NOT a mathematical one, but a common discourse logic (rhetoric). Section Four will examine Dow’s very similar claims. Section Five will cover Chick’s claims about Keynes’s common discourse logic. Section Six will conclude the paper.

II. Keynes’s Formal Approach to Logic in His A Treatise on Probability

Pace Carabelli and Dow, there is no common discourse logic in Keynes’s A Treatise on Probability (TP, 1921). All Post Keynesian, Institutionalist and Heterodox economists, as well as all orthodox economists, who have published work on Keynes in the 20th and 21st centuries, have simply overlooked Boole’s contribution to the basic foundations of Keynes’s work.

George Boole, and not J M Keynes in his 1921 A Treatise on Probability, put forth the first technically advanced, mathematical and logical treatment of a logical theory of probability in 1854 in his The Laws of Thought (LT,1854). It was based on a logic of propositions about events or outcomes and not the events or outcomes themselves. Boole’s logic is a mathematical logic and has absolutely nothing to do with an ordinary discourse human logic, which involves the use of commonsense language between humans or the intuitionism/Platonic, metaphysical speculation currently dominant in discussions in academia about Keynes.

Given that Keynes built his A Treatise on Probability directly on the mathematical and logical approach and foundation of G Boole’s Boolean algebra and logic, it is simply impossible for Keynes’s approach in his A Treatise on Probability to have been based on a logic of ordinary language as claimed by Carabelli (1985, 1988, 2003), Chick (1998), and Chick and Dow (2001).

Keynes is supposed to have had some kind of a unique, unclear and peculiar approach to logical analysis, which was supposed to have been based on an intuitive approach that can’t be discerned, according to Anna Carabelli (1985, 1988, 2003) and other heterodox economists. Carabelli argues that Keynes was anti-logicist, anti-empiricist, anti-positivist, anti-rationalist, and anti-formalist in his method, as well as being anti-mathematical. It is quite impossible for Keynes to have opposed all of these positions completely and still write Parts II, III and V of the A Treatise on Probability, which provide formal,
mathematical, and logicist underpinnings to his approach of inexact measurement and approximation that leads directly to Keynes’s specification of lower and upper bounds on all probabilities and outcomes in Part II of the TP. The only exception was for areas of application that involved his Principle of Indifference and relative frequencies that had passed an application of the Lexis-Q test for the stability of the frequency, an early version of exploratory data analysis and/or goodness of fit tests.

Keynes’s inductive logic in Part III of the TP is built directly on the method of inexact measurement and approximation of Part II of the TP. This involves Keynes’s use of a modified version of Boole’s Problem X that he solved on pp.192-194 of the TP and used on pp. 233-238 and 254-257 of Part III. Keynes’s development of the concept of finite probability, applicable to both numerical and non-numerical probabilities, was a necessary prerequisite for understanding Keynes’s work in Part III on induction and analogy. Given Keynes’s work on the relationship between probability and induction in Part III of the TP, Keynes’s work in Part III of the TP is then a prerequisite for his work in Part V of the TP. We can now see that it is impossible to grasp Keynes’s work in Part III of the TP unless Part II of the TP is understood first, and it is not possible to grasp Part V unless Part III of the TP has been digested. Heterodox economists, in general, study only chapters I-III, IV and VI of Part I of the TP. No Heterodox economist has ever studied Part II of the TP where Keynes covers Boole’s contributions in the appendix to chapter XIV, XV, XVI and XVII.

III. On Carabelli’s Assessment of Keynes’s Views on Logic as Presented in His TP

What is any reader to make of the following assessments of Keynes’s inexact/imprecise approach to mathematics and logic based on Boole made by Carabelli?

“3.1.1. Keynes’s view of probability, whose basic aspects were considered in the previous chapter, was centred (sic) on some general key doctrines. As I have already noted, these doctrines were not always explicit and expressed in univocal and coherent form. Hence the necessity not only of a close reading of Keynes’s text, but also of a sort of systematic reconstruction of Keynes’s approach to key epistemological topics, together with an attempt to clarify his position within his historical intellectual context. Such a task, which will be attempted in the present section, will enable one, for instance, to grasp the fact overlooked in a superficial reading of the Treatise, that Keynes (as we will see in Chapter 8) did not usually adopt the term ‘logical’ in the sense of formal logic, but in the sense of ordinary language logic, that is, in a sense which was actually antithetical to it. This explains the above-mentioned uncritical ranking of Keynes within the so-called logicist approach to probability.” (Carabelli, 1988, p.23).

Contrary to Carabelli, Keynes makes it clear that he is building on Boole in Part II of the TP.

Her major conclusion is the following one:

“...the logicist interpretation of Keynes’s theory appears to be based on a hasty reading of Keynes’s text. In various passages Keynes did indeed speak of the ‘logical character of his notion of probability. But this fact does not mean that...it was a logic of the formal type. In fact, it was an ordinary discourse logic.’ (Carabelli, 1988, p.145).

The only citation that Carabelli can offer is a complete misinterpretation and misunderstanding of what Keynes is doing in the TP, which is building on Boole’s relational, propositional logic. Keynes is not deploying an “ordinary discourse logic”: “In the ‘Treatise’ the priority of ordinary language over mathematical language was unquestioned. ‘I shall not cut myself [sic]’, Keynes wrote, ‘from the convenient’ but looser, expressions which have been habitually employed by previous writers and have the advantage of being immediately intelligible to the reader’. In the footnote, he praised ordinary language, in terms of its semantical character, contrasting it to the pure syntactical one of artificial mathematical language (Keynes, 1921, pp. 18-19).” (Carabelli, 1985, p.166)

Carabelli completely bypasses the rest of chapters I and II, which are directly based on chapters I, XI and XII of Boole’s The Laws of Thought. In fact, what Keynes is saying is that his exposition in the TP will not be based on the same ‘rigor’ as Russell’s work and not what is claimed by Carabelli below:

“Just for its organic characteristics, its open structure and the non-finite number of propositions, its compatibility with contradiction and its semantical character, ordinary language permitted one to deal with phenomena presenting [sic] the attribute of complexity”. (Carabelli, 1985, p.166)

Carabelli has not changed her views in nearly 35 years. In her latest contribution in 2021, we find the same claims made in 1988 in her following statements made about Keynes’s approach in 2021:

“He...prefers ordinary discourse...He is interested in exactness, not precision.” (Carabelli, 2021, p.8-the reader should note that exactness is precision))

and

“My interpretation of Keynes’s method stresses his logical way of reasoning as a non-demonstrative logic, based on his concept of probability, persuasion and ordinary language.” (Carabelli, 2021, p.10)

and

“Keynes belongs to the tradition of Aristotelian practical reasoning and (justified) realistic common sense...using ordinary language.”(Carabelli, 2021, p.15).

There is simply no support anywhere in Keynes’s TP for any of Carabelli’s claims, as Keynes’s non demonstrative logic is identical to Boole’s formal, mathematical, symbolic, relational, propositional logic
that he used throughout the *A Treatise on Probability*. Keynes's non-demonstrative logic is NOT a common discourse logic associated with Aristotle.

Carabelli’s erroneous understanding of Keynes can be traced back at least to 1985:

‘...this mixture of anti-empiricism and anti-rationalism was the core of Keynes's peculiar epistemological position and makes it difficult to describe his position in simple terms’ (Carabelli, 1985, pp. 151).

It is actually quite easy to describe Keynes’s position—it is very similar to Boole’s position, which emphasized inexact measurement, approximation, interval valued probability and a relational, propositional logic.

Carabelli’s position is restated in every published contribution she has made in the literature since 1985. On page 145 of Carabelli (1988, p.145), we find the cause of her misclassification of Keynes as being anti-mathematical (Carabelli, 1988, p.140), anti-logicist (Carabelli, 1988, p.10, pp.145-46, 280 and p.134- “What, therefore, was Keynes’s peculiar view on logic...”), anti-formalist (Carabelli, 1988, p.139) and anti-empiricist (Carabelli, 1988, p.94). She completely misrepresents the nature of the Keynes-Boole connection, which underlies the entire TP.

She severely misreads paragraph 5 in chapter I of LT and some claimed, unspecified reading from chapter 21 of LT, which actually appears on pages 422-423 in the last chapter of LT, chapter 22. Carabelli severely misinterprets Boole’s linking of mathematical reasoning with correct reasoning as being an anti-Keynes mathematical view, which Carabelli implicitly contrasts with her misbelief that Keynes was anti-mathematical. For Boole, logic always came first, and mathematical modeling only came after the nature of the variables being considered was fully understood. This is identical to the position taken by Keynes. In fact, the entire TP is built on Boole’s original logic and algebra. It is no wonder that Carabelli is unable to identify Keynes’s method or views on logic, mathematics and formal exposition correctly, as she is unable to identify what a formal, mathematical, symbolic logic is.

IV. S. Dow

Consider the following statements by Dow:

“...much recent Keynes scholarship has been devoted to outlining the particular, alternative logic that Keynes developed in the Treatise on Probability (Keynes, 1973b). This “human logic” or “ordinary logic” was required to apply to a (nonergodic) world of which most knowledge is held with uncertainty. The economic system is nonergodic. Keynes argued that, in practice (in ordinary life, as in science), we need to establish reasonable grounds for belief in propositions as the basis for action, in spite of uncertainty. According to his “ordinary logic,” we do this by using judgment to combine direct knowledge, indirect (theoretical) knowledge, conventional knowledge, and animal spirits or intuition.” (Dow, 2005, p.387; italics added)

and

“It is also compatible with Keynes's ordinary logic that combines different sources of knowledge in order to increase confidence in propositions. In modern terminology, it is compatible, with pluralism. Indeed, Keynes was at pains to develop a different logic that was more rigorous in having more direct application to the real world than classical logic. (Dow, 2005, p.388; italics added).

Nowhere in Keynes’s *A Treatise on Probability* is there an ordinary or human logic presented or applied as claimed by Dow. Dow fails to provide a single citation to any page or chapter of Keynes’s book that would demonstrate any support for her contentions. The logic Keynes applied (he never developed any logic) was Boole’s original, relational, propositional logic that he presented in 1854 in *The Laws of Thought*.

Note that in order to discuss and apply the concepts of ergodicity and non-ergodicity, one must accept the limiting frequency interpretation of probability, which was rejected by Keynes except as a special case. Finally, Keynes’s *A Treatise on Probability* has nothing to do with pluralism or methodological pluralism.

The italicized materials in her statements above have nothing to do with Keynes’s Boolean, relational, propositional logic. Dow has simply made all of her claims up, which is why there are no footnotes to any parts or pages of the TP.

Let us move on to another article. Consider the following statement:

“While closed systems are the province of classical logic, open systems are the province of a broader system of logic – ordinary logic, or human logic, as exemplified by Keynes (1973). While including classical logic as a special case for application under conditions of certainty, ordinary logic can also be applied to conditions of uncertainty, as pertains in open systems.” (Dow 2012, p.1).

Again, Keynes’s logic in the TP has nothing to do with some “…broader system of logic – ordinary logic, or human logic, as exemplified by Keynes (1973).”

Consider the following statement:

“...and referring back to the Treatise on Probability which laid the philosophical foundations for Keynes’s use of the concept. There we argued that animal spirits were a critical element of a framework for decision making under uncertainty which was rational in a broader sense, an argument by which we continue to stand …” (Dow and Dow, 2011, p.1).

There is no foundation or mention in Keynes’s TP for animal spirits. Keynes’s two paragraphs, on pp.161-162 of the *General Theory* (GT), are his attempt to incorporate what he had left out of the TP, as it was strictly of secondary importance and relevance. Keynes’s discussion of animal spirits is equivalent to
Boole’s discussion on pp.242, 272 in Boole’s The Laws of Thought (1854).

Consider the following statement:

“The reason that the investment decision relies on animal spirits is that rational quantitative calculation alone cannot justify action under uncertainty. This argument, building on Keynes’s Treatise on Probability, provides the basis for the broader interpretation of animal spirits in the Post Keynesian literature, one captured by Kregel (1987) in the term ‘rational spirits.’” (Dow and Dow, 2011, p.6).

Again, there is nothing in Keynes’s TP even remotely related to what Dow and Dow are claiming, which is why there are not any citations to any pages or paragraphs from the TP. Keynes follows G. Boole’s interval valued approach, which Keynes covered in chapters 15, 16, and 17 of the TP. Chapters I and II of the TP are Keynes’s versions of Boole’s chapters I, XI, and XII.

The investment decision DOES NOT rely on animal spirits. It is based on Keynes’s non-numerical probabilities (Boole’s interval probabilities) and Keynes’s Evidential Weight of the Evidence. In cases of decision making under Ignorance, decision makers put off making a decision and remain liquid. The broader interpretation of “animal spirits” of Keynes’s extremely brief coverage of animal spirits in the Post Keynesian literature has nothing to do with Keynes and has nothing to do with Keynes’s TP.

“As Carabelli (1988: 237) puts it:

‘What he [Keynes] appeared to think was that different types of rationality, or rules of reasonableness, existed, according to the different cognitives [sic] domains, which characterized the different economic groups.’

Since animal spirits enter into the perception of uncertainty surrounding knowledge, they affect the process of judgement (Davis 2003).” (Dow and Dow, 2011, p.9)

The claims of Dow and Dow, based on Carabelli, have nothing to do with Keynes’s use of Boole’s relational, propositional logic in his TP. This relational, propositional logic is a mathematical, formal, symbolic logic, which Dow and Dow have absolutely no understanding of. It is objective because the decision maker is able to assign a rational degree of belief based on the content of the h and the a propositions, where the h propositions supply the evidence to support the a proposition conclusion. It doesn’t matter whether the facts are different or perceived differently by different decision makers. Relative to their choice of what counts as evidence, they will be able to identify a rational degree of belief, expressed either as a numeral or an interval. This has nothing to do with some amorphous, ordinary language logic.

V. The Dow and Chick Claims about Keynes’s “Anti-Formalist” Position Completely and Totally Ignores the Boole–Keynes Connection

This section will be divided into two subsections, (5a) and (5b). (5a) will cover Chick while (5b) will cover Dow.

(5a) Chick and Dow (2001)

The work of Chick (1998) and Chick and Dow (2001) is based directly on Carabelli’s (1988). Chick and Dow make exactly the same identical mistake made by Carabelli in 1985 and 1988, which is to skip the Boolean foundation of Keynes’s inexact method and substitute a claim that Keynes was engaged in some type of ordinary language logic. Such a “logic” appears nowhere in the TP and is not mentioned anywhere by any reviewer of the TP. Every reviewer of the TP who mentioned the word “logic” in the TP recognized it as a formal, mathematical logic, although they all failed to connect Keynes’s approach to Boole and incorrectly concluded that Keynes had developed some new type of logical analysis that depended on intuition. In the Treatise on Probability, Keynes sought a

“...logic...to be applicable to everyday belief and action, as well as to ‘scientific’ knowledge and resulting policy recommendations. In the common sense tradition...based on(ordinary) logic...Ordinary logic in turn generates knowledge which is imperfect, partial or vague...Thus, an organicist understanding of the subject matter justifies the adoption of an ordinary (human) logic...” (Chick and Dow, 2001, p.711).

Such an ordinary human logic exists nowhere in the TP or GT. It apparently is based on a misquotation and misunderstanding of Keynes’s initial, introductory statements made in chapter II of the TP on pp.18-19, which is what is cited by Carabelli.

(5b) Chick (1998)

Chick is completely unaware that Keynes, following Boole, had developed mathematical techniques to analyze “complex “systems long before Zadeh was born. Her ignorance of Keynes’s mathematical approach to probability and statistics, using imprecision, approximation and upper and lower bounds for probabilities that Keynes identifies in his TP as inexact measurement, can be seen in her next assessment of Keynes’s TP:

“Keynes accepted the fact of irreducibly limited and imperfect knowledge, and warned against misplaced precision: Much economic theorizing to-day suffers, I think, because it attempts to apply highly precise and mathematical methods to material which is itself much too vague to support such treatment. (Keynes, CW XIV, p. 379.) Our precision will be a mock precision if we try to use such partly vague and non-quantitative concepts [net real output and the general price level] as the basis of a quantitative
analysis. (Keynes, CW VII, p. 40.) ‘Mock precision’ is a debatable judgement and, naturally, does not apply to everything. Formal methods are admired precisely because they eliminate vagueness and imprecision, but they eliminate them only in the theory; theory can never eliminate vagueness inherent in the data or objects of study. A debate on the nature of the objects of study is surely past due, as mathematicians and engineers now have techniques to deal in a more formal way with vagueness (fuzzy logic). In the face of imprecise knowledge, Keynes favoured informal, verbal exposition of economic theory…” (Chick, 1998, p.1864).

Contrary to Chick, Keynes NEVER favored an approach using “…informal, verbal exposition of economic theory.” He favored an approach based on his Inexact and imprecise approach to measurement and approximation, which was based on Boolean logic and algebra of which Chick is ignorant. Chick apparently never knew of the fact that Keynes had based his approach on Boole, who had developed “…techniques to deal in a more formal way with vagueness (fuzzy logic)”, in 1854.

VI. Conclusions

Carabelli, Chick, and Dow are simply ignorant of what Keynes was doing in the TP. We are led inexorably to the same conclusion first reached by Hishiyama in 1969—that the A Treatise on Probability had not been read. The only possible modification that I might consider making to Hishiyama’s 1969 summary is that only very tiny, small pieces of chapters III, IV, VI and XXVI of the TP were examined, in a very poor fashion by economists and philosophers like Ramsey, Good, Misak and Bateman, who were looking for some “new” angle that they could stick in an article that they had submitted for journal publication. After publication, they would become “experts” on Keynes’s logical theory of probability and A Treatise on Probability.

My judgment is that Carabelli, Chick, and Dow looked through the TP in a random fashion, much like the approach used of F P Ramsey in 1922 and 1926 which led him to claim that there was an axiom I in the TP that is completely fictional and existed only as a figment of Ramsey’s imagination (Ramsey, 1922, p.3). This would lead them to believe that Keynes had made all types of mistakes, was unclear or ambiguous, or confused:

“As I have already noted, these doctrines were not always explicit and expressed in univocal and coherent form. Hence the necessity not only of a close reading of Keynes’s text, but also of a sort of systematic reconstruction of Keynes’s approach to key epistemological topics, together with an attempt to clarify his position within its historical intellectual context.” (Carabelli, 1988, p.23)

Carabelli’s “systematic reconstruction” led her to assert that Keynes’s new logic, first noted by Hishiyama, was an ordinary discourse language logic written out in English. This is exactly the WRONG way to read Keynes’s TP.

Until Keynes’s technical and analytic structure is grasped and mastered (from Boole), I do not see how it is possible for any historian, philosopher or economist in the rest of the 21st century to have any chance at all of grasping what is actually a masterpiece, which was based on a previous masterpiece by Boole. One is left to read the incomprehensible assertions contained in the work of, for a few examples, the Bateman’s, Blackburn’s, Davis’s, Clarke’s, Gerrard’s, Misak’s, and Winslow’s. ALL of this work is based on the illusions, delusions, and hallucinations of F P Ramsey about some supposed Axiom I that never existed in reality. Again see Ramsey, 1922, p.3.

Acknowledgement

I want to thank the referees for their detailed comments.

References Références Referencias