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Fuzzy Linear Programming on Portfolio Optimization: Empirical Evidence from FTSE 100 Index

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Fuzzy Linear Programming on Portfolio Optimization: Empirical Evidence from FTSE 100 Index

Fatih Konak [α] & Buğra Bağcı [σ]

Abstract- Portfolio is a list of securities that the investor has. The main objective of portfolio management is to maximize return while minimizing unsystematic risk. Firstly, fundamental definitions are given about theory of fuzzy logic and fuzzy logic approach is stated in this study. In the model of fuzzy logic price/earnings ratio and accumulation/distribution index which are added by the model that Werner improved. Taking all into consideration a new model is developed at the last part of this research.

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I. Introduction

Investors are aiming to increase and protect their income in various ways by taking into account every condition that they encounter. For this reason, one way of applying this is using their incomes in financial markets. However, financial markets are affected by financial and social events, this causes a vague structure. To decide under this uncertainty is one of the hardest challenges for the investors.

Besides, investor's knowledge and experience are very important during making the decision process. To use investors experience in the model will provide more realistic results. Fuzzy set theory is used to let experience and uncertain conditions as linear programming technique participate in the decision making process. The aim of this study is in this direction.

By adding fuzzy theory to the linear programming models fuzzy linear programming models are created. Fuzzy linear programming is recommended for solutions to the problems which have fuzzy parameters and can be modeled by using linear functions. It provides easier solutions to developed models and allows decision makers to express their demands in a flexible way.

In the first part of the study, decision under fuzzy environment technique is discussed. Membership functions are introduced and the structure of the purpose function is described. In the second part of the study, the model for choosing the portfolio using fuzzy linear programming method is discussed. In the third and last part, suitable portfolios are created according to investors behaviors from FTSE-100 shares.

II. Decision Making in the Fuzzy Environment

Mathematical formulation of fuzzy set theory was created for the first time in 1965 by Zadeh. Zadeh introduced a way where uncertain conditions can be modeled mathematically. (Mansur, 2002:1) Linear programming problems divided into three main components. Those components are decision variables, restricts and purpose function. In fuzzy linear programming, purpose function and purpose function coefficient are named as fuzzy target and represented by G. Fuzzy restricts are represented by C. For conclusion, decision for fuzzy target and fuzzy restricts is called fuzzy decision. Fuzzy decision is represented by D and μ D (x) is the membership function of the fuzzy decision. Functions related to fuzzy targets is represented by μG (x), functions related to fuzzy restricts is represented by μC (x).

Membership function related to targets is represented by μ G (x) ∈ [0, 1] and valued from 0 to 1. If the membership function equals to 1 then target is fully achieved, if function equals to 0 target is not fully achieved. However, if membership function equals a value between 0 and 1 then target is partially achieved. Membership function related to restricts is represented as μC(x) ∈ [0, 1] and valued from 0 to 1. When membership function is equal to 0 then related restrict is not fully relevant, when equals to 1 then related restrict is fully relevant. When between 0 to 1, related restrict is partially relevant. Fuzzy decision is described as fuzzy target and fuzzy restricts are provided together. This is described as,

$$D = G \cap C \qquad (1)$$

Using equality membership functions in (1)

$$\mu D(x) = \mu G(x) \wedge \mu C(x) = [\mu G(x), \mu C(x)] \qquad (2)$$

can be written (Terano et al, 1992). For more general description equalities in (1) ve (2) G1 , G2 ,..., Gn n number of fuzzy targets and C1 , C2 ,..., Cm m number of fuzzy restricts,

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$$D = G1 \cap G2 \cap \ldots \cap Gn \cap C1 \cap C2 \cap \ldots \cap Cm \quad (3)$$

with membership functions (Bellman and Zadeh, 1970: 141-164).

$$\mu D(x) = \min [\mu G1(x), \mu G2(x), \ldots, \mu Gn(x), \mu C1(x), \mu C2(x), \ldots, \mu Cm(x)] \quad (4)$$

In order to achieve optimum decision in the problem, the highest degree of the element in the fuzzy decision set should be determined. This is calculated as (Terano et al 1992).

$$\mu D(xM) = \max \mu D(x) \quad (5)$$

The equality in (5) is known as max-min processor. Max-min processor is a reliable method to choose the best solution between the worst cases. Extendedly Max-min processor is written as,

$$\max \mu D(x) = \max(\min(\mu G(x), \mu C(x))) \quad (6)$$

III. Fuzzy Linear Programming and Portfolio Analysis

Investors are aiming to increase and protect their income in various ways by taking into account every condition that they encounter. For this reason, one way of applying this is using their incomes in financial markets. However, financial markets are affected by financial and social events, this causes a vague structure. To decide under this uncertainty is one of the hardest challenges for the investors. Uncertainty in this environment brings lots of risk parameters for the investors. Investors are trying to reduce risk factors into minimum by using different instruments for their assets. By creating portfolio and managing it, risk is already reduced. Because, risk of the portfolio as a whole is smaller than risks that every share possesses individually. But, over diversification can be harmful while creating the portfolio. While doing over diversification, low-performance investment instruments are included in the portfolio. Also, it can be harder to provide information about investment tools when the number is increased. Generally, portfolio is a new entity, which has measurable qualities in relation with together to fulfill certain purposes (Ceylan and Korkmaz, 1998). Portfolio is a pool in where at least two instruments are in it in order to reduce risk and provide the highest income due to that risk (Ercan ve Ban, 2005).

Markowitz's modern portfolio approach put forward in 1952 by at least risk level needed to reach the targeted level of investor returns and begin to determine the structure of the model portfolio risk level (Ulucan, 2004). Although it is theoretically appropriate, Markowitz portfolio optimization model is not preferred in practice for especially large-scale portfolios. The most important reason behind the practical usefulness of the Markowitz model poses challenges emerging in the solution of quadratic programming problems with large-scale covariance matrix.

Sharpe (1967, 1971) developed alternative methods to Markowitz. Konno (1990) used linear programming in his study instead of quadratic programming approach. Konno and Yamazaki (1991) used absolute deviation risk function instead of Markowitz risk function. Simaan (1997) Compared average variance model between average absolute deviation models in his study. Speranza (1993) used semi absolute deviation portfolio model in his study to measure portfolio risk.

As a model portfolio of functional formulation requires the return of the shares that make up the portfolio and estimation of the distribution of the risk. Information on the selected shares during the time interval, the return and risk distribution of the portfolio is random therefore managers of the portfolios should have reviews regarding shares provides great importance.

These different interpretations by different portfolio managers can be caused from the same set of information. Having different interpretations of the portfolio by the managers will be transferred to the portfolio models created with use of fuzzy set theory.

Followed by the development of fuzzy decision theory by Bellman and Zadeh (1970:141-164) took the form of a tool that can be used for portfolio optimization fuzzy linear programming. Ramaswamy (1998) created portfolio selection model using fuzzy decision theory. We can encounter same studies in Östermark (1996: 243-254) and Leon and others (2002: 178-189) Östermark created dynamic portfolio management model in his study. Watada (2001: 141-162) also created portfolio selection model using the same theory. Tanaka and Guo (1999) used probability theory in order to study uncertainty. Lai and others (2002), Wang and Zhu (2002) used linear interval programming model to choose portfolio in their studies.

In this study, based on recommended model by Konno and Yamazaki (1991: 515-531) Fang and others (2005:879-893 will try to create optimum portfolio. This model is explained below.

Here, ρM0 (expected fuzzy income amount), is in the closed interval of τ, tolerance value known amount of expected [ρM0, ρM0+τ]. ρM0+τ, is determined by decision maker as an upper value of expected income.

$$minw(x) = \frac{\sum_{t=1}^{T} y_t}{T}$$

$$y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0\,, \quad t = 1,2,\ldots,T$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0\,, \quad t = 1,2,\ldots,T$$

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$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0 + \tau$$

$$0 \leq x_j \leq 1, \ \ j = 1,2,\dots,n$$

$$\sum_{j=1}^{n} x_j = 1$$

Here, ρM0 (expected fuzzy income amount), is in the closed interval of τ, tolerance value known amount of expected [ρM0, ρM0+τ]. ρM0+τ, is determined by decision maker as a upper value of expected income.

This model can be used to determine how much to invest in to different stocks by using α [0, 1] for different levels of expectation. Besides, decision makers at this level can determine target income and risk values at specified level.

However, the main purpose of this model is to achieve an optimum solution from a variety of combinations of return and risks are not fully adequate. Werners have suggested that the objective function due to blurred and fuzzy inequality constraints sources may also be fuzzy. As in Verdegay's approach, every fuzzy source tolerance is assumed to be known. In order to apply Werner's approach to the model is solved for ρM0 (α=0) and ρM0 τ (α=1) expected income and function values are found as Z0 and Z1 (minimized risk values). As the expected income value in the model is increased, the minimized risk value will also increase and therefore Z1>Z0. As the investors are sensitive to risk, when risk is increased, satisfaction will decrease. When the membership functions are introduced in the linear programming model, fuzzy target DP model becomes standard DP model below:

$$Maks.\alpha$$

$$\sum_{t=1}^{T} \frac{y_t}{T} + \alpha(Z^1 - Z^0) \leq Z^1$$

$$y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ \ t = 1,2,\dots,T$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ \ t = 1,2,\dots,T$$

$$\sum_{j=1}^{n} r_j - \alpha\tau \geq \rho M_0$$

$$0 \leq x_j \leq 1, \ \ j = 1,2,\dots,n$$

$$\sum_{j=1}^{n} x_j = M_0$$

IV. Application

In this part of the study, with the help of proposed model, creation of an optimal portfolio for FTSE 100 stocks included in the index will be calculated for portfolio risk and return of amounts obtained. After calculation of monthly income of stocks expected income is (The average rate of return on average equity, ρ) 0, 02 (% 2) and the maximum expected rate of return can be obtained from stock, the maximum of the average returns of stock (ρ max) is found as 0,055 (%5, 5). The tolerance of expected income tolerance (τ=ρ max-ρ) is 0,035 (%3, 5). By taking membership function as $M_0 = 1$, table is created as below.

$$\mu_K(x) = \begin{cases} 0 & , \ \sum_{j=1}^{n} r_j x_j < \rho M_0 \\ \left[\sum_{j=1}^{n} r_j x_j - \rho M_0\right] / \tau, & \rho M_0 \leq \sum_{j=1}^{n} r_j x_j \leq \rho M_0 + \tau \\ 1 & , \ \sum_{j=1}^{n} r_j x_j > \rho M_0 + \tau \end{cases}$$

$$\mu_K(x) = \begin{cases} 0 & , \ \sum_{j=1}^{30} r_j x_j < 0{,}02 \\ \left[\sum_{j=1}^{30} r_j x_j - 0{,}02\right] / 0{,}035 & , \ 0{,}02 \leq \sum_{j=1}^{30} r_j x_j \leq 0{,}055 \\ 1 & , \ \sum_{j=1}^{30} r_j x_j > 0{,}055 \end{cases}$$

Here τ is the tolerance value of expected rate of return.

After that, parametric equation is solved accordingly for expected incomes ρM0 (α=0) and ρM0+τ (α=1), by doing this Z0 and Z1 (minimized risk values) target function values are found.

$$min w(x) = \frac{\sum_{t=1}^{T} y_t}{T}$$

$$y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ \ t = 1,2,\dots,T$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ \ t = 1,2,\dots,T$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0 + \tau$$

$$0 \leq x_j \leq 1, \ \ j = 1,2,\dots,n$$

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$$\sum_{j=1}^{n} x_j = M_0$$

By solving with this model, Z0=0.0080 and Z1=0.0113 values are found.

After finding Z0 and Z1 values, target membership function, when α=0 is Z0 and when α= 1 Z1 values are used to determine target membership function like below.

$$\mu_z(x) = \begin{cases} 1, & Z < Z_0 \\ 1 - \dfrac{Z - Z_0}{Z_1 - Z_0}, & Z_0 \leq Z \leq Z_1 \\ 0, & Z > Z_1 \end{cases}$$

$$\mu_Z(x) = \begin{cases} 1, & Z < Z_0 \\ 1 - \dfrac{Z - 0.0080}{0.0033}, & 0.0080 \leq Z \leq 0.0113 \\ 0, & Z > 0.0113 \end{cases}$$

By putting membership functions into their places, fuzzy target and sourced DP model becomes standard DP model.

$$Max\ \alpha$$

$$\sum_{t=1}^{T} \frac{y_t}{T} + \alpha(Z_1 - Z_0) \leq Z_1$$

$$y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0$$

$$y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0$$

$$\sum_{j=1}^{n} r_j x_j - \alpha\tau \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq 1$$

$$\alpha \in [0,1]$$

$$j = 1,2,3,\dots,n \qquad t = 1,2,3,\dots,T$$

By solving this model, α=0.51 is found. Minimum risk ratio related to this α value is found using membership function as below:

$$\mu_z(x) = \alpha = 1 - \frac{z - 0.0080}{0.0033}$$

$$0.51 = 1 - \frac{z - 0.0080}{0.0033}$$

$$\frac{z - 0.0080}{0.0033} = 0.49$$

$$z - 0.0080 = 0.001617$$

$$z = 0.009617$$

With α=0.51 satisfaction level minimized risk ratio z is calculated as approximately % 9.6. In this satisfaction level expected rate of return is;

$$Expected\ Return = \rho M_0 + \alpha\tau$$

$$= 0.000325 + 0.51 * 0.00076$$

$$= 0.000325 + 0.0003876$$

$$= 0.0007126$$

With α=0.51 satisfaction level, with taking %9.6 as risk, expected rate of return is around %0.7. The table below shows that after solving the recommended model, stocks which should be present in the portfolio and the amount of stocks in the portfolio (%).

Table 1 : Weights of Shares in the Target Portfolio

x_j	Shares	Weight
x_1	ANGLO AMERICAN	0
x_2	BRITISH AMERICAN TOBACCO	0.6065
x_3	CARNIVAL	0
x_4	DIAGEO	0
x_5	EXPERIAN	0
x_6	FRESNILLO	0
x_7	GLAXOSMITHKLINE	0
x_8	HIKMA PHARMACEUTICALS	0.3935

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x_9	HSBC HDG.	0
x_{10}	IMPERIAL TOBACCO GP.	0

In the portfolio created using the values, %60.65 of BRITISH AMERICAN TOBACCO stocks and %39.35 of HIKMA PHARMACEUTICALS stocks should be presented.

V. Conclusion

Behind the portfolio concept, idea of risk minimization lies. For this reason, in order to invest the assets into only one instrument, it is beneficial to invest a portfolio which consists of more than one instrument. This diversification should be done by comparing the stocks in the portfolio or in the sector they are in. By doing this, expected rate of return could be achieved easily. The study which Markowitz conducted in 1952 created new horizons for the investors. In the meantime, new assumptions and approaches are created after Markowitz's work. Linear programming model by Konno-Yamazaki, which is an approach to this model, was fuzzed by Werners and other researchers. In this study, Werner's model using fuzzy linear programming for portfolio optimization is taken as a basis.

This model is now examining the situation and the past performance of stocks in the sector, which is one of the main methods of analysis Price / Earnings ratio of technical analysis and collection - distribution index is created as a new model by adding constraints. By solving the proposed model by economic package program, portfolio is created. In this portfolio, there should be BRITISH AMERICAN TOBACCO stock by %60.65 and HIKMA PHARMACEUTICALS stocks by %39.35. This portfolio is expected to have a rate of return of % 0.7 with %9.6 risk.

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