Global Journal of Management and Business Research: C
Finance
Volume 14 Issue 5 Version 1.0 Year 2014
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4588 & Print ISSN: 0975-5853

Estimating the Volatility of Brazilian Equities Using Garch-Type Models and High-Frequency Volatility Measures

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Abstract- Financial markets require an accurate estimate of asset volatility for various purposes such as risk management, decision-making and portfolio selection. Moreover, for risk management, volatility estimation is critical in Value-at-Risk (VaR) calculation models. However, there is still no consensus on a model that performs best in estimating volatility. This study proposes comparing volatility measures based on high-frequency data, such as RV and RRV, with heteroskedastic volatility models that use squared daily returns and daily closing prices. Four GARCH type models were implemented to estimate heteroskedastic volatility for the two most actively traded shares on the Brazilian stock exchange, using skewed generalized t (SGT) distribution and allowing flexibility for modeling the empirical distribution of these asymmetric financial data.

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GJMBR-C Classification: *JEL Code: E60*

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Keywords: volatility; garch-type models; high-frequency volatility measures; value at risk.

I. Introduction

Financial markets require an accurate estimate of asset volatility for various purposes such as decision-making and portfolio selection. Moreover, for risk management, volatility estimation is critical in Value-at-Risk (VaR) calculation models.

According to Liu, Chiang and Cheng (2012), the debate on estimating volatility is intense and has been frequently explored in various academic studies. However, there is still no consensus on a model that performs best in estimating volatility. This may be explained by a failure to correctly specify true volatility.

A common practice, although one that has been questioned, is the use of squared daily returns as the most appropriate measure of true volatility. Studies like those of Andersen and Bollerslev (1998), McMilan and Speight (2004), and Angelidis and Degiannakis (2008) suggest that realized volatility (RV), which is based on squared intra-day returns, would be a more appropriate measure of true volatility.

Other empirical studies, like that of Garman and Klass (1980), suggest an alternative volatility estimator derived from the highest and lowest trading prices of each intra-day interval as well as the opening and closing prices. Martens and van Dijk (2007) adapted this concept. They proposed the use of squared returns for each intra-day period, considering the highest and lowest price of the period, with the aim of creating an estimator based on the realized range volatility (RRV), which they claim is more efficient than the RV in an ideal world.

The positioning of models in exercises comparing their performance in volatility forecasting has been highly dependent on each model's degree of measurement. Most studies of this type consider a single measure of volatility, which may result in a faulty evaluation of model performance. This suggests that there is a need for research evaluating the accuracy of estimates from several adaptations of GARCH models, using not only the RV, but also the RRV as measures of volatility.

This study proposes comparing volatility measures based on high-frequency data, such as RV and RRV, with heteroskedastic volatility models that use squared daily returns and daily closing prices. Among the models used to estimate heteroskedastic volatility are the GARCH (symmetric), EGARCH (asymmetric), CGARCH (long memory), and TGARCH (threshold-asymmetric) models.

The article is organized as follows: (i) a brief literature review will be presented in section 2; (ii) section 3 describes the methodology and the model estimates; (iii) the data used to estimate the RV and the RRV will be described in section 4; (iv) the results obtained will be presented in section 5; and (v) section 6 discusses the study's conclusions.

II. Brief Literature Review

Based on the theory that the measure of volatility converts to a genuine measure of latent volatility when the frequency of observations increases to an infinitesimal interval, Andersen and Bollerslev (1998) proposed using RV as a measure of intra-day volatility. After checking measures of regression errors and the coefficient of determination (R2), using different interval volatility measures, the authors concluded that intra-day

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volatility measures improved the measurement of latent volatility.

Martens and van Dijk (2007) adapted RV when they considered the square of daily returns using the highest and lowest price of each daily interval, thus creating the RRV. The authors conducted an empirical analysis of the Standard and Poor's (S&P) 500 and S&P 100 indexes to confirm the RRV's potential, and concluded, through simulations, that the RRV presented a mean squared error that was less than that of the RV.

Both RV and RRV are alternative means to measure the volatility of assets. Various studies have used these alternative measures to analyze the performance of volatility forecasting models.

Hsieh (1991) presented one of the first estimates of daily returns using 15-minute interval intra-day returns from the S&P 500 index. The research was informal in the sense that there was no association with the concept of the quadratic variation.

Andersen and Benzoni (2008) also addressed the concept of RV and its possible applications. The authors identified four areas of related research: (i) volatility forecasting, with emphasis on research focused on improving the performance of such forecasting, in literature related to detecting jumps and in research on problems related to the microstructure in forecast performance; (ii) implications for the distribution of returns for the no-arbitrage condition; (iii) multivariate measures of the quadratic variation; and (iv) realized volatility, specification, and the estimation of models.

Considering the research areas highlighted by Andersen and Benzoni (2008), this article can be classified among the first research area, since its aim is to evaluate improved performance in volatility forecasting by using RV and RRV measures.

The literature discussed below are classified and also relevant in this research area.

Andersen et al. (2003) created a framework for integrating high-frequency data in the measurement, modeling and projection of volatility, and the distributions of returns. Based on the theory of the arbitrage-free process and the theory of quadratic variation, the authors made a correlation between realized volatility and the conditional covariance matrix. In the study, the authors used data based on the German mark/dollar and the Japanese yen/dollar exchange rates.

Andersen et al. (2005) developed a model with adjustment procedures to calculate unbiased volatility based on realized volatility. According to these authors, the procedures are easy to implement and highly accurate in empirical situations.

Martens and van Dijk (2007) proposed creating a new indicator, RRV, based on changes in RV. The study was conducted using an empirical analysis of the S&P 500 and S&P 100 indexes. The authors concluded that the RRV was a better measure of volatility than the RV when the same sample was used.

Maheu and McCurdy (2011) proposed a bivariate model of returns and RV and explored which characteristics of temporal series models contributed to density forecasts for horizons of one to 60 days out of sample. This forecast structure was used to investigate the importance of intra-day information incorporated in the RV, the functional form for the dynamic log (RV), the time of information availability, and the distribution assumed for both the returns and the log. The study used data from the S&P 500 stock index and IBM shares.

Liu et al. (2012) compared the performance of GARCH-type models using the RV and the RRV of the S&P 500 stock index as volatility measures. Furthermore, the authors calculated the VaR for each model analyzed.

Dufour et al. (2012) provided evidence for two alternative mechanisms of interaction between returns and volatility: the effect of leverage and the effect of volatility. The authors emphasized the importance of distinguishing between realized volatility and implied volatility, and concluded that implied volatility is essential to evaluating the effect of volatility. Moreover, they introduced the concept of variance risk premium, which is equal to the difference between implied volatility and realized volatility, and concluded that a positive variance risk premium has more impact on returns than a negative one.

Zhang and Hu (2012) examined whether RV can provide additional information about the volatility process for the GARCH and EGARCH models, using data from the Chinese stock market. The authors concluded that RV adds information to the volatility process for some shares, but adds no additional information for a significant number of shares as well. The RV calculated for 30-minute intervals outperformed the measures taken at other intervals. The size of the company, the turnover rate, and amplitude partially explained the difference in the RV's explanatory power among companies. Although the authors concluded that there were doubts about the RV's additional information, they argued that the implied volatility was, at the least, the same information offered by the RV.

Vortelinos and Thomakos (2013) used daily, high-frequency data to test and model seven new volatility estimators for six international stock indexes. The authors concluded that the selection of the realized volatility estimator has a significant impact on the detection of jumps, magnitude, and modeling. The elements that each estimator is intended to incorporate affect the detection, magnitude, and properties of the jumps.

III. Methodology

The aim of this article is to compare the volatility estimated by the GARCH, EGARCH, CGARCH, and TGARCH models with the RV and RRV volatility measures, evaluating the performance of the models in implementing VaR for the Petrobras (PETR4) and Vale (VALE5) shares.

The models were estimated incorporating skewed generalized t (SGT) distribution, allowing flexibility for modeling the empirical distribution of asymmetric financial data with fat tails and leptokurtosis for the daily and weekly volatility estimates of the preferred shares of Petrobras and Vale. These two companies have the most traded shares on the Brazilian stock exchange. The buy options for these companies together represent more than 90% of the volume of options traded in the Brazilian market.

a) Estimated models

Bollerslev's (1986) symmetric GARCH(1,1) model is given by:

$$h^2_t = \omega + \alpha\varepsilon^2_{t-1} + \beta h^2_{t-1} \tag{3.1}$$

This model implies high volatility persistence. The impact of past information on forecasting future volatility decreases very slowly.The EGARCH model, proposed by Nelson (1991), is a GARCH-type model able to handle asymmetric volatility in response to asymmetric shocks, expressed by:

$$\ln\left(h^2_t\right) = \omega + \alpha\left(\nu\varepsilon_{t-1}/h_{t-1} + \left|\varepsilon_{t-1}\right|/h_{t-1} - \sqrt{2/\pi}\right) + \beta\ln\left(h^2_{t-1}\right) \tag{3.2}$$

The coefficient v captures the asymmetric impacts of new information, with the negative shocks having a greater impact than the positive shocks with the same magnitude of v < 0; the effect of volatility clustering is captured by a significant **α**.

The primary objective of the CGARCH model of Engle and Lee (1999) is to separate the permanent (or long-term) and transitory (or short-term) components of the effects of volatility with the following specifications:

$$h^2_t = q_t + \alpha\left(\varepsilon^2_{t-1} - q_{t-1}\right) + \beta\left(h^2_{t-1} - q_{t-1}\right) \tag{3.3}$$

$$q_t = \omega + \tau q_{t-1} + \phi\left(\varepsilon^2_{t-1} - h^2_{t-1}\right) \tag{3.4}$$

Here q represents the long-term volatility (or tendency); the estimation error serves as a driving force behind the movement of the trend dependent on time; and the difference between the conditional variance and its tendency is the transitory component of conditional variance.

Based on the study by Engle (1982), errors are assumed to be normally distributed. Thus, for the empirical distribution of the series of returns exhibiting fat tails, leptokurtosis, and asymmetry, this article uses the SGT distribution for the errors proposed by Theodossiou (1998) as follows:

$$f\left(z_t;N;\kappa,\lambda\right) = C\left(1 + \frac{\left|z_t+\delta\right|^k}{\left(\left(N+1\right)/k\right)\left(1+sign\left(z_t+\delta\right)\lambda\right)^k\theta^k}\right)^{-(N+1)/k} \tag{3.5}$$

where:

$$C = 0.5\kappa.\left(\frac{N+1}{\kappa}\right)^{-1/\kappa}.B\left(\frac{N}{\kappa},\frac{1}{\kappa}\right)^{-1}.\theta^{-1} \tag{3.6}$$

$$\theta = \left(g-\rho^2\right)^{\frac{-1}{2}}$$

$$\rho = 2\lambda.B\left(\frac{N}{\kappa},\frac{1}{\kappa}\right)^{-1}.\left(\frac{N+1}{\kappa}\right)^{\frac{1}{\kappa}}.B\left(\frac{N-1}{\kappa},\frac{2}{\kappa}\right) \tag{3.7}$$

(3.8)

$$g = \left(1+3\lambda^2\right).B\left(\frac{N}{\kappa},\frac{1}{\kappa}\right)^{-1}.\left(\frac{N+1}{\kappa}\right)^{\frac{2}{\kappa}}.B\left(\frac{N-2}{\kappa},\frac{3}{\kappa}\right) \tag{3.9}$$

$$\delta = \rho.\theta \tag{3.10}$$

The parameter θ is obtained through the quasi-maximum likelihood (QMLE) method, as suggested by Bollerslev and Wooldridge (1992), maximizing the following function:

$$LL(\theta) = \sum_{t=1}^{T} \ln f(\theta) \tag{3.11}$$

The TGARCH model captures the asymmetry of the volatility:

$$\zeta_t^{\,2} = \omega + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right) + \sum_{j=1}^{q} \beta^j \zeta_{t-j} \tag{3.12}$$

b) *Volatility measures based on intra-day returns and intervals*

To compare the forecasting ability of each model, we consider two volatility measures: RV, as proposed by Andersen and Bollerslev (1998); and RRV, introduced by Martens and van Dijk (2007).

Andersen and Bollerslev (1998) define RV as the sum of the squared returns of five-minute intra-day intervals, as follows:

$$\hat{\sigma}^2_{RV,t} = \sum_{d=1}^{D} \left[100x \left(\ln\left(P_{t,d}\right) - \ln\left(P_{t,d-1}\right) \right) \right]^2 \tag{3.13}$$

Here P (t,d) is the price of the asset at time d in five-minute intervals observed during trading day t.

Martens and van Dijk (2007) substituted each squared intra-day return for the interval's highest and lowest prices, creating the RRV:

$$\hat{\sigma}^2_{RRV,t} = \frac{1}{4\ln 2} \sum_{d=1}^{D} \left[100x \left(\ln\left(H_{t,d}\right) - \ln\left(L_{t,d}\right) \right) \right]^2 \tag{3.14}$$

where H (t,d) and L(t,d) denote the asset's highest and lowest prices observed during a period of five minutes on day t.

c) *Evaluating the performance of volatility forecasting*

The three popular statistical functions Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and Logarithmic Loss Error (LLE) were employed to evaluate the accuracy of the competing models in forecasting volatility for daily and weekly horizons. These metrics are expressed below:

$$RMSE_k = \left[\frac{1}{T} \sum_{n=1}^{T} \left(h^2{}_{n,k} - \hat{\sigma}_n^2 \right)^2 \right]^{\frac{1}{2}} \tag{3.15}$$

$$MAPE_k = \frac{1}{T} \sum_{n=1}^{T} \left| \frac{\left(h^2 - \sigma^2 \right)}{\sigma^2} \right| \tag{3.16}$$

$$LLE_k = \frac{1}{T} \sum_{n=1}^{T} \left[\ln\left(h^2{}_{n,k} \right) - \ln\left(\hat{\sigma}_n^2 \right) \right]^2 \tag{3.17}$$

In practice, each market participant gives a different importance to overestimation and underestimation. For this reason, it is best to use the mean error (MME) statistic, as it allows potential asymmetry in the loss function (Liu et al., 2012).

UP (n,k) is defined as the potential loss from underestimation generated by model k for day n, and OP (n,k) as the potential loss from overestimation, as follows:

$$UP_{n,k} = \begin{cases} \left|\hat{\sigma}_n^2 - h^2{}_{n,k}\right| & \text{if} \quad \sigma^2{}_n - h^2{}_{n,k} \le 0 \\ \left(\hat{\sigma}_n^2 - h^2{}_{n,k}\right)^{0.5} & \text{if} \quad 0 < \sigma^2{}_n - h^2{}_{n,k} \le 1 \\ \left(\hat{\sigma}_n^2 - h^2{}_{n,k}\right)^2 & \text{if} \quad \sigma^2{}_n - h^2{}_{n,k} > 1 \end{cases} \tag{3.18}$$

$$OP_{n,k} = \begin{cases} \left|\hat{\sigma}_n^2 - h^2{}_{n,k}\right| & \text{if} \quad \sigma^2{}_n - h^2{}_{n,k} \ge 0 \\ \left(\hat{\sigma}_n^2 - h^2{}_{n,k}\right)^{0.5} & \text{if} \quad -1 \le \sigma^2{}_n - h^2{}_{n,k} < 0 \\ \left(\hat{\sigma}_n^2 - h^2{}_{n,k}\right)^2 & \text{if} \quad \sigma^2{}_n - h^2{}_{n,k} < -1 \end{cases} \tag{3.19}$$

The MME for volatility model k that harshly penalizes underestimation, MME(U)k, as well as overestimation, MME(O)k, are expressed as follows:

$$MME(U)_k = \frac{1}{T}\sum_{n=1}^{T} UP_{n,k} \tag{3.20}$$

$$MME(O)_k = \frac{1}{T}\sum_{n=1}^{T} OP_{n,k} \tag{3.21}$$

Value at Risk - VaR

The VaR estimate based on the GARCH model for one and five days is calculated according to the following formula:

$$Var_n^k(1;\alpha_1) = \mu + F(Z_n;\alpha_1).h_{n,k} \tag{3.22}$$

$$Var_n^k(5;\alpha_1) = 5\mu + F(Z_n;\alpha_1).h_{n,k}.\sqrt{5} \tag{3.23}$$

Here F (Z;α) corresponds to the quantile of the SGT distribution (99° or 99.5°) with specific parameters (N, κ and λ) and h(n,k) is the square root of the estimate of the conditional variance generated by the model k, calculated in time n.

In this study, with the aim of back-testing the VaR result, we employed the likelihood ratio test developed by Kupiec (1995), LR (uc), to determine whether the actual loss probability is statistically consistent with the theoretical probability given by the VaR model. The null hypothesis of the loss probability, p, is tested against the alternative hypothesis that the loss probability differs from p. The test uses the following formula:

$$LR_{uc} = -2\ln\left[\frac{p^{n1}(1-p)^{n0}}{\hat{\pi}^{n1}(1-\hat{\pi})^{n0}}\right] \sim \chi^2 \tag{3.24}$$

where $\pi = n^1/(n^0 + n^1)$ is the maximum likelihood estimate of p, and n is a Bernoulli random variable representing the number of times that the realized loss in Brazilian reals exceeds the estimated VaR for the period beyond the sample.

The conditional coverage test (LRcc), developed by Christoffersen (1998), jointly investigates whether the number of losses is equal to the expected number, and if the loss process of the VaR exceptions displays serial independence.

Initially, an indicator (It) should be defined with a value equal to one if a violation occurred, and equal to zero if a violation did not occur. This indicator is used for determining the variable n, as in the table below:

	$I_{t-1}=0$	$I_{t-1}=1$	
$I_t=0$	n_{00}	n_{10}	$n_{00}+n_{10}$
$I_t=1$	n_{01}	n_{11}	$n_{01}+n_{11}$
	$n_{00}+n_{01}$	$n_{10}+n_{11}$	N

π, in turn, represents the probability of observing a conditional exception the previous day:

$$\pi = \frac{n_{01}}{n_{00}+n_{11}}, \quad \pi_1 = \frac{n_{11}}{n_{10}+n_{11}} \text{ and } \pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} \tag{3.25}$$

By the null hypothesis of an independent loss process with loss probability, p, against the alternative hypothesis of a Markov loss process with a different probability transition matrix, the statistical test of the likelihood ratio is expressed as:

$$LR_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\ \pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\ \pi_0^{n_{01}}\ (1-\pi_1)^{n_{10}}\ \pi_1^{n_{11}}}\right) \tag{3.26}$$

According to Nieppola (2009), the Kupiec and Christoffersen tests are combined to test the actual loss rate and the independence of the exceptions; the test is as follows:

$$LR_{cc} = LR_{POF} + LR_{ind} \tag{3.27}$$

IV. Preliminary Data and Analysis

This study uses tick-by-tick trading prices of the PETR4 and VALE5 shares. The data was supplied by BM&FBOVESPA and covers the period between July 1, 2011 and August 31, 2013.

For each trading day, we selected trades that took place between 10:05 am and 4:54 pm, in order to exclude the auction period. The trades selected were classified into five-minute intervals. Thus, for each trading day, we set 84 intervals and for each interval we highlighted the highest, lowest, and last values traded to calculate the RV, RRV, and return. As a final result, for each trading day, there was one RV, one RRV, and one return.

To estimate the models, we calculated the returns, considering the first and last trades of the day, excluding the auction trades, as follows:

$$R_t = 100 \text{ x } \ln\left(\frac{P_t}{P_0}\right) \tag{3.28}$$

The returns were calculated in this way to avoid any inconsistency with the RV and RRV calculations, which were calculated considering only the prices of the referenced trading day.

Table 1 shows the descriptive statistics for the daily estimated RV and RRV of PETR4 and VALE5, using five-minute intervals. The results show that distributions of both shares are asymmetric on the right and exhibit fatter tails than those in a normal distribution.

Table 1 : Descriptive Statistics of the RV and RRV of PETR4 and VALE5

	RV Petro	RV Vale	RRV Petro	RRV Vale
Mean	1,63	1,42	1,64	1,41
Median	1,55	1,35	1,57	1,35
Maximum	4,82	7,20	4,09	4,53
Minimum	0,60	0,51	0,68	0,53

Std. Dev.	0,51	0,49	0,43	0,42
Skewness	1,72	3,85	1,35	2,27
Kurtosis	9,40	39,98	6,95	14,77
Jarque-Bera	1.180,85	31.918,95	511,81	3.558,94
Probability	0,00	0,00	0,00	0,00
Sum	873,41	764,12	878,26	759,78
Sum Sq. Dev.	139,95	128,63	97,50	93,13
Observations	537	537	537	537

Figure 1 shows the RV and RRV series of Petrobras and Vale over the period analyzed.

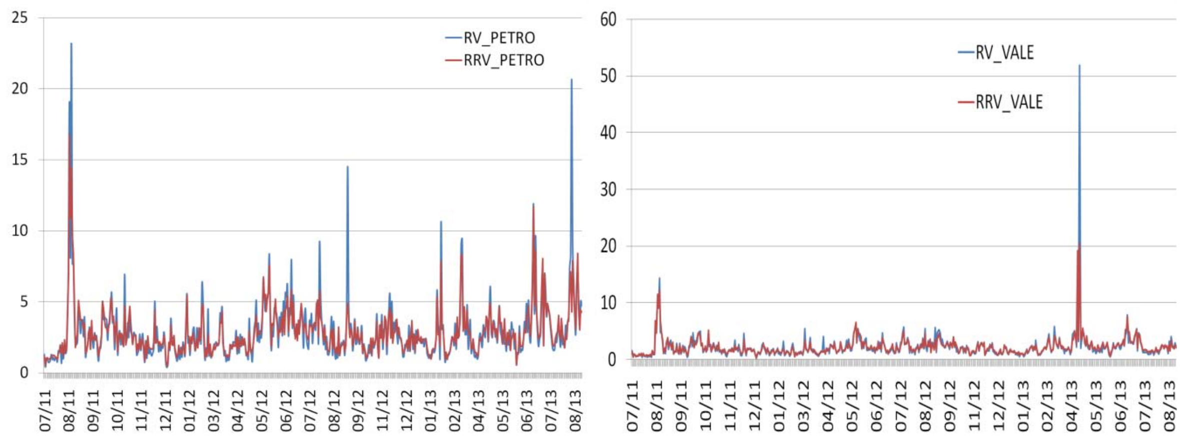

Figure 1 : The RV and RRV Series of Petrobras and Vale

V. Empirical Results

In this section, we present the results for the estimated models. From a sample of 537 observations, the last 165 were considered out of sample, i.e., they were not considered for estimating the parameters.

Table 2 shows the model estimates for the Petrobras shares. With the exception of the TGARCH model, all the conditional mean parameters are not statistically significant. The conditional variance is significant at a level of 90% for all the models.

Parameter β of the GARCH model is close to one and is significant at a level of 1%, which implies a high degree of volatility persistence.

The asymmetry parameter (ν) of the EGARCH model is positive and significantly different from zero at a level of 1%, indicating that negative shocks have a greater impact on volatility than positive shocks.

The sum of parameters α and β of the CGARCH model is less than the sum of the same parameters of the GARCH model, indicating that the short-term volatility component is not strong.

The long-term volatility component (τ) of the CGARCH model is equal to 0.95, indicating that the permanent component of the conditional variance shows that there is strong volatility persistence.

The Akaike Information Criterion (AIC) and the Log Likelihood, although very close for all the models, indicate that the TGARCH model suits the data most effectively.

Table 2 : Estimates of the Models- Petrobras

Parameters/Models	GARCH	EGARCH	CGARCH	TGARCH
μ	-0,08	-0,08	-0,1	-0,11**
ω	0,16***	0,08**	3,09*	0,11*
α	0,02	-0,01**	0,15*	-0,07*
β	0,93*	-0,05***	0,55**	0,07*
ν	-	0,93*	-	1*
τ	-	-	0,95*	-
Φ	-	-	-0,01	-
Log Likelihood	-723,04	-721,25	-1042,96	-718,77
Akaike	3,91	3,91	3,91	3,90

*, ** e *** indicate rejection of the null hypothesis at a significance level of 1%, 5% and 10%, respectively.

Table 3 presents the model estimates for the Vale shares. The conditional mean parameters of all the models are not statistically significant. The conditional variance is significant at a level of 95% for the CGARCH and TGARCH models.

Parameter β of the CGARCH model is close to one and is significant at a level of 1%, which implies a high degree of volatility persistence.

The asymmetry parameter (ν) of the EGARCH model is positive and significantly different from zero at a level of 1%, indicating that negative shocks have a greater impact on volatility than positive shocks.

The sum of parameters α and β of the CGARCH model is less than the sum of the same parameters of the GARCH model, indicating that the short-term volatility component is not strong.

The AIC and the Log Likelihood indicate that the TGARCH model suits the data most effectively.

Table 3 : The Estimates of the Models – Vale

Parameters/Models	GARCH	EGARCH	CGARCH	TGARCH
μ	-0,01	-0,01	0	-0,01
ω	0,93	0,17	2,2*	0,58**
α	0,05	0,02	0	-0,07**
β	0,53	-0,14**	-0,99*	0,21**
ν	-	0,76*	-	0,7*
τ	-	-	0,53	-
Φ	-	-	0,04	-
Log Likelihood	-672,37	-667,77	-670,95	-665,44
Akaike	3,64	3,62	3,65	3,61

*, ** e *** indicate rejection of the null hypothesis at a significance level of 1%, 5% and 10%, respectively.

a) *Errors*

In order to evaluate the accuracy of the models, we used the RMSE, MAPE, LLE, and MME measures[1], for both daily and weekly forecasts. The smaller these measures, the closer the models' volatility estimates are to real volatility. Tables 4 and 5 show the calculation of these measures for the two forecasts.

Analyzing the results of Table 4 and using the RMSE, MAPE, and LLE measures to evaluate the daily volatility forecasts of the Petrobras shares, for both the RV and the RRV, the CGARCH model displays the most accurate forecasts, followed by the GARCH, EGARCH, and TAGRCH models, respectively. However, the measures considering RRV indicate minor errors. We found the same results for the weekly forecasts (except for the MAPE measure).

The MME (UP) and MME (OP) measures enable the inclusion of potential asymmetry in the loss function. The MME (UP) measure penalizes undervalued volatility forecasts, while the MME (OP) measure penalizes overvalued volatility forecasts. Thus, they are considered important, as market participants can assign different degrees of importance to the undervaluation or overvaluation of volatility.

For the daily forecast, with the exception of the MME (OP) measure using the RRV, the model that is penalized the least for undervaluing or overvaluing volatility forecasts is the CGARCH model. This model

[1] The daily volatility forecasts come from each model, while the weekly volatility forecasts are generated by multiplying the daily volatility forecast by five. This occurs for each formula used in this study. This simplification was used in the study by Corrado and Truong (2007). The weekly measures of real volatility, RV and RRV, were obtained by adding together the volatility of the last five days, as in the study by Liu, Chiang and Cheng (2012).

has the most accurate forecasts, followed by the GARCH, EGARCH, and TAGRCH models, respectively.

For the weekly forecast, the rank for the MME (OP) is the same considering RV and RRV: the GARCH model is indicated as the model that overvalues volatility the least, followed by the CGARCH, EGARCH, and TGARCH models, respectively.

Additionally, for the weekly forecast, the MME (UP) indicates that the CGARCH model undervalues volatility the least, followed by the GARCH model, for both the RV and the RRV, although the ranking of the third and fourth models is different.

The error measures indicate that the model which forecasts volatility most accurately for Petrobras is the long memory model, CGARCH, suggesting that the ability to capture a long memory of volatility is more crucial than modeling asymmetry or high volatility persistence.

Table 4 : Errors and Ranks of the Models – Petrobras

Models	RMSE	Rank	MAPE	Rank	LLE	Rank	MME(UP)	Rank	MME(OP)	Rank
Daily volatility										
RV										
GARCH	2,118	2	0,575	2	0,322	2	4,312	2	1,625	2
EGARCH	2,209	3	0,643	3	0,376	3	4,456	3	1,960	3
CGARCH	1,958	1	0,523	1	0,275	1	3,646	1	1,567	1
TGARCH	4,657	4	1,435	4	0,892	4	5,318	4	19,651	4
RRV										
GARCH	1,603	2	0,451	2	0,226	2	2,511	2	1,311	1
EGARCH	1,713	3	0,527	3	0,279	3	2,659	3	1,643	3
CGARCH	1,502	1	0,422	1	0,198	1	2,163	1	1,328	2
TGARCH	4,502	4	1,271	4	0,770	4	4,149	4	19,287	4
Weekly volatility										
RV										
GARCH	7,221	2	0,368	2	0,166	2	41,531	2	15,355	1
EGARCH	8,219	3	0,450	3	0,232	3	47,852	4	25,315	3
CGARCH	6,843	1	0,337	1	0,144	1	35,563	1	15,835	2
TGARCH	22,413	4	1,200	4	0,744	4	47,810	3	469,728	4
RRV										
GARCH	6,025	1	0,321	2	0,129	2	27,229	2	13,217	1
EGARCH	6,968	3	13,037	4	0,189	3	30,627	3	22,986	3
CGARCH	6,135	2	0,309	1	0,123	1	26,228	1	15,642	2
TGARCH	22,188	4	1,128	3	0,680	4	37,841	4	469,437	4

Table 5 shows the forecasting errors of the implemented models. In the case of Vale, the indications of error measures are more divergent. Considering the MAPE and LLE measures for evaluating the daily volatility forecast, using both the RV and the RRV, the GARCH model provides the most accurate forecasts, followed by the TGARCH model. Ranking third and fourth are the EGARCH and CGARCH models (with an exception for the LLE measure considering the RV). It is worth noting that the error measures considering RRV are lower.

For the daily forecast, the RMSE measure indicates that the TGARCH model has the most accurate forecasts, followed by the EGARCH, GARCH, and CGARCH models, respectively. Thus, it provides a different model ranking when compared to the other measures.

For the weekly forecast, with the exception of the RMSE measure, the GARCH model provides the most accurate forecasts, followed by the EGARCH, TGARCH, and CAGRCH models, respectively. As with the daily forecast, the error measures considering the RRV are also lower.

Additionally, for the weekly forecast, the RMSE measure indicates that the TGARCH model has the most accurate forecasts, but diverges with regard to the other rankings when the RV or RRV is considered. The measures considering the RRV are also lower when compared to those considering the RV.

The MME (UP) measure using both the RV and the RRV, with either the daily or weekly forecast, indicates that the TGARCH model is penalized the least for undervaluing volatility forecasts, followed by the EGARCH, GARCH, and CAGRCH models, respectively.

For the daily forecast considering the MME (OP), the GARCH model is indicated as the model that overvalues volatility the least, followed by the TGARCH, EGARCH, and CGARCH models, in that order. When the weekly forecast is evaluated, the GARCH model is also indicated as being the model that overvalues volatility the least, although in that instance it is followed by CGARCH, EGARCH, and TGARCH, respectively.

Most error measures indicate that the GARCH model has the greatest accuracy in forecasting both the daily and weekly volatility of Vale shares. This suggests, in the case of Vale, that the ability to capture either long memory volatility, model asymmetry, or high persistence is not crucial.

Table 5 : Errors and Ranks of the Models _ Vale

Models	RMSE	Rank	MAPE	Rank	LLE	Rank	MME(UP)	Rank	MME(OP)	Rank
Daily volatility										
RV										
GARCH	2,538	3	0,587	1	0,337	1	6,510	3	1,170	1
EGARCH	2,521	2	0,602	3	0,389	4	6,382	2	1,247	3
CGARCH	2,577	4	0,609	4	0,363	3	6,674	4	1,249	4
TGARCH	2,480	1	0,591	2	0,353	2	6,163	1	1,237	2
RRV										
GARCH	1,581	3	0,514	1	0,275	1	2,609	3	1,017	1
EGARCH	1,574	2	0,527	3	0,297	3	2,536	2	1,079	3
CGARCH	1,632	4	0,538	4	0,299	4	2,740	4	1,089	4
TGARCH	1,540	1	0,518	2	0,289	2	2,416	1	1,076	2
Weekly volatility										
RV										
GARCH	7,576	3	0,403	1	0,210	1	51,588	3	9,930	1
EGARCH	7,519	2	0,418	2	0,223	2	49,033	2	11,708	3
CGARCH	7,799	4	0,424	4	0,234	4	53,816	4	11,342	2
TGARCH	7,261	1	0,420	3	0,227	3	44,835	1	12,120	4
RRV										
GARCH	5,869	2	0,388	1	0,187	1	28,683	3	9,516	1
EGARCH	5,869	3	0,404	2	0,200	2	26,801	2	11,499	3
CGARCH	6,126	4	0,412	4	0,209	4	30,566	4	10,971	2
TGARCH	5,654	1	0,410	3	0,204	3	23,799	1	12,071	4

b) Value-at-Risk - VaR

Forecasting the volatility of assets is a crucial element in the area of finance, particularly for risk management. Consequently, in this study, we use volatility forecasts generated by the GARCH, EGARCH, CGARCH, and TGARCH models to evaluate each model's performance in calculating VaR.

Table 6 shows the mean value of the VaR of the Petrobras shares for each model implemented and the exceptions when compared with the RV and RRV.

Considering the mean value of the VaR, the CGARCH model presents the lowest VaR mean and the lowest number of exceptions for both daily and weekly estimates; followed by the GARCH, EGARCH, and TGARCH models, respectively. It should be noted that this is the same order indicated by the error measures.

When the Kupiec Test is applied to the models presenting exceptions, all were rejected for daily forecasting with 95% and 99% confidence. The rejection on the tests indicates that the models' loss probabilities are not compatible with theoretical probability.

All the models for which it was possible to apply the Kupiec and Christoffersen joint test were rejected. This indicates that the exceptions are not independent and that when market volatility changes rapidly the models are slow to change the VaR value.

Based on the two volatility estimators used, the results indicate that the models were not suitable for estimating the VaR of PETR4.

Table 6 : Mean VaR and Exceptions – Petrobras

Models	Mean VaR	RV		RRV	
Daily - 95% confidence		Nº Exceptions	Exceptions (%)	Nº Exceptions	Exceptions (%)
GARCH	2,991	10	1,9%	6	1,1%
EGARCH	3,010	9	1,7%	9	1,7%
CGARCH	2,852	8	1,5%	5	0,9%
TGARCH	3,523	15	2,8%	13	2,4%
Daily - 99% confidence					

GARCH	4,538	1	0,2%	0	0,0%
EGARCH	4,543	2	0,4%	0	0,0%
CGARCH	4,227	1	0,2%	0	0,0%
TGARCH	5,144	3	0,6%	1	0,2%
Weekly - 95% confidence					
GARCH	6,476	6	1,1%	5	0,9%
EGARCH	6,515	10	1,9%	6	1,1%
CGARCH	6,099	6	1,1%	5	0,9%
TGARCH	7,559	22	4,1%	23	4,3%
Weekly - 99% confidence					
GARCH	9,935	0	0,0%	0	0,0%
EGARCH	9,943	0	0,0%	0	0,0%
CGARCH	9,173	0	0,0%	0	0,0%
TGARCH	11,184	2	0,4%	0	0,0%

Table 7 shows the mean value of the VaR of the Vale shares for each model implemented and the exceptions when compared with RV and RRV.

Considering the mean value of the weekly VaR, at a confidence level of 99%, the TGARCH model presents the lowest mean VaR, despite having the highest number of exceptions, for both the daily and weekly estimates, followed by the EGARCH, CGARCH, and GARCH models, respectively.

When the Kupiec Test is applied to the models presenting exceptions, all were rejected for daily forecasting with 95% and 99% confidence. The rejection of the tests indicates that the models' loss probabilities are not compatible with theoretical probability.

All the models for which it was possible to apply the Kupiec and Christoffersen joint test were rejected. This indicates that the exceptions are not independent and that the models are slow to change the VaR value when market volatility changes rapidly.

The results indicate that the models were not suitable for predicting the VaR, using the RV and RRV volatility estimators.

Table 7 : Mean VaR and Exceptions – Vale

Models	Mean VaR	RV		RRV	
Daily - 95% confidence		Nº Exceptions	Exceptions (%)	Nº Exceptions	Exceptions (%)
GARCH	2,637	6	1,1%	6	1,1%
EGARCH	2,531	8	1,5%	7	1,3%
CGARCH	2,623	5	0,9%	6	1,1%
TGARCH	2,519	11	2,1%	9	1,7%
Daily - 99% confidence					
GARCH	3,972	1	0,2%	2	0,4%
EGARCH	3,691	2	0,4%	2	0,4%
CGARCH	3,925	1	0,2%	2	0,4%
TGARCH	3,674	2	0,4%	1	0,2%
Weekly - 95% confidence					
GARCH	5,873	8	1,5%	10	1,9%
EGARCH	5,641	10	1,9%	11	2,1%
CGARCH	5,866	8	1,5%	8	1,5%
TGARCH	5,607	12	2,2%	5	0,9%
Weekly - 99% confidence					
GARCH	8,856	0	0,0%	0	0,0%
EGARCH	8,233	0	0,0%	0	0,0%
CGARCH	8,778	0	0,0%	0	0,0%
TGARCH	8,190	0	0,0%	0	0,0%

VI. Conclusions

Modeling the volatility of assets in finance is essential for asset allocation, portfolio selection, option pricing, and risk management. This study's contribution is to present alternate adaptations of GARCH-type models to forecast daily and weekly volatility, using intra-day volatility measures. While these types of measures have been used in studies from other countries, they are still rarely used with data from Brazilian companies.

When comparing the RV with the RRV for both Petrobras and Vale shares, the RRV was proven a more efficient volatility estimator, since it had the lowest error measures.

The applied tests indicate that the CGARCH model, in the case of Petrobras, and the GARCH and TGARCH models, in the case of Vale, presented the most accurate volatility forecasts compared with the other models. In the case of Petrobras, capturing long volatility memory appeared to be more important than asymmetry or volatility persistence. In the case of Vale, volatility persistence appeared to be less relevant since the symmetric and asymmetric threshold models presented the best results.

In the case of Petrobras, the MME (OP) measure suggests that the CGARCH model overestimates volatility the least. Thus, it is a useful model for option sellers of these shares because if the volatility were overestimated, the option's price would be overestimated as well. From the perspective of option buyers, the GARCH model would be more useful, since it underestimates volatility the least, and would thus be the least likely to lead to an underestimation of the option's price.

In the case of Vale, the MME (UP) measure suggests that the TGARCH model underestimates volatility the least, and would thus be the least likely to lead to an underestimation of the option's price. The GARCH model overestimates volatility and, consequently, overestimates the option's price.

The implemented tests did not indicate that the RV and RRV volatility estimators obtained a better performance than the GARCH family estimated models.

Moreover, both the RV and RRV estimators and the GARCH models showed unsatisfactory performance in estimating the daily and weekly VaR.

One possible extension of this study is the use of models that estimate volatility based on the high-frequency estimators used here. Moreover, it is possible that with a larger sample, the performance of the models in estimating the VaR would be improved.

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