Productivity Growth and Its Influence on the Dollar/Euro Real Exchange Rate

By Ordean Olson
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Productivity Growth and Its Influence on the Dollar/Euro Real Exchange Rate

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1. Introduction

The euro greatly depreciated against the dollar during the period 1995-2001. This decline has often been associated with relative productivity changes in the United States and the euro area over this time period. During this time period in particular, average labor productivity accelerated in the United States, while it decelerated in the euro area. Economic theory suggests that the equilibrium real exchange rate will appreciate after an actual or expected shock in average labor productivity in the traded goods sector. Such an equilibrium appreciation may be influenced in the medium term by demand side effects. Thus, productivity increases raise expected income, which leads to an increased demand for goods. However, the price of goods in the traded sector is determined more by international competition. By contrast, in the non-traded sector, where industries are not subject to the same competition, goods prices tend to vary widely and independently across countries.

The work of Harrod (1933), Balassa (1964), and Samuelson (1964) show that productivity growth will lead to a real exchange rate appreciation only if it is concentrated in the traded goods sector of an economy. Productivity growth that has been equally strong in the traded and non-traded sectors will have no effect on the real exchange rate.

This paper analyses the impact of relative productivity developments in the United States and the euro area on the dollar/euro exchange rate. This paper then provides evidence on the long-run relationship between the real dollar/euro exchange rate and productivity measures with and without the oil prices and government spending variables. Importantly, to...
center around the Balassa-Samuelson model, portfolio balance considerations as well as the uncovered (real) interest rate parity condition. This study will focus on the role of productivity differentials in the determination of the dollar/euro exchange rate.

According to the Balassa-Samuelson framework, the distribution of productivity gains between countries and across tradable and non-tradable goods sectors in each country is important for assessing the impact of productivity advances on the real exchange rate. The intuition behind the Balassa-Samuelson effect is rather straightforward. Assuming, for instance of simplicity, that productivity in the traded goods sector increases only in the home country, marginal costs will fall for domestic firms in the traded-goods sector. This leads (under the perfect competition condition) to a rise in wages in the traded goods sector at given prices. If labor is mobile between sectors in the economy, workers shift from the non-traded sector to the traded sector in response to the higher wages. This triggers a wage rise in the non-traded goods sector as well, until wages equalize again across sectors. However, since the increase in wages in the non-traded goods sector is not accompanied by productivity gains, firms need to increase their prices, which do not jeopardize the international price competitiveness of firms in the traded goods sector Harrod (1933), Balassa (1964) and Samuelson (1964).

Tille, Stoffels and Gorbachev (2001) revealed that nearly two-thirds of the appreciation of the dollar was attributable to productivity growth differentials (using the traded and nontraded differentials). However, it is important to note that Engel (1999) found that the relative price of non-traded goods accounts almost entirely for the volatility of US real exchange rates.

Accordingly, there should be a proportional link between relative prices and relative productivity. Labor productivity, however, is also influenced by demand-side factors, though their effect should be of a transitory rather than of a permanent nature. In particular, as the productivity increases raise future income, and if consumers value current consumption more than future consumption, they will try to smooth their consumption pattern as argued by (Bailey and Wells 2001). This leads to an immediate increased demand for both traded and non-traded goods. The increase in demand for traded goods can be satisfied by running a trade deficit. The increased demand for non-traded goods, however, cannot be satisfied and will lead to an increase in prices of non-traded goods instead. Thus, demand effects lead to a relative price shift and thereby to a real appreciation.

a) The Asymptotically Stationary Process of the Model

This section presents evidence in favor of stable long-run relationships between the real dollar/euro exchange rate, the productivity measure, and the other variables. One model specification was estimated for the productivity measure. The sample covers the period from 1985 to 2007. The general model includes all variables discussed above as well as deterministic components.

The results of the autocorrelations and partial autocorrelations in figures 1-3 show that the autocorrelations typically die out over time with increasing time as in the GDP, oil prices and US productivity variables. The dashed lines are just +/- 2/√T lines; consequently, they give a rough indication of whether the autocorrelation coefficients may be regarded as coming from a process with true autocorrelations equal to zero. A stationary process for which all autocorrelations are zero is called white noise or a white noise process. Clearly, all of the series are not likely to be generated by a white noise process because the autocorrelations reach outside the area between the dashed lines for more than 50% of the time series. On the other hand, all coefficients at higher lags are clearly between the lines. Hence, the underlying autocorrelation function may be in line with a stationary data gathering process. The partial correlations convey basically the same information on the properties of the time series.
Lutkepohl (2004) states that autocorrelations and partial autocorrelations provide useful information on specific properties of a data gathering process other than stationarity. Consistency and asymptotic normality
of the maximum likelihood estimators are required for
the asymptotic statistical theory behind the tests to be
valid. The results of these tests are shown in the
appendix (table 6). They consist of an LM test of no error
autocorrelation, an LM-type test of no additive
nonlinearity, and another LM-type test of parameter
constancy.

\[
F_y(\Psi) = (2\Pi)^{-1} \sum y/e^{-i\Psi(j)} = (2\Pi)^{-1} \left[ \sum y_j \cos(\Psi j) \right]
\]

(1)

Where \( I = \sqrt{-1} \) is the imaginary unit, \( \Psi \) is the frequency, that is, the number of cycles in a
unit of time measured in radians, and the \( y_j \)’s are the
autocovariances of \( y_t \) as before. It can be shown that

\[
Y_j = \int F_y(\Psi) d\Psi
\]

(2)

Thus, the autocovariances can be recovered
from the spectral density function integral as follows:

\[
Y_0 = a^2 \int (\Psi) d\Psi
\]

(3)

Graph 1 shows the log of the smoothed
spectral density estimator based on a Bartlett window
with window width \( M_r = 20 \).

Many economic time series have characteristics
incompatible with a stationary data gathering process.
However, Lutkepohl (2004) recommends the use of
simple transformations to move a series closer to
stationarity. A logarithmic transformation may help
stabilize the variance. In figure 4 the logarithms of the
US productivity, M2, oil prices, US GDP, US/euro
exchange rate and government spending are plotted.
The logarithm is used as it ensures that larger values
remain larger than smaller ones. The relative size is
reduced, however. The series has an upward trend and
a distinct seasonal pattern. The series clearly has
important characteristics of a stationary series

b) Unit Roots.

Fuller (1976) and Dickey & Fuller (1979)
proposed the augmented Dickey-Fuller (ADF) test for
the null hypothesis of a unit root. It is based on the \( t \)-
statistic of the coefficient \( \hat{\phi} \) from an OLS estimation
(see table 1). Schmidt & Phillips (1992) propose
another group of tests for the null hypothesis of a unit
root when a deterministic linear trend is present.

<table>
<thead>
<tr>
<th>Sample Range</th>
<th>Lagged Difference</th>
<th>Critical Values</th>
<th>Test Values</th>
<th>Schmidt &amp; Phillips Critical Values</th>
<th>Test Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro Prod 1985-2008</td>
<td>2</td>
<td>-4.1978</td>
<td>3.96</td>
<td>-17.3112</td>
<td>18.1**</td>
</tr>
<tr>
<td>US GDP 1985-2008</td>
<td>2</td>
<td>-5.4389</td>
<td>3.41</td>
<td>-11.5869</td>
<td>18.1**</td>
</tr>
</tbody>
</table>
The empirical analysis employs cointegration tests as developed by Johansen (1995). In the present setting, some variables would theoretically be expected to be stationary, but appear to be near-integrated processes empirically. The presence of the cointegration relationships is tested in a multivariate setting. Table 2 and 3 show the results of the cointegration tests. Over all, the results suggest that it is reasonable to assume a single cointegration relationship between the variables and suggest being viewed as an order of I(1).

**Table 2**

<table>
<thead>
<tr>
<th>Cointegration Without Oil</th>
<th>Period</th>
<th>Specification</th>
<th>LR Ratios</th>
<th>Critical Ratios &amp; Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Prod</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>3.72</td>
<td>16.22***</td>
</tr>
<tr>
<td>Euro Prod</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>2.7</td>
<td>12.45**</td>
</tr>
<tr>
<td>US GDP</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>2.23</td>
<td>12.53**</td>
</tr>
<tr>
<td>Euro GDP</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>3.32</td>
<td>9.14**</td>
</tr>
<tr>
<td>US CPI</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>10.59</td>
<td>12.45**</td>
</tr>
<tr>
<td>Euro CPI</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>2.48</td>
<td>12.45**</td>
</tr>
</tbody>
</table>

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikonen and Lutkepohl.

**Table 3**

<table>
<thead>
<tr>
<th>Cointegration With Oil</th>
<th>Period</th>
<th>Specification</th>
<th>LR Ratios</th>
<th>Critical Ratios &amp; Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Prod</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>15.34</td>
<td>25.73**</td>
</tr>
<tr>
<td>Euro Prod</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>31.68</td>
<td>42.77**</td>
</tr>
<tr>
<td>US GDP</td>
<td>1985-2008</td>
<td>2 lags</td>
<td>13.61</td>
<td>16.22***</td>
</tr>
</tbody>
</table>

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikonen and Lutkepohl.
c) Data for Variables

For the period prior to 1999, the real dollar/euro exchange rate was computed as a weighted geometric average of the bilateral exchange rates of the euro currencies against the dollar. In addition, the model was estimated controlling for several other variables, which included US productivity, M2, oil prices, government spending and US GDP. As regards the real price of oil, its usefulness for explaining trends in real exchange rates is documented. For example, Amano and Van Norden (1998a and 1998b) found strong evidence of a long-term relationship between the real effective exchange rate of the US dollar and the oil price. As regards government spending, the fiscal balance constitutes one of the key components of national saving. In particular, Frenkel and Mussa (1985) argued that a fiscal tightening causes a permanent increase in the net foreign asset position of a country, and consequently, an appreciation of its equilibrium exchange rate in the long term. This will occur provided that the fiscal consolidation is considered to have a long-run effect.


This study shows how much of the decline of the euro against the US dollar during the 1995-2001 period can be attributed to relative changes in productivity in the United States and the euro area. While the estimation covers the period 1985-2007, the following analysis concentrates on two distinct periods.

Period 1 (1995-2001) covers the US dollar appreciation against the euro. Moreover, it encompasses the period during which the productivity revival in the United States has taken place. Over this period, the dollar appreciated by almost 41% against the euro area currency. During the first three years (1998-2001) of the euro, it depreciated by almost 30% against the US dollar. Figure 5 shows the impact of a change in relative productivity developments over these periods on the equilibrium real exchange rate. The contribution of the relative developments in productivity on the explanation of the depreciation of the euro against the US dollar since 1995 is significant. However, these developments are far from explaining the entire euro decline. Figures 6 and 7 show the impact of a change in relative US GDP and Euro GDP on the equilibrium dollar/euro real exchange rate.

Period 2 (2001-2007) covers the US dollar depreciation against the euro. Figure 8 also shows the impact of a change in relative productivity developments over these periods on the equilibrium real exchange rate. The impact of productivity on the real exchange rate is significant. The contributions of the oil prices, US GDP, M2 and US government spending on the explanation of the volatility of the euro against the US dollar since 1995 are also shown in Figures 9-12.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Period</th>
<th>Lags</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro GDP</td>
<td>1985-2008</td>
<td>2</td>
<td>26.07</td>
<td>30.67***</td>
</tr>
<tr>
<td>US CPI</td>
<td>1985-2008</td>
<td>2</td>
<td>17.82</td>
<td>25.73**</td>
</tr>
<tr>
<td>Euro CPI</td>
<td>1985-2008</td>
<td>2</td>
<td>16.62</td>
<td>30.67**</td>
</tr>
</tbody>
</table>

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikkonen and Lutkepohl.
Figure 7: US GDP > USD/EURO Exchange Rate.
Figure 8: US Prod > Dollar/Euro Exchange Rate.
Figure 9: Oil Prices > dollar/Euro Exchange Rate.
Figure 10: US Government Spending > Dollar/Euro Exchange Rate.
Figure 11: GDP > Dollar/Euro Exchange Rate.
Figure 12: M2 > Dollar/Euro Exchange Rate.
III. Estimation and The Structural VECM

Lutkepohl (2004) suggests the following basic vector autoregressive and error correction model (neglecting deterministic terms and exogenous variables):

For a set of K times series variables

\[ y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + \mu_t \]  

(4)

The VAR model is general enough to accommodate variables with stochastic trends, it is not the most suitable type of model if interest centers on the cointegration relations because they do not appear explicitly. The following VECM form is a more convenient model setup for cointegration analysis:

\[ y_t = \Pi y_{t-1} + I_{\Delta} \eta_{t-1} + \ldots I_{p-1} \Delta \eta_{t-p+1} + \mu_t \]  

(5)

a) Deterministic Terms

Several extensions of the basic model are usually necessary to represent the main characteristics of a data set. It is clear that including deterministic terms, such as an intercept, a linear trend term, or seasonal dummy variables, may be required for a proper representation of the data gathering process. One way to include deterministic terms is simple to add them to the stochastic part,

\[ y_t = \mu_t + x_t \]  

(6)

Here, \( \mu \) is the deterministic part and \( x_t \) is a stochastic process that may have a VAR or VECM representation.

A VAR representation for \( y_t \) is as follows:

\[ y_t = \nu_0 + \nu_t + A_t \Delta y_{t-1} + \ldots A_p y_{t-p} + \mu_t \]  

(7)

A VECM \((p-1)\) representation has the form

\[ y_t = \nu_0 + \nu_t + \Pi \Delta y_{t-1} + \ldots \Pi_{p-1} \Delta \eta_{t-p+1} + \mu_t \]  

(8)

b) Exogenous Variables

Lutkepohl (2004) recommends further generalizations of the model to include further stochastic variables in addition to the deterministic part. A rather general VECM form that includes all these terms is

\[ y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots \Gamma_{p-1} \Delta \eta_{t-p+1} + CD_{t} + Bz_t + \mu_t \]  

(9)

where the \( z_t \) are unmodeled stochastic variables, \( D_t \) contains all regressors associated with deterministic terms, and \( C \) and \( B \) are parameter matrices. The \( z \)’s are considered unmodeled because there are no explanatory equations for them in the system.

c) Estimation of VECM’s

Under Gaussian assumptions estimators are ML estimators conditioned on the presample values (Johansen 1988). They are consistent and jointly asymptotically normal under general assumptions,

\[ V^{-1} \text{VEC} \left( \begin{bmatrix} \Gamma_1 & \ldots & \Gamma_{p-1} \end{bmatrix} - \begin{bmatrix} \Gamma_{p-1} & \ldots & \Gamma_1 \end{bmatrix} \right) \rightarrow_d N(0, \Sigma) \]  

(10)

Reinsel (1993) gives the following:

\[ \text{VEC}(\beta_{p,n}) \equiv N \left( \text{VEC}(\beta_{p,n}), \left\{ y^2_{1}, MY^2_{1} \right\}^{-1} \phi \left( \alpha \Sigma_n^{-1} \alpha \right)^{-1} \right) \]  

Adding a simple two-step (S2S) estimator for the cointegration matrix, \( \beta \)

\[ y_t - \Pi y_{t-1} - \Gamma x_{t-1} = \Pi_2 y_{1,2}^2 + \mu_t \]  

(12)

The restricted estimator \( \beta_{p,n}^R \) obtained from VEC \((\beta_{p,n})^R = \begin{bmatrix} \Pi_1 \end{bmatrix} h \) a restricted estimator of the cointegration matrix is

\[ B_R = [I_1, B_{K,2}] \]  

(13)

d) Estimation of Models with more General Restrictions and Structural Forms.

The first stage estimator \( \beta \) is treated as fixed in a second-stage estimation of the structural form because the estimators of the cointegrating parameters converge at a faster rate than the estimation of the short-term parameters (Lutkepohl-2004). In other words, a systems estimation procedure may be applied to

\[ A\Delta y_t = a^* \beta^* y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots \Gamma_{p-1} \Delta \eta_{t-p+1} + C*D_t + B^*z + \nu_t \]  

(14)

As suggested by King et al (1991) the following procedure is used for the estimation of the model: Using economic theory we can infer that all three variables should be I(1) with \( r = 2 \) cointegration relations and only one permanent shock. The variables in this model include government spending, US productivity and oil prices. Because \( k^* = 1 \), the permanent shock is identified without further assumptions (\( k^* -1)/2 = 0 \). For identification of the transitory shocks a further restriction is needed. If we assume that the second transitory shock does not have an instantaneous impact of the first one, we can place the permanent shock in the \( e \) vector. These restrictions can be represented as follows in this framework:

\[ \Xi B = \begin{bmatrix} [*00] & B & [***] \\ [*00] & [*0*] \\ [*00] & [***] \end{bmatrix} \]

Asterisks denote unrestricted elements. Because \( \Xi B \) has rank 1, the new zero columns represent two independent restrictions only. A third
Wherever possible we see that the processes, it is easy to understand the subject matter theory. The robustness of the results with respect to the ordering of variables if no particular ordering is suggested by the literature. Occasionally, interest centers on the accumulated effects of the impulses. They are easily obtained over all periods. The total long-run effects are given by

\[ \phi_s = \Sigma \phi_s = (I_s - A_s - \ldots A_s)^{-1} \]

This matrix exists if the VAR process is stable.

Lutkepohl (2004) criticizes the forecast error impulse response method in that the underlying shocks are not likely to occur in isolation if the components of \( \mu \) are instantaneously correlated. Therefore, orthogonal innovations are preferred in an impulse response analysis. One way to get them is to use a Choleski decomposition of the covariance matrix \( \Sigma \). If \( B \) is a lower triangular matrix such that \( \Sigma = B' \mu \), we obtain the following:

\[ y_t = \phi_0 e_t + \phi_1 e_{t-1} + \ldots, \]

Sims (1981) recommends trying various triangular orthogonalizations and checking the robustness of the results with respect to the ordering of the variables if no particular ordering is suggested by subject matter theory.

f) Impulse responses analysis of nonstationary VAR’s and VECM’s

Although the Wold representation does not exist for nonstationary cointegrated processes, it is easy to see that the \( \phi \) impulse response matrices can be computed in the same way based on VAR’s with integrated variables or the levels version of a VECM as proposed by Lutkepohl (1991) and Lutkepohl & Reimers (1992). In this case, the \( \phi \) may not converge to zero as \( S \rightarrow \infty \); consequently, some shocks may have permanent effects. Of course, one may also consider orthogonalized or accumulated responses. However, assumes this model.

\[ V_t = B_1 \mu_{t-1} + \ldots + B_h \mu_{h-1} + \text{error}_t \]

For the purpose of this model the VECM form is as follows:

\[ \Delta y_t = a_0 \bar{y}_t \Delta y_{t-1} + \ldots + \Delta y_{t-h} + \mu_t \]

From Johansen’s (1998a) version of Granger’s Representation Theorem it is known that if \( y \) is generated by a reduced form VECM

\[ \Delta y_t = a_0 \bar{y}_t \Delta y_{t-1} + \ldots + \Delta y_{t-h} + \mu_t \]

it has the following MA representation

\[ y_t = \Xi \sum \mu_i + \Xi^*(x) \mu_i + y^*_0 \]

VI. Tests for Nonnormality

Given the residuals \( \mu_t \) (\( t = 1, \ldots, T \)) of an estimated VECM process, the residual covariance matrix is therefore estimated as

\[ \Sigma^\mu = \Xi^\top \left( \mu_t - \mu^\top \right) \left( \mu_t - \mu^\top \right) \]

and the square matrix \( \Sigma'^{1/2} \) is computed.

The standardization of the residuals used here was proposed by Doornik & Hansen (1994) and Lutkepohl (1991). An alternative way of standardization is based on a Choleski decomposition of the residual covariance matrix.

Refer to the appendix (table 6) for tests for nonnormality, which include the Chow tests, Portmanteau test, LM-type test for autocorrelation, Jarque & Bara test, Multivariate ARCH-LM test, the Kernal Density Estimation and the CUSUM test.

a) Forecasting VECM Processes

Once an adequate model for the data gathering process of a system of variables has been constructed, it may be used for forecasting as well as economic analysis. The concept of Granger-causality, which is based on forecast performance, has received considerable attention in the theoretical and empirical literature. Granger (1969) introduced a causality concept whereby he defines a variable \( y_2 \) to be causal for a time series variable \( y_1 \) if the former helps to improve the forecasts of the latter.

In Table 5 the test for Granger-Causality reveals none of the \( p \)-values are smaller than 0.05. Therefore, using a 5% significance level, the null hypothesis of noncausality cannot be rejected. However, in the test for instantaneous causality there is weak evidence of a Granger-causality relation from US productivity differentials \( \rightarrow \) dollar/euro exchange rate because the \( p \)-value of the related test is at least less than 10%.

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This procedure can be used if the cointegration properties of the system are unknown. If it is known that all variables are at most I(1), an extra lag may simply be added and the test may be performed on the lag-augmented model. Park & Phillips (1989) and Sims et al (1990) argue that the procedure remains valid if an intercept or other deterministic terms are included in the VAR model. Forecasting vector processes is completely analogous to forecasting univariate processes. It is assumed the parameters are known.

The identification of shocks using restrictions on their long-run effects are popular. In many cases, economic theory suggests that the effects of some shocks are zero in the long-run. Therefore, the shocks have transitory effects with respect to some variables. Such assumptions give rise to nonlinear restrictions on the parameters which may turn be used to identify the structure of the system.

The impulse responses obtained from a structured VECM usually are highly nonlinear functions of the model parameters. This should be considered when drawing inferences related to the impulse responses.

\[b) \text{ Estimation of Structural Parameters}\]

Following the procedure recommended by Lutkepohl (2004), the estimation of the SVAR model is equivalent to the problem of estimating a simultaneous equation model with covariance restrictions. First, consider a model without restrictions on the long-run effects of the shocks. It is assumed that \(\varepsilon_t\) is white noise with \(\varepsilon_t \sim N(0, I_k)\) and the basic model is a VAR; thus the structural form is

\[A_y = A[A_1, \ldots, A_p] Y_{t-1} + B_1\]

The concentrated log-likelihood is as follows:

\[l_c(A, B) = \text{constant} + T/2 \log |A|^2 - T/2 \log \{B\} - T/2 m (A'B^{-1} A\Sigma)^{-1}\]

(24)

where \(\Sigma_\mu = \tau^{-1}(Y - A'Z)(Y - AZ)\) is just the estimated covariance matrix of the VAR residuals as argued by Breitung (2001).

Lutkepohl (2004) recommends that continuation of the algorithm stops when some prespecified criterion are met. An example would be a relative change in the log-likelihood and the relative change of the parameters. The resulting ML estimator is asymptotically efficient and normally distributed, where the asymptotic covariance matrix is estimated by the inverse of the information matrix. Moreover, the ML estimator for \(\Sigma_\mu\) is

\[\Sigma_\mu = A^{-1} B^{-1} A\Sigma A^{-1}\]

(25)

can be used for estimating the structural parameters A and B. If no restrictions are imposed on the short-run parameters, the \(\Sigma_\mu\) matrix represents the residual covariance matrix obtained from a reduced rank regression. If the short-run parameters are restricted or restrictions are placed on the cointegration vectors, some other estimator may be used instead of the ML estimator, and \(\Sigma_\mu\) may be estimated from the corresponding residuals.

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(23)

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\[l_c(A, B) = \text{constant} + T/2 \log |A|^2 - T/2 \log \{B\} - T/2 m (A'B^{-1} A\Sigma)^{-1}\]

(24)

Where \(A^{-1}\) and \(B^{-1}\) are estimators of A and B, respectively. Note that \(\Sigma_\mu\) only corresponds to the reduced-form estimate \(\Sigma_\mu\) if the SVAR is exactly identified. In the presence of over-identifying restrictions, an LR test statistic for these restrictions can be constructed in the usual way as

\[LR = T(\log I|\Sigma_\mu| - \log I|\Sigma_\mu|)\]

(26)

For VECM’S the concentrated likelihood function

\[l_c(A, B) = \text{constant} + T/2 \log |A|^2 - T/2 \log \{B\} - T/2 m (A'B^{-1} A\Sigma)^{-1}\]

(27)

Generally, if long-run identifying restrictions have to be considered, maximization of the above formula is a numerically difficult task because these restrictions are typically highly nonlinear for A, B, or both. In some cases, however, it is possible to express these long-run restrictions as linear restrictions, and maximization can be done using the scoring algorithm defined above. When considering a cointegrated VECM where \(A = I_k\) it follows that the restrictions on the system variables can then be written in implicit form as

\[\text{Table 5}\]

\[\text{TEST FOR GRANGER-CAUSALITY: } H_0: \text{"US_PROD Differentials" do not Granger-cause "US_EURO}\]

Test statistic: \(l = 0.9604\) \(pval-F(l; 1, 86) = 0.3298\)

\[\text{TEST FOR INSTANTANEOUS CAUSALITY:}\]

\(H_0: \text{No instantaneous causality between "US_PROD Differentials" and "US_EURO"}\)

Test statistic: \(c = 3.3221\) \(pval-Chi(c; 1) = 0.0684\)
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Where $R \Xi$ is an appropriate restriction matrix. Following the suggestions of Vlaar (1998) we can reformulate these restrictions as

$$R \Xi (l_0 \, \hat{\Xi} \, vec(B) = R \Xi vec(\Xi B) = 0 \quad (29)$$

Replacing $\Xi$ by an estimator obtained from the reduced form we obtain $R B, l = R \Xi (l_0 \, \hat{\Xi})$, which is a stochastic restriction matrix. These implicit restrictions can be derived. Here $t^{y/2}$ and $t^{1-y/2}$ are the $y/2$ and $(1 - y/2)$ equations, respectively, of the empirical distribution of $(\hat{\omega} \cdot - \bar{\omega})$

c) Impulse Responses

Figures 13-17 display the impulse responses of the dollar/euro exchange rate to a one standard deviation change in the US productivity, M2, oil prices, and government spending. The responses are significant at the 95% level. Table 8 (in the appendix) displays the point estimates of the impulse responses of the real exchange rate to the one-standard deviation US productivity shocks. Also note that the results are relatively robust with the individual impulse responses falling within the 5% significant tests. Figure 13 shows that for the exchange rate these shocks have a highly significant impact over the 10-year time period and the correlation between these impulse responses is high. They show that productivity shocks have a very significant long-run impact on the dollar/euro exchange rate. The results follow those of Clarida and Galf (1992). The point estimates in table 8 show that for each percentage point in the US-Euro area productivity differential there is a three percentage point real change in the dollar/euro valuation. This suggests that fundamental real factors are significant in the long-run fluctuations in real exchange rates.

Refer to the appendix (figures 31-44) for the US and Euro productivity differentials. Figure 31 shows the long-run impact of productivity shocks on the dollar/euro real exchange rate. Figure 35 shows the significance of large gaps in the euro and US productivity differentials especially around the years 2000-2001 when the dollar started to depreciate against the euro.

Figure 13: US Productivity $\rightarrow$ US/EURO Exchange Rate.

![Figure 13](image1)

Figure 14: Government Spending $\rightarrow$ US/EURO Exchange Rate

![Figure 14](image2)
d) Forecast error variance decomposition

Forecast error variance decomposition is a special way of summarizing impulse responses. Following Lutkepohl (2004) the forecast error variance decomposition is based on the orthogonalized impulse responses for which the order of the variables matters. Although the instantaneous residual correlation is small in our subset VECM, it will have some impact on the outcome of a forecast error variance decomposition.

Lutkepohl (2004) suggests the forecast error variance as

$$
\hat{\sigma}^2_k(h) = \sum(\hat{\Psi}_{kl,n}^2 + \ldots + \hat{\Psi}_{kn}^2) = \Psi_{kjo}^2 + \ldots \Psi_{kh-1}^2
$$

(30)

The term \(\hat{\Psi}_{kn}^2 + \ldots \hat{\Psi}_{kn}^2\) is interpreted as the contribution of variable j to the h-step forecast error variance of variables k. This interpretation makes sense if the disturbance terms can be viewed as shocks in variable i.

Dividing the preceding by \(\hat{\sigma}^2_k(h)\) gives the percentage contribution of variable j to the h-step forecast error of variable h.

$$
\frac{\hat{\Psi}_{kjo}^2 + \ldots \Psi_{kh-1}^2}{\hat{\sigma}^2_k(h)}
$$

(31)

Chart 1 shows the proportion of forecast error in the dollar/euro accounted for by US productivity, government spending, M2, oil prices and US GDP. The US productivity accounts for 28% over the 20 year time interval with a sharp rise of 21% during the first 5 years. This shows that productivity shocks have a very significant short-run impact on the dollar/euro exchange rate while the long-run impact is more transitory in nature. Figures 23-26 show the time series forecasts of the system for the years 2007-2011 with 95% forecast intervals indicated by dashed lines. That all observed variables are within the approximately 95% forecast intervals is viewed as an indication of model adequacy for forecasting purposes.
**Chart 1**

VECM FORECAST ERROR VARIANCE DECOMPOSITION
Proportions of forecast error in "bUS_EURO"
accounted for by:

<table>
<thead>
<tr>
<th>forecast horizon</th>
<th>aUS_PROD_B</th>
<th>bUS_EURO</th>
<th>cOil_prices</th>
<th>dm2</th>
<th>g_spend_q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.90</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.11</td>
<td>0.89</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.11</td>
<td>0.89</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
<td>0.88</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>0.87</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>0.13</td>
<td>0.87</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>0.14</td>
<td>0.86</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>0.14</td>
<td>0.85</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
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<td>0.84</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
<td>0.83</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>0.16</td>
<td>0.82</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>12</td>
<td>0.17</td>
<td>0.81</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>13</td>
<td>0.18</td>
<td>0.80</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>14</td>
<td>0.19</td>
<td>0.79</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>15</td>
<td>0.19</td>
<td>0.78</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>16</td>
<td>0.20</td>
<td>0.76</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>17</td>
<td>0.21</td>
<td>0.75</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>18</td>
<td>0.22</td>
<td>0.74</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>19</td>
<td>0.22</td>
<td>0.72</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>20</td>
<td>0.23</td>
<td>0.71</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Figure 19:** Time Series Forecast Oil Prices.

**Figure 20:** Time Series Forecast Gov Spending.
Productivity Growth and Its Influence on the Dollar/Euro Real Exchange Rate

Figure 21: Time Series Forecast Dollar/Euro Exchange Rate

Figure 22: CUSUM statistics for oil_prices equation

Figure 23: CUSUM statistics for q_spend_q equation

Figure 24: CUSUM statistics for US_PROD equation

Figure 25: CUSUM statistics for m2 equation

Figure 26: Time Series Forecast (CI 95.0%) of USD_EURO
V. DISCUSSION OF THE RESULTS

This paper provides evidence on the long-run relationship between the real dollar/euro exchange rate and productivity measures, controlling for the real price of oil, relative government spending and M2. However, the results imply that the productivity measure can explain only about 27% of the actual amount of depreciation of the euro against the US dollar for the period 1995-2001. This outcome is confirmed by a specification in this study. Figure 18 shows that the productivity can explain only about 28% of the appreciation of the euro during the period 1995-2007 (appendix table 6 for point estimate).

Evidently, productivity is not the only variable affecting the real exchange rate in the model specified. The other variables identified also affected the dollar/euro exchange rate. In particular, the surge in oil prices since early 1999 seems to have contributed to the weakening of the euro. The magnitude of the long-run impact of changes in the real price of oil on the dollar/euro exchange rate is certainly significant. Between 1997 and 2001, the model indicates on the average that the equilibrium euro depreciation related to oil prices developments could have been around 20% (refer to table 8 for point estimate and figure 21). These results are based on long-term relationships.

Overall, the model is surrounded by significant uncertainty, reflecting the inherent difficulty of modeling exchange rate behavior. While we find that in 1995-2001 the euro traded well below the central estimates derived from these specifications, this uncertainty precludes any quantification of the precise amount of over or under valuation at any point in time. This point is also made clear by Detken and Dieppe (2002), who employed a wide range of modeling strategies to show that the deviation from the estimated equilibrium differs widely across models and is surrounded by some uncertainty. Moreover, the results provided by Maeso-Fernandez and Osbat (2001) find various reasonable but non-encompassing specifications leading to different exchange rate equilibria. Again, this suggests a very cautious interpretation of the magnitude of over/under valuation.

REFERENCES RÉFÉRENCES REFERENCIAS


Appendix

The data for this study was collected from the following sources:

Economic Data Base (FRED) of the Economic Research Department of the Federal Reserve Bank of St. Louis. The PPI and CPI are used as proxies for tradable and nontradable goods.


Candelon and Lutkepoh (2001) recommended using bootstrap versions for the Chow tests to improve sample properties. The bootstrap is set up with modifications to allow for residual vectors rather than univariate residual series. Table 9 shows the results of a possible break date for 2001 in which the government changed to the euro.

On the basis of the appropriate p-values, the bootstrap findings of the sample-split. Chow tests do not reject stability in the model even with the structural break in 2001.

**Test for Nonnormality**

The following test for residual autocorrelation is known as the Portmanteau test statistic. The null hypothesis of no residual autocorrelation is rejected for large values of $Q_h$ (test statistic). The p-value is relatively large: consequently, the diagnostic tests indicate no problem with the model.

Lomnicki (1961) and Jarque & Bera (1987) propose a test for nonnormality based on the skewness and kurtosis for a distribution. The Jarque & Bera tests in table 9 show some nonnormal residuals for two variables (oil prices and government spending (u4 and u6).

Lutkepohl (2004) states that if nonnormal residuals are found, this is often interpreted as a model defect. However, much of the asymptotic theory on which inference in dynamic models is based works also for certain nonnormal residual distributions. Still, nonnormal residuals can be a consequence of neglected nonlinearities. Modeling such features as well may result in a more satisfactory model with normal residuals. Sometimes, taking into account ARCH effects may help to resolve the problem. With this in mind a multivariate ARCH-LM test was performed. The results shown in Table 10 indicate the p-value is relatively large: consequently, the diagnostic tests indicate no problem with the model.
Table 6

*** Sun, 26 Jul 2009 07:38:32 ***
PORTMANTEAU TEST (H0:Rh=(r1,...,rh)=0)

<table>
<thead>
<tr>
<th>tested order</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>test statistic</td>
<td>419.1197</td>
</tr>
<tr>
<td>p-value</td>
<td>1.0000</td>
</tr>
<tr>
<td>adjusted test statistic</td>
<td>505.9513</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9746</td>
</tr>
<tr>
<td>degrees of freedom</td>
<td>570.0000</td>
</tr>
</tbody>
</table>

*** Sun, 26 Jul 2009 07:38:33 ***
LM-
TYPE TEST FOR AUTOCORRELATION with 5 lags

| LM statistic | 301.5520 |
| p-value      | 0.0000 |
| df           | 180.0000 |

Table 7

*** Sun, 26 Jul 2009 07:10:23 ***
CHOW TEST FOR STRUCTURAL BREAK
On the reliability of Chow-type tests.


| sample range: | [1996 Q3, 2008 Q2], T = 48 |
| tested break date: | 1999 Q4 |
| (13 observations before break) |

| break point Chow test: | 83.7823 |
| bootstrapped p-value: | 0.0000 |
| asymptotic chi^2 p-value: | 0.0000 |
| degrees of freedom: | 27 |

| sample split Chow test: | 9.3234 |
| bootstrapped p-value: | 0.2500 |
| asymptotic chi^2 p-value: | 0.1562 |
| degrees of freedom: | 6 |

| Chow forecast test: | 1.3188 |
| bootstrapped p-value: | 0.0000 |
| asymptotic F p-value: | 0.2388 |
| degrees of freedom: | 210, 20 |

Cusum Tests

The standardization of the residuals used in this model was proposed by Doornik & Hansen (1994) and Lutkepohl (1991). An alternative way of standardization is based on a Choleski decomposition of the residual covariance matrix.

Lutkepohl (2004) recommends checking the time invariance of a model by considering recursively estimated quantities. Plotting the recursive estimates together with their standard or confidence intervals can give useful information on possible structural breaks. The recursive estimates of the model are shown in Figures 27-30. They appear to be somewhat erratic at the sample beginning which would reflect greater uncertainty. However, even when taking this into account one finds that the recursive estimates do not indicate parameter uncertainty. The erratic behavior of the recursive estimates at the beginning could be attributed to the change over to the euro in 2001.

The results of the CUSUM tests of the system with 99% level critical bounds (for sample periods 1985-2007) also indicate that government spending, GDP, US productivity, oil prices and M2 recursive estimates are all outside the critical bounds for the CUSUM statistics. This would suggest some stability problems even though they are only outside the critical bounds for the years of 2005-2008. They are all well within the uncritical region for the years up to 2005. For VECMs with cointegrating variables, Hansen & Johansen (1999) have recommended recursive statistics for stability analysis. Figure 35 shows the eigenvalue with a cointegrating rank r =1. The recursive eigenvalue
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appears to be within the 95% confidence intervals. Also, the tau statistic $T(\phi)$ is plotted in Figure 36 and the results indicate that the eigenvalue is stable. Therefore, there is no indication of instability of the system.

Table 8

*** Mon, 2 Nov 2009 11:22:23 ***
VECM Orthogonal Impulse Responses

Selected Confidence Interval (CI):
a) 95% Hall Percentile CI (B=100 h=20)

Selected Impulse Responses: "impulse variable -> response variable"

<table>
<thead>
<tr>
<th>time</th>
<th>aUS_PROD_B -&gt; bUS_EURO</th>
</tr>
</thead>
<tbody>
<tr>
<td>point estimate</td>
<td>-0.0174 CI a) [-0.0310, -0.0021]</td>
</tr>
<tr>
<td>1 point estimate</td>
<td>-0.0185 CI a) [-0.0336, -0.0037]</td>
</tr>
<tr>
<td>2 point estimate</td>
<td>-0.0197 CI a) [-0.0356, -0.0040]</td>
</tr>
<tr>
<td>3 point estimate</td>
<td>-0.0209 CI a) [-0.0381, -0.0044]</td>
</tr>
<tr>
<td>4 point estimate</td>
<td>-0.0221 CI a) [-0.0412, -0.0041]</td>
</tr>
<tr>
<td>5 point estimate</td>
<td>-0.0234 CI a) [-0.0446, -0.0035]</td>
</tr>
<tr>
<td>6 point estimate</td>
<td>-0.0248 CI a) [-0.0482, -0.0027]</td>
</tr>
<tr>
<td>7 point estimate</td>
<td>-0.0263 CI a) [-0.0519, -0.0029]</td>
</tr>
<tr>
<td>8 point estimate</td>
<td>-0.0278</td>
</tr>
</tbody>
</table>

Figure 29

Figure 30

(No: this eigenvalue is stable) Tau statistic for eigenvalue 1
Productivity Growth and Its Influence on the Dollar/Euro Real Exchange Rate

Figure 31: Time Series Euro Productivity and US Productivity Dollar/Euro Real Exchange Rate.

Figure 32: Time Series Euro Traded and Nontraded Goods.

Figure 33: Time Series US Productivity Differentials.

Figure 34: US Traded Goods US Nontraded Goods.

Figure 35: Time Series Euro and US Productivity Differentials.
Figure 36: Time Series US PPI and CPI.

Figure 37: Time Series Euro Productivity Differentials.

Figure 38: Time Series Euro PPI and CPI.

Figure 40: Euro Productivity US Productivity.

Figure 39: US PPI and CPI.

Figure 41: Time Series Euro and US Productivity Differentials.
Figure 42: Time Series US PPI and CPI (Index).

Figure 43: Time Series Euro Productivity Differentials.

Figure 44: Time Series Euro PPI and CPI.