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Linear Programming on Portfolio Optimization: Empirical Evidence from Bist Mining Industry Index

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Linear Programming on Portfolio Optimization: Empirical Evidence from Bist Mining Industry Index

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Abstract- A lot of methods are improved for the portfolio optimization within classical approach. Quadratic programming, one of these methods, has many disadvantages, so alternative methods are studied to improve. MAD Method, an improved new method, is converted portfolio optimization problem into a linear programming problem. MAD Method is demonstrated and a case study is done by using stock certificate which belongs to BIST Mining Sector.

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I. Introduction

Many models have been developed in the financial literature under the heading of portfolio optimization and these models were named as traditional portfolio and modern portfolio optimization. While attempting to minimize the risk of the portfolio through diversification of securities too much, not minding interrelationship between them, in traditional portfolio management approach; portfolio optimization has been made through mean- variance model (Markowitz, 1952:77-91) in modern portfolio management approach.

Harry Markowitz is called as the founder of the theory of modern investment with his study, by the name of "Portfolio Selection", that he presented as phd dissertation in the year 1952. In this study, Markowitz targeted the selection of the lowest-risk portfolio corresponding to a certain return on the basis of mean variances.

Various scientists attempted to develop portfolio selection model on the basis of mean -variance model. Tobin (1958), Sharpe (1964) ve Lintner (1965) adapted real-life constraints to the model, such as investor's decision on percentage of portfolio consisting of risky assets, borrowing- lending situation, short-term sales, transaction costs and taxes. Brennan (1971) investigated the subject of borrowing and lending; Turnbull (1977) investigated the subjects of personal taxation, uncertain inflation, nonmarket assets. Levy (1983) and Schnabel (1984) dealt with short-term sales problem.

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The difficulties caused by the increase in the number of securities for expected return of optimum portfolio and determination of variance were overcome by single index model developed by Sharpe (1963) and multiple index models developed by Perold (1984).The studies conducted on mean- variance model revealed the Capital Assets Pricing Model (CAPM), which is both mathematical and logical extension of the mean-variance model (Harrington, 1983). Sharpe (1964), Lintner (1965) and Mossin (1966) added moving risk-free financial asset to the model on the basis of Markowitz's studies.

Konno and Yamazaki (1991) proposed the mean absolute deviation (MAD) model, which is also a portfolio optimization model, alternatively to the mean-variance portfolio optimization model of Markowitz. MAD Model has used mean absolute deviation instead of variance intended to be minimized in the objective function of mean-variance model. Thus, portfolio selection problem was degraded from a quadratic program to a linear program (Simaan, 1997: 1437).

In this study, it is intended to inform about MAD model proposed by Konno and Yamazaki (1991) for also solution of large-scale portfolio optimization problems that can't be solved with Markowitz's classical mean -variance model and investigate its some properties. In the practice section, portfolio optimization was performed through MAD model for trading securities at BIST.

II. Mean Absolute Deviation (MAD) Model

MAD model is an alternative method simplifying Markowitz's classical formulation by using absolute deviation as a risk scale. When these two mathematically equivalent formulas have been considered in terms of calculation, significant differences are noticed between them. As well as approach of risk measurement through variance converts the problem to quadratic programming problem, absolute deviation approach degrades the problem to linear programming problem (Konno ve Koshizuka, 2005: 893).

Konno and Yamazaki revealed that mean absolute deviation of normal distribution was proportional with standard deviation of that. Consequently, MAD model and Markowitz model show the same activity under the multi variability of return of assets. Provided that these returns of (R_1,R_2,…,R_n) suggest multi variability normal distribution, then these two measures are the same. In other words, when returns of (R_1, R_2,…,R_n) suggest multi variability normal distribution, it means that minimizing the function $w(x)$ is minimizing the function σ (x) at the same time (Simaan, 1997: 1437). Furthermore, Rudolf, Wolter and Zimmermann (1999) revealed that minimizing mean deviation was equivalent to maximizing expected utility in case of avoiding risk (Rudolf et al. 1999: 85 103).

a) *Mathematical Model*

$R_j, (j = 1,2, \ldots n)$, $j.$ represents the random variable implying return of asset and x_j, $(j = 1,2, \ldots, n)$ $j.$ representing the ratio to be invested to asset, total return of portfolio consisting of assets is calculated as follows:

$$R(x) = \sum_{j=1}^{n} R_j x_j$$

Here, $j.$ represents return of asset, F_t ; asset's price at the end of the period, F_{t-1} ; asset's price at the beginning of the period $R_j = \frac{(F_t - F_{t-1})}{F_{t-1}}$, is calculated with this formula.

Standard deviation used as the scale of variance and risk in standard portfolio analysis is calculated as follows:

$$V(x) = E[(R(x) - E[R(x)])^2]$$
$$\sigma(x) = \sqrt{V(x)}$$

Mean absolute deviation used as a measure of the risk in MAD model is defined as follows (Konno ve Yamazaki, 1991: 523 524).

$$w(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

This function is also the objective function that will be minimized.

$$min w(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

$$\sum_{j=1}^{n} E(R_j) x_j \geq \rho$$

$$\sum_{j=1}^{n} x_j = 1$$

$$0 \leq x_j \leq 1, \quad j = 1,2, \ldots, n$$

ρ: mimimum return desired by investor

$r_{jt} : t$ is the acquired return for time period $t = (1,2, \ldots, T)$ and it is assumed that this return could be acquired from historical data or predictions for future; besides, expected value of random variable would converge these data resulted mean

$$let \; r_j = E(R_j) = \sum_{t=1}^{T} \frac{r_{jt}}{T}$$

$w(x)$ converges in the way given below :

$$E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right] = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt} - r_j)x_j\right|$$

$$let \; a_{jt} = r_{jt} - r_j; \; j = 1,2, \ldots, n; \; t = 1,2, \ldots, T$$

In this case, the problem is converted to the minimizasyon problem given below:

$$min w(x) = \frac{\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{jt} x_j\right|}{T}$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho$$

$$\sum_{j=1}^{n} x_j = 1$$

$$0 \leq x_j \leq 1, \quad j = 1,2, \ldots, n$$

By means of equations mentioned above, objective function became linear. This model will be equivalent to such a linear programming problem given below.

$$min w(x) = \frac{\sum_{t=1}^{T} y_t}{T}$$

$$y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0, \quad t = 1,2, \ldots, T$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0, \quad t = 1,2, \ldots, T$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho$$

$$0 \leq x_j \leq 1, \quad j = 1,2, \ldots, n$$

$$\sum_{j=1}^{n} x_j = 1$$

III. Aplication

In this section, portfolio optimization was carried out through that MAD model was applied to the actual data obtained from BIST. In the application part of this study, returns of mining sector concerned shares included in the SIST index between the dates 04.01.2010 to 4.12.2014 were calculated on the basis of daily closing prices and MAD model was applied to these data. It was assumed in the model that investors would create their portfolios with fully risky investments and risk-free investment and short selling wouldn't be allowed. MAD model was applied to the data by being written in an econometric package program. The shares used in the study belong to İhlas Mining (IHMIN), Ipek Natural Energy (IPEKE), Koza Mining (KOZAA), Koza Gold (KOZGO), Park Electric and Mining (PRKM).The statistics concerning these 5 shares have been given in Table 1.

Table 1 : Descriptive Statistics

Shares	Mean	Std. Dev.	Skw.	Kur.
IHMAD	0,038	0,050	1,325	-5,341
IPEKE	-0,045	0,031	1,291	0,274
KOZAA	0,049	0,090	5,594	-2,584
KOZAL	0,089	0,046	3,863	1,209
PRKM	0,058	0,025	1,580	1,158

For an investor targeting different returns and will make an investment on the basis of MAD model; in which ratio from which shares he should invest to his portfolio have been shown in Table 2.

Table 2 : Minimum Risk Ratio of Shares by a different Target Return

HİSSE	ρ					
	0.0001	**0.0002**	**0.0003**	**4E-04**	**0**	**6E-04**
IHMAD	0	0	0	0	0	0
IPEKE	0	0	0	0	0	0
KOZAA	0	0	0	0	0	0.004
KOZAL	0	0	0	0	0	0
PRKM	1	1	1	1	1	0.996
Min.Risk	0.0173	0.0173	0.0173	0.017	0.02	0.018

Table 2 (Con't) : Minimum Risk Ratio of Shares by a different Target Return

HİSSE	ρ						
	0.0007	**0.0008**	**0.0009**	**0.001**	**0**	**0.003**	**0.004**
IHMAD	0	0	0	0	0	0	0
IPEKE	0	0	0	0	0	0	0
KOZAA	0.027	0.0501	0.0732	0.096	0.33	0.558	0.789
KOZAL	0	0	0	0	0	0	0
PRKM	0.973	0.9499	0.9268	0.904	0.67	0.442	0.211
Min.Risk	0.0183	0.0191	0.0199	0.021	0.03	0.037	0.0455

The expected return and variances of these portfolios acquired for different returns using MAD model have been shown in Table 3.

Table 3 : The expected returns and variances of efficient portfolios by employing MAD model

ρ	Exp. Return	Variance
0.0001	0.000583	0.000651
0.0002	0.000583	0.000651
0.0003	0.000583	0.000651
0.0004	0.000583	0.000651
0.0005	0.000583	0.000651
0.0006	0.000599	0.0068
0.0007	0.000699	0.000852
0.0008	0.000799	0.001024
0.0009	0.0009	0.001196
0.001	0.001	0.001369
0.002	0.001999	0.00309
0.003	0.002999	0.004812
0.004	0.004	0.006535

a) *Evaluation of the Consequences Acquired from the Model*

Having constituted optimum portfolios with different minimum returns acquired through application of MAD Model; in order to test the reliability of this model, Tablo4 and Tablo5 will be examined.

Table 4 : Closing Share Prices and Returns by Years

	SHARES									
	IHMAD		IPEKE		KOZAA		KOZAL		PRKM	
Date	Close Price	Return	Close Price	Return	Close Price	Return	Close Price	Return	Close Price	Return
04.01.2010	0.87	0.33	3.22	0.15	1.7	0.35	15.7	0.23	2.93	0.34
31.12.2010	1.16		3.72		2.3		19.4		3.93	
03.01.2011	1.16	1.41	3.72	-0.4	1.6	0.12	19.5	0.59	3.87	-0.09
30.12.2011	2.8		2.2		1.8		31.1		3.49	
02.01.2012	2.77	0.84	2.18	1.75	1.8	0.11	32	0.33	3.47	0.78
31.12.2012	5.12		6		2		42.6		6.18	
02.01.2013	5.12	0.48	6.08	-0.48	2	-0.15	43	-0.51	6.14	-0.19
31.12.2013	7.6		3.12		1.7		21		4.95	
02.01.2014	7.78	-0.82	2.88	-0.44	1.8	0.05	20.75	-0.3	4.92	-0.17
24.12.2014	1.36		1.6		1.9		14.41		4.07	
Return	0.56		-0.5		0.11		-0.08		0.38	

Table 5 : Closing Share Prices, Min., Max. and Differences

	Min.	Max.	Difference
IHMAD	0.64	9.8	9.16
IPEKE	1.45	6.18	4.73
KOZAA	1.48	2.32	0.84
KOZAL	12.15	48	35.85
PRKM	2.34	7.28	4.94

When examined the tables, it has been seen that the shares that will make contribution to portfolio in terms of profit that will be created for investor's desire and natural expectation are IHMIN, KOZGO and PRKM yielding positive return. Accordingly, it is obvious that other two shares will not make any contribution for profit growth. Two mainly recommended shares in the portfolios created by MAD model are KOZGO and PRKM. The reason why IHMIN securities haven't been included in the optimum portfolios is that there is so much risk due to excessive fluctuations between beginning of period and end of period related market closing prices of the years selected. Remembering that through MAD model it is intended to minimize the equation that is objective function, yielding mean absolute deviation; naturally, the shares to be selected are supposed to minimize the risk as well as increase profit. Therefore, not only the shares with positive returns but also the ones with minimum risk were selected in optimum portfolios created by MAD model. When these considerations taken into account, it has been noticed that MAD model has yielded positive results and can be used in daily life.

On the other hand, comparing the variation coefficients (coefficient of change), another criterion in the selection of shares; it can be decided that which shares should be included in the portfolio that will be created and which ones shouldn't. Variation coefficient is defined as follows.

$$Variation\ Coefficient = \frac{Risk}{Return}$$

Since standard deviation has been used as a risk scale in portfolio optimization,

$$Variation\ Coefficient = \frac{Standard\ Deviation}{Return}$$

Table 6 includes standard deviation and value of returns of the shares belonging to mine sector in question and the results of variation coefficient calculated on the basis of these values.

Table 6 : Standard Deviations, Returns and Variation Coefficients of Shares

	Std. Dev.	Return	Variation Coeff.
IHMAD	0,050	0,00388	129
IPEKE	0,031	-4,5E-05	-704,72
KOZAA	0,090	0,0049	18,31
KOZAL	0,046	0,00089	52,22
PRKM	0,025	0,0058	43,725

It was found in the evaluations mentioned above that IPEKE and KOZGO securities' returns were negative, accordingly they shouldn't be included in the portfolios created. On the basis of that, when other three shares compared, the risks of the ones that must be included in portfolio, are supposed to be small as much as possible. When their variance coefficients were compared, the order from high to low value would be respectively IHMIN, KOZAA, and PRKM. On the basis of that securities with small variation coefficient should be included in the optimum portfolio for portfolio optimization, since IHMIN return's variation coefficient is so high, KOZAA and PRKM securities are supposed to be included in the optimum portfolio to be created as much as possible. When all of these taken into account, it is seen that portfolio optimization carried out with MAD model comply with daily life and not conflict with other portfolio selection criteria or methods in finance sector.

IV. Conclusion

Mean-Variance Model creating major changes in Markowitz's portfolio selection understanding is a currently used quadratic programming model revealing interrelationships between assets through risk-return variation, accordingly, taking into account diversification and the evaluation of entire portfolio. MAD model proposed by Konno and Yamazaki is one of the models proposed in time to overcome several problems encountered in the selection of portfolio. In MAD model, which is a linear programming model, risk is expressed with mean absolute deviation, not with variance.

In this study, MAD model was theoretically introduced. In the study performed with actual data, daily values of returns of the securities between January 2010- December 2014 of mine sector being included in SIST's index were used and portfolios were acquired on the basis of different target returns through application of the model.

Model was tested firstly comparing the fluctuations between values of returns of actual data and market closing prices; secondly, variation coefficient comparison method, which is another criterion used for selection of share in portfolio optimization, was used. It was seen according to both these two considerations that there wasn't any conflict with the consequences of the portfolios created through MAD model.

MAD model brought a new perspective to the classic portfolio optimization problem and degraded the problem to linear programming problem by defining the risk on the basis of mean absolute deviation. Thus, model has brought along the advantages such as transaction easiness, not requiring distribution assumption, ability to be reformulated for various constraints. The only disadvantage of MAD model encountered in the literature is that it can lead to prediction error due to not taking covariance matrix into account. When theoretical benefits and application performance of the model have been considered together, it has showed itself as a preferable portfolio optimization model.

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