Impact Of Weiner Process On Exchange Rate Forecasting

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\textit{Abstract-} Hedging techniques whether in portfolio or foreign exchange management require accurate forecasted underlying assets’ prices and rates not only to quantify the expected losses but also for computing the optimum premium payable and in deciding the number of contracts needed to neutralise the expected losses. Most of the previous studies in this area prove the efficiency and accuracy of the forecasting models through intensive algebra and a few tested on the real underlying assets’ prices and exchange rates. To overcome this problem we tested Black-Scholes and Euler-Maruyama models on Indian Rupee and Malaysian Ringgit rates of eight country currencies. Our results show that the Black-Scholes and Euler-Maruyama models forecast exchange rates close to each other though they work on different principles. These two forecasted rate trajectories converge almost perfectly. But when compared to real Indian Rupee rates the models are inefficient as the real path of exchange rates of these eight countries diverge very much. In case of Malaysian Ringgit rates for almost six months, Black-Scholes and Euler-Maruyama models forecasted exchange rates with lesser error and their paths converge with the real rates. This finding will be useful in exchange rate risk management.

I. INTRODUCTION

Foreign exchange rates assume importance in the present global financial crisis. US dollar continuously depreciated during the last three years against Euro and other major world currencies including Indian Rupee. To protect the values of imports, exports and foreign assets multinational corporations (MNCs) use derivative instruments like forward and futures contracts. These contracts work alike in protectecting value of financial assets. The futures contracts are standardised contracts traded on the exchange while forward contracts are mostly customised to the MNC’s needs and the participating banks. The futures contracts work on the marking to market principles to reduce default risk. Both futures and forwards are highly inflexible and they are not resilient when the financial assets move favourably. On the other hand the options contracts offer flexibility in the form of exercise right to the buyer of the contract to execute or not in the future. This valuable right given by the option contracts is the reason for the popularity of these contracts in all spheres of finance especially hedging. Traditionally the MNCs use forward contracts to protect the foreign financial assets. Modern derivative instruments include option contracts and swap contracts. These contracts are very attractive as they are standardised, cheap in terms of price, flexibility and liquidity as they are traded on derivative exchanges. Strict regulation allays fears of default, fraud and breach of agreements. Reasonable commission, transparency in dealings and on line trading are the other additional attractions for the option contracts. Despite the above positive points these options contracts are elusive not only in pricing but also in quantitying their risk, even experienced brokers find it difficult to price. They use market standard models like Black and Scholes (BS) to price them. Model like BS will work only when the underlying asset’s price movements are accurately forecasted, if not errors creep in and lead to either over or under pricing consequently the cost of hedging increases. The real challenge is in forecasting the movement of the underlying assets price and the co-movement of these contracts’ prices. The prices of these contracts are to be efficient to avoid arbitrage opportunities in spot and forward markets.

The Black Scholes method is the pioneering effort in pricing the European type options, which could be exercised only on maturity. The later models like Longstaff-Schwartz and other methods modify the BS method to accommodate the pricing of American options and the exotic options like Bermuda, Asian, Barrier options. The challenge lies not in pricing techniques but in forecasting the future movements of financial assets’ prices like share and foreign exchange. Almost all models use Brownian motion (Weiner process) to quantify random movements in financial asset prices. Even the BS model uses the Weiner process and the logarithmic returns to forecast the movement of underlying values (David, 2009, De Los Rios, 2009). A new branch of calculus is applied to estimate random movements in prices. The stochastic calculus which incorporates the Weiner process for estimating the random movements in prices of financial assets is the main theme of the financial engineering. Ito’s lemma is mainly applied in stochastic calculus which is a subset of Taylor series is the main branch advocated by several researchers to capture the evolution of prices of financial assets, with Brownian motion.

In this paper we attempt to compare empirically the forecasting accuracy of the BS model which is based on natural log returns and works on Geometric Brownian Motion (GBM) with the Euler Maruyama method (EMM) which works on Arithmetic Brownian Motion (ABM). The stochastic behaviour and movement of almost all financial time series are based on two parameters namely mean (drift) and variation (diffusion) (McMillan, 2009). While ABM computes future movement by adding the returns the GBM computes the future movement by exponentiation. We test these motions on six different exchange rates of Indian Rupees (INR) and Malaysian Ringgit (MYR). Most of the
research papers published in this area takes Monte Carlo Simulated data to prove the property, pattern and behaviour of time series data (Giles et al, 2009, Higham, 2001). Another group of researchers explain the above properties through pure algebra. Pure algebra and Monte Carlo simulated data both produce the expected behaviour perfectly and researchers conclude that the stochastic calculus is ideal for forecasting the underlying prices and thus good for option valuation (Chenggui, 2006, Golightly, 2009, Jacob, 2009, So Mike, 2007). Our paper is different from others as it tests the Weiner process stochastic behaviour of exchange rates with real exchange rates. In our paper we attempt to compare the real exchange rate with Monte Carlo Simulated diffusion based BS and EMM to find out their convergence and divergence.

Reminder of this paper is organised as follows. Section two discusses the Weiner process and its importance in stochastic calculus. The methodology is given in section three. The sources of data and its scope are given in section four. Section five gives results of data analysis and interpretation. Section six concludes the paper.

II. BROWNIAN MOTION - WEINER PROCESS

Botanic Scientist Brown (1827) found pollens floating in water were moving in different directions haphazardly due to the random bombardment of water molecules. Weiner (1926) refined it with time dimension and established trajectories of drift to the random movements. This analogy is applied in finance and in forecasting the movements of various financial asset prices (Kiani, 2009, Wu, 2009). In financial engineering the water molecules are equated with the transactions in the capital market (demand and supply of financial assets) and prices move at random due to the buying and selling activities which are equivalent to molecular bombardment. The trading transactions in the capital market and foreign exchange market are innumerable small independent events which collectively determine the price of financial assets. Prices of shares, bonds, units, rates like interbank offered rate and exchange rate are all fall in this domain, (Adkins et al, 1999).

III. METHODOLOGY

Assume that N transactions independently occur at a time interval Δt, with mean λ, we can define the process as Poisson process. In a time interval (t,t+Δt) the number of transactions that would happen is equal to \( \Delta N_t = N_t + \lambda \Delta t - N_t \) i.e. the expected number of transactions is \( E(\Delta N_t) = \lambda \Delta t \) and the variance is \( Var(\Delta N_t) = \lambda \Delta t \) as per the Poisson process. If the number of transactions are sufficiently larger (>20) then the Poisson Process will become normal Gaussian process (Gaussian Distribution) or noise. Gaussian Process (otherwise known as Weiner process or Brownian motion) plays an important role in modelling returns (Hall et al, 2008). The transactions, like water molecules, hit the underlying asset prices. The fundamental reason for their importance is that the volatility in prices of financial assets is the direct consequence of stochastic nature of orders received in capital market (Hooper et al, 2009, Adkins et al, 1999). If ask orders are more than the bid orders the price decreases and vice versa (Beine et al, 2009). The volatility in prices generated by orders is collectively characterised by Gaussian probability density function which is:

\[
p(x) = \frac{1}{\sqrt{2\pi}\sqrt{\Sigma}} \exp \left[ -0.5(X - \bar{X})' \Sigma^{-1} (X - \bar{X}) \right]
\]

\( \Sigma \) is the covariance matrix of \( X=(x_1, x_2, ..., x_n) \) n orders at times of t1, ... tn

The normal cumulative probability distribution function \( F(x) \) gives the cumulative probability or area of the normal curve for a given size of standard deviation. The volatility is stochastic as discussed earlier and may be any random number which can be simulated through Monte Carlo method. The normal cumulative distribution function is to find probability is:

\[
F(x) = \int_{-\infty}^{x} p(t) \, dt = 1
\]

(3)

Natural variables such as population of all species, financial assets like shares bonds etc grow continuously. But nature always controls the growth of these variables with random events like pandemic, natural calamities as tsunami, earthquake etc. Technically speaking to the deterministic models a noise is added to control the continuous positive growth. This growth is equivalent to drift (mean) and the diffusion is equivalent to (variance) which could be modelled by stochastic differential Equation (Chenggui, 2006, Golightly, 2009, Jacob, 2009, So Mike, 2007). Financial time series data are to be differentiated to get stationary data. The stationary data (returns) has the property of constant mean and variance which could be easily modelled through Weiner process. Thus

\[
dx_t = \sigma x_t \, dW_t
\]

(4)

\( dN_t = \mu \Delta t + \sqrt{\lambda \Delta t} \, dz \)

(1)

In other words the number of transactions in the next time interval is approximately equal to mean transactions plus square root of variance (volatility) multiplied by a random number drawn from the standard normal distribution (Gaussian Distribution) or noise.
enters the stochastic Calculus (Itô Calculus) is applied as the normal calculus is not suitable for differentiation and integration. The underlying financial asset’s price will change from x0 to xt. In integral form equation four becomes

\[ X(t) = X_0 + \int_0^t f(X(s))ds + \int_0^t g(X(s))dW(s), \quad 0 \leq t \leq T \]

\[ (5) \]

f = represents mean
\( g = \) represents volatility
\( s = \) small time in terms of years
\( t = \frac{\Delta t}{12} \), \( \Delta t \) is the smallest time in years

The above equation five is based on arithmetic progression. But the celebrated Black Scholes (BS) proves that the growth is based on Geometric Brownian motion (Berkowitz, 2010, Madan, 2009) which argues that the growth is a compounded rate and hence the log returns are to be the basis for any modelling. Itô calculus emerging from Taylor series models the geometric growth as:

\[ dF = \frac{df}{dS}dS + \frac{1}{2}\sigma^2 S \frac{d^2F}{dS^2} dt \]

\[ (6) \]

By manipulation we get

\[ \ln \frac{F(t)}{F(0)} = \left( \mu - \frac{1}{2}\sigma^2 \right) t + \sigma S \frac{dF}{dS} \]

\[ \ln \frac{X(t)}{X(0)} = \left( \mu - \frac{1}{2}\sigma^2 \right) t + \sigma S \frac{dF}{dS} \]

\[ (7) \]

\[ S(t) = S(0)e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma S \frac{dF}{dS} (X(t) - X(0))} \]

\[ (8) \]

\[ X_t = X_0 e^{(r - 0.5\sigma^2) \Delta t + \sigma \Delta W} \]

\[ (9) \]

\[ X_{t+1} = X_{t} + \Delta t \left( f(X_t) + g(X_t) \right) \]

\[ (10) \]

The above equation is the celebrated Black Scholes equation which works on geometric compounded growth principles. Another model which works on arithmetic progression is the EMM. Two main differences can be attributed to this EMM compared to Black Scholes model. Firstly it is based on integration by parts with the initial spot price. Secondly it works on longer time interval say a week (Chenggui, 2008, Garrison, 2005). The proposed model is given below.

\[ X_{j} = X_{j-1} + f \left( X_{j-1} \right) \Delta t + g \left( X_{j-1} \right) \left( W_{j} - W_{j-1} \right) \]

\[ (11) \]

The time limit \( \Delta t \) in the above equation \( \lim_{\Delta t \to 0} \) the frequency of transactions will increase to infinity. This infinitive solution is desirable but the time, effort and computations not worth it. Even in longer time intervals with fewer transactions the same answer to the SDE is arrived. The above equation in differential form is as follows.

\[ X(t) = X_0 + \int_0^t f(X(s))ds + \int_0^t g(X(s))dW(s), \quad 0 \leq t \leq T \]

\[ (12) \]

\( X_0 \) is the price of the underlying at time \( t_0 \), \( s \) is the \( s \) is the smallest time in years \( f \) and \( g \) are the mean and standard deviations of returns. The first integral is the deterministic growth (drift) and the second integral is the Brownian motion in a longer time interval \( \Delta t \) times standard deviation. The solution \( x(t) \) will give the trajectory of any financial underlying asset.

The above equation will be given in differential form is as follows

\[ dX(t) = f(X(t))dt + g(X(t))dW(t), \quad X(0) = X_0, \quad 0 \leq t \leq T \]

\[ (13) \]

The same equation for numerical computation is

\[ X_j = X_{j-1} + f \left( X_{j-1} \right) \Delta t + g \left( X_{j-1} \right) \left( W_{j} - W_{j-1} \right) \]

\[ (14) \]

\[ \text{aerr} = \frac{1}{n} \sum_{j=1}^{n} (X_r - X_{BS/EM})^2 \]

\[ (15) \]

\( \text{aerr} = \) average error
\( X_r = \) Real price of underlying asset
\( X_{BS} = \) BS rate or EMM rate

Another way of measuring the error is to take the end point data of both real and theoretical rates to find the absolute error at T. This will give only the error at the end of the trajectory.

\[ \text{err} = ep | X_r - X_{BS/EM} | \]

\[ (16) \]

\( ep = \) prices of underlying at the end of trajectory

With the above methodology we write MATLAB m file (given in appendix) to empirically generate rates and visualise the paths generated by the Black-Scholes and Euler-Maruyama Method SDEs.

IV. DATA

To empirically test the above SDEs under BS and EMM we took Indian Rupee (INR) and Malaysian Ringgit (MYR) as the base currencies and downloaded data of exchange rates of eight currencies for 2008 and 2009. Excluding African continent we have two currencies each for every continent. American continent is represented by USD, CAD, Europe is represented by EUR, GBP, Australia is represented by AUD, NZD and Asia is by JPY, KRW. Presently the trade is not limited to any closed boundary due to globalisation and the present global financial crisis also emphasises the importance of exchange rates in quantifying currency risks.
As the exchange rates growing in prominence in every sphere of business hedging decision is also grow in proportion. We proceed to test the BS and EMM SDEs with real exchange rates. Using 2008 data for BS we computed log returns and for EMM we used normal returns and computed descriptive parameters like mean and standard deviations. Exchange rate data of 2009 is used as a holdout sample to validate the Black-Scholes and Euler-Maruyama theoretical rates.

V. ANALYSIS AND INTERPRETATION

Applying the above methodology we simulated random volatility using Monte Carlo technique for every day exchange rate and applying the parameters computed on 2008 data, we forecasted exchange rates for 2009. If the SDEs are efficient in deriving the trajectory they should provide a trajectory closer to real exchange rate path. If the theoretical rates do not converge to real rates, errors emerge. We have quantified the errors in sums of squares and in absolute deviation for both BS and EMM. The results of our analysis are presented below.

<table>
<thead>
<tr>
<th>Currency</th>
<th>Mean Rate</th>
<th>Mean Return</th>
<th>SD</th>
<th>Mean Rate</th>
<th>Mean Return</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian Rupee</td>
<td></td>
<td></td>
<td></td>
<td>Malaysian Ringgit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USD</td>
<td>48.615</td>
<td>0.001</td>
<td>0.007</td>
<td>3.310</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>CAD</td>
<td>39.698</td>
<td>0.000</td>
<td>0.011</td>
<td>3.334</td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>EUR</td>
<td>67.668</td>
<td>0.001</td>
<td>0.010</td>
<td>4.878</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>JPY</td>
<td>0.536</td>
<td>0.002</td>
<td>0.013</td>
<td>0.030</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>GBP</td>
<td>71.043</td>
<td>-0.001</td>
<td>0.010</td>
<td>6.562</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>AUD</td>
<td>33.942</td>
<td>0.000</td>
<td>0.016</td>
<td>2.917</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>NZD</td>
<td>28.261</td>
<td>0.000</td>
<td>0.013</td>
<td>2.559</td>
<td>0.000</td>
<td>0.008</td>
</tr>
<tr>
<td>KRW</td>
<td>0.039</td>
<td>0.000</td>
<td>0.017</td>
<td>0.004</td>
<td>0.000</td>
<td>0.003</td>
</tr>
</tbody>
</table>

SD = Standard Deviation

The mean rates for hard currencies are greater than Rupees 30 while for soft currencies it is less than one Rupee. Japanese yen though it is less than one Rupee it is still a hard currency. The British Pound shows a small negative return while other currencies show an average closer to zero. Once the rates are converted to returns they become stationary. In terms of volatility Korean Won and Australian Dollar show a wider variation of 0.017 and 0.016. USD shows very less volatility. It implies that USD is more stable in 2008 than other exchange rates. In case of MYR the average returns are zero for all currencies which indicates that 50% of the time the exchange returns (ExR) are positive and vice versa. This shows that when the rates are converted to returns they become stationary and useful for further analysis. NZD, AUD and JPY show somewhat higher variation. Other currencies returns are less volatile and stable. INR is soft when compared to MYR and hence the means and volatility are higher for INR. This is consistent with the prevailing economic situation and shows the traders confidence in a country’s currency.

The parameters mean and standard deviation computed with 2008 ExR are applied in forecasting the BS and EMM exchange rates. Monte Carlo simulated random noise was included while estimating the exchange rates under BS and EMM. Table two below gives the real ExR mean, BS mean and EMM mean with their respective volatilities in terms of standard deviations.
Table 2 Rupee and various exchange rates’ mean and standard deviation for 2009

<table>
<thead>
<tr>
<th></th>
<th>Real Mean</th>
<th>Real SD</th>
<th>BS Mean</th>
<th>BS SD</th>
<th>EMM Mean</th>
<th>EMM SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>48.352</td>
<td>1.429</td>
<td>47.554</td>
<td>2.190</td>
<td>47.435</td>
<td>2.249</td>
</tr>
<tr>
<td>CAD</td>
<td>42.451</td>
<td>1.907</td>
<td>36.167</td>
<td>2.018</td>
<td>35.944</td>
<td>2.075</td>
</tr>
<tr>
<td>EUR</td>
<td>67.276</td>
<td>2.193</td>
<td>69.304</td>
<td>3.044</td>
<td>68.843</td>
<td>3.196</td>
</tr>
<tr>
<td>JPY</td>
<td>0.517</td>
<td>0.015</td>
<td>0.523</td>
<td>0.029</td>
<td>0.517</td>
<td>0.030</td>
</tr>
<tr>
<td>GBP</td>
<td>75.546</td>
<td>3.251</td>
<td>67.124</td>
<td>3.771</td>
<td>66.325</td>
<td>4.057</td>
</tr>
<tr>
<td>AUD</td>
<td>38.150</td>
<td>3.696</td>
<td>40.081</td>
<td>7.045</td>
<td>39.407</td>
<td>6.616</td>
</tr>
<tr>
<td>NZD</td>
<td>30.602</td>
<td>3.080</td>
<td>36.931</td>
<td>5.564</td>
<td>36.526</td>
<td>5.351</td>
</tr>
<tr>
<td>KRW</td>
<td>0.038</td>
<td>0.002</td>
<td>0.042</td>
<td>0.003</td>
<td>0.041</td>
<td>0.003</td>
</tr>
</tbody>
</table>

SD = Standard Deviation

The first two columns give real INR average and standard deviations, column four and five give the mean and standard deviation of BS method and the final two columns give the same parameters for EMM for the selected eight currencies. USD, CAD and GBP show lesser mean rate than the real mean and higher standard deviations both in BS and EMM rates. But Euro, Yen, AUD, NZD and KRW show the opposite. Their BS and EMM mean rates are higher than real mean rates and show lesser volatility. These results do not convey any fixed pattern and it seems the currencies move at random and not based on any co-movement or regional links.

The mean rate movement against MYR is given in the following table for the same eight currencies. Except Japanese Yen all other currencies both in BS and EMM show higher mean rates than the real mean rates. In terms of volatility the forecasted volatilities are less than the real volatilities of these eight currencies. For Yen the opposite behaviour could be observed.

Table 3 Ringgit and various exchange rates 2009, means and standard deviations

<table>
<thead>
<tr>
<th></th>
<th>Real Mean</th>
<th>Real SD</th>
<th>BS Mean</th>
<th>BS SD</th>
<th>EMM Mean</th>
<th>EMM SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>3.332</td>
<td>0.149</td>
<td>3.375</td>
<td>0.033</td>
<td>3.372</td>
<td>0.033</td>
</tr>
<tr>
<td>CAD</td>
<td>3.136</td>
<td>0.134</td>
<td>3.632</td>
<td>0.160</td>
<td>3.623</td>
<td>0.154</td>
</tr>
<tr>
<td>EUR</td>
<td>4.888</td>
<td>0.173</td>
<td>4.927</td>
<td>0.150</td>
<td>4.919</td>
<td>0.153</td>
</tr>
<tr>
<td>JPY</td>
<td>0.032</td>
<td>0.003</td>
<td>0.028</td>
<td>0.001</td>
<td>0.028</td>
<td>0.001</td>
</tr>
<tr>
<td>GBP</td>
<td>6.154</td>
<td>0.375</td>
<td>6.461</td>
<td>0.110</td>
<td>6.454</td>
<td>0.115</td>
</tr>
<tr>
<td>AUD</td>
<td>2.828</td>
<td>0.270</td>
<td>3.050</td>
<td>0.158</td>
<td>3.040</td>
<td>0.152</td>
</tr>
<tr>
<td>NZD</td>
<td>2.370</td>
<td>0.201</td>
<td>2.579</td>
<td>0.127</td>
<td>2.571</td>
<td>0.124</td>
</tr>
<tr>
<td>KRW</td>
<td>0.003</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
</tbody>
</table>

SD = Standard Deviation

This shows the importance of base currency on which ExRs are computed. INR is softer by thirteen times and hence the multinational corporations and traders dependent more on harder currencies while pricing their products and services. Stronger currencies not only bring good margin of profits but also they are chosen frequently for pricing goods and services it seems.

The errors are deviations of BS and EMM rates from the real rates. They show the convergence or divergence of the rates forecasted. The errors are as follows.
Table 4 Black-schools and Euler-Maruyama average error and Absolute error

<table>
<thead>
<tr>
<th>Currency</th>
<th>Mean BS Error</th>
<th>Absolute BS Error</th>
<th>Mean EMM Error</th>
<th>Absolute EMM Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian Rupees</td>
<td>USD 3.08</td>
<td>363</td>
<td>96.2</td>
<td>1,638</td>
</tr>
<tr>
<td></td>
<td>CAD 47.96</td>
<td>1,584</td>
<td>408.4</td>
<td>1,353</td>
</tr>
<tr>
<td></td>
<td>EUR 24.64</td>
<td>786</td>
<td>203.2</td>
<td>3,333</td>
</tr>
<tr>
<td></td>
<td>JPY 0.00</td>
<td>7</td>
<td>1.8</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>GBP 106.57</td>
<td>2,179</td>
<td>593.6</td>
<td>4,276</td>
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<tr>
<td></td>
<td>AUD 19.16</td>
<td>835</td>
<td>177.9</td>
<td>1,172</td>
</tr>
<tr>
<td></td>
<td>NZD 49.93</td>
<td>1,595</td>
<td>376.7</td>
<td>771</td>
</tr>
<tr>
<td></td>
<td>RW 0.00</td>
<td>1</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Malaysia Ringgit</td>
<td>USD 0.02</td>
<td>34.34</td>
<td>8.1</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>CAD 0.31</td>
<td>125.29</td>
<td>7.1</td>
<td>29.7</td>
</tr>
<tr>
<td></td>
<td>EUR 0.02</td>
<td>25.43</td>
<td>17.2</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>JPY 0.00</td>
<td>0.99</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>GBP 0.21</td>
<td>78.48</td>
<td>26.8</td>
<td>16.8</td>
</tr>
<tr>
<td></td>
<td>AUD 0.18</td>
<td>73.84</td>
<td>5.7</td>
<td>16.6</td>
</tr>
<tr>
<td></td>
<td>NZD 0.14</td>
<td>71.57</td>
<td>3.9</td>
<td>16.7</td>
</tr>
<tr>
<td></td>
<td>RW 0.00</td>
<td>0.12</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The errors shown by INR exchange rates are higher when compared to MYR. Two reasons could be attributed for this larger error in INR. The INR is a soft currency and hence the exchange rates are higher in number. Therefore by volume the error is larger. Secondly the error also could be attributed to the deviation of the forecasted rates from actual rates. Due to non convergence also the error could arise. For INR the mean and absolute errors are lesser in BS computations than the EMM computations. The EMM errors are several times higher because the BS model computes 252 rates for each day and it is an average while the EMM computes 63 rates each for one week in a year, to be precise rates for every four days. Hence the magnitude of the errors is more in case of EMM. Similar explanation could be given for MYR error levels.

The predicted or forecasted exchange rates and the error level will not reveal the true convergence of the rates. A graph in the form of line chart will visually show the convergence. The following figure shows the convergence of these rates.

The above figure gives the real rate trajectories of eight currencies with the BS and EMM rates’ trajectories. The trajectories of BS and EMM converge closely in all eight figures. This proves the BS method which works on geometrical Brownian motion and EMM which works on arithmetical Brownian motion do not vary in forecasting the exchange rates. Despite this the actual exchange rates do not converge with the theoretical trajectories produced by BS and EMM. In all figures the actual exchange rates go above the BS and EMM rates. They all show substantial divergence. The USD, Euro, JPY and KRW come closer to the forecasted trajectories in the middle of 2009. All other rates walk above the expected rates. In all currencies the tail end rates show a very large divergence than beginning rates. All trajectories start at spot rates prevailed at the beginning of 2009 but in two months time they go in different paths. The BS and EMM paths fall sharply and increase in the middle of the year and again fall at the end of 2009.

The MYR rates are not as divergent as seen in INR. The USD for almost six months in 2009 goes closer to the forecasted paths. The CAD goes hand in hand throughout 2009. The Euro and Japanese Yen capture the trend but the real rate goes above the theoretical rates. GBP, AUD and NZD all move with the forecasted trajectories for almost nine months and then they show some divergence. The real exchange rate at the tail end falls sharply and steeply than the BS and EMM rates. The Korean Won only for two months moves with the actual rates and later it goes below the forecasted path. It further declines steeply showing a wider divergence at the tail end.
VI. CONCLUSION

In foreign exchange management hedging decisions play a significant role in controlling the transaction and translation risks. Hedging instruments pricing is dependent on the underlying asset’s pricing as they are cointegrated. For pricing and as well as for finding matching instrument in terms of price and the number of contracts needed to hedge are all dependent on the forecasted rates. Any mispricing will lead to inefficient hedging which not only increases the cost but also could not avoid the expected losses fully. Several attempts have been made in the past to forecast the expected price path of the underlying assets. We also attempted to forecast the exchange rates through the two prominent models BS and EMM. Financial engineering mostly relies on stochastic differential equations in deriving the financial assets’ prices.

We took eight exchange rates in INR and MYR to test the forecast accuracy through SDEs. Our results are mixed. In the case of INR these stochastic models produce lower rates than the actual exchange rates. The actual rates’ path is well above the forecasted trajectories. The same technique when applied on MYR the results are good. In almost all exchange rates the BS and EMM converge for almost 6 months and later it diverges. Even this divergence is not so significant. We conclude that for strong currencies these models work well and for weak currencies’ exchange rates some adjustments are needed to make them converge. This finding is useful in foreign exchange risk management especially in transaction risk management.

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VII. REFERENCES


MATLAB M file for simulating Black- Scholes and Euler– Maruyama trajectories

```matlab
MATLAB M file for simulating Black-Scholes and Euler-Maruyama trajectories

close all
clear all
clc

format short
load INRMY

data1=price2ret(data); % Take all returns

fore=[]; bserr=zeros(16,1); d8=zeros(16,1);
emerr=zeros(16,1); d6=zeros(16,1); d7=zeros(16,1);
d9=zeros(16,1); d16=zeros(16,1);
d10=zeros(16,1); d12=zeros(16,1);

for i=1:16 % for loop for eleven currencies

d1=data1(1:250,i); % Take log returns of 2008

d2=data1(251:502,i);% Take log returns of 2009

d3=data1(1:251,i); % Take Exchange rates of 2008

d4=data(252:503,i); % Take Exchange rates of 2009

xmean=mean(d1); % Find mean
xstd=std(d1); % Find Standard Deviation

xzero=data(252:252,i); % Take 1-Jan-2009 data

T=1; N=252; dt=T/N; t=dt:dt:T; % Set the time variables
randn('state',1) % Set the same random numbers for all
xwei=randn(1,N); % Simulate Brownion motion 252 times
xcwei=cumsum(xwei); % Integrate Brownion motion

xbs=xzero*exp((xmean-.5*xstd^2)*t+xstd*xcwei); % Black Scholes Rates
xbssr(:,i)=xbs; % Store Black Scholes Rates

R=4; Dt=R*dt; L=N/R; tt=Dt:Dt:T; % Set the time variables for E-M Method
xem=zeros(1,L); % Create memory storage

temp=xzero; % Keep the 1-Jan-2008 in temporary

for j = 1:L % for loop for finding E-M rates with a wider time interval

winc=sum(xwei(R*(j-1)+1:R*j)); % Integrate Brownion motion in wider time

d5=d4(R*(j-1)+1);
temp=temp + temp*Dt*xmean + temp*xstd*winc; % Find rate in every time interval

xem(j)=temp; % Store each Euler-Maruyama Rates
end

fore=[fore; xem]; % Store all Euler-Maruyama Rates

figure
plot(t,xbs); hold on; plot(tt,xem,'-r'); % Plot various exchange rates

hold on; plot(t,d4,'-g')

title(textdata(i))
xlabel('Year 2009')
ylabel('Exchange Rate - MYR')
legend('Black Scholes Rate', 'Euler-Maruyama Rate',...
'Actual Exchange Rate',3)

bserr(i)=sum((d4-xbs).^2); % Find the sums of squares-Black Scholes

emerr(i)=sum((d5-xem).^2); % Find the sums of squares - Euler Maruyama

d6(i)=xzero;
d7(i)=xmean;
d8(i)=xstd;
d9(i)=mean(xbsr(:,i));
d10(i)=mean(fore(:,i));
d16(i)=std(xbsr(:,i));
d12(i)=std(fore(:,i));

end

xerr=[bserr./2.56 emerr./64]; % Find error percentage
xdesr=[d6 d7 d8];
xdesbs=[d9 d10 d16 d12];
```