Stress Testing and Risk Management of the Equity Market in New Zealand

By Andrew Maree

Reserve Bank of New Zealand

Abstract- The banking supervision sets out the need to conduct stress tests for the financial institutions in New Zealand. For the purpose of stress tests, the paper develops a methodology to calculate a series of severe but plausible economic scenarios. Five widely-used statistical distributions are investigated in fitting the return series of NZ 50. We show that the Skewed *t* distribution has the best goodness of fit and generates the most suitable stress test scenarios. Our approach could be an important component of sound risk management for the Reserve Bank of New Zealand. The financial institutions are expected to continue to develop their stress testing frameworks, and to use the results in our paper to inform their capital management and risk appetite setting processes.

Keywords: tail risk; goodness of fit; stress test scenarios.

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Stress Testing and Risk Management of the Equity Market in New Zealand

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Abstract- The banking supervision sets out the need to conduct stress tests for the financial institutions in New Zealand. For the purpose of stress tests, the paper develops a methodology to calculate a series of severe but plausible economic scenarios. Five widely-used statistical distributions are investigated in fitting the return series of NZ 50. We show that the Skewed *t* distribution has the best goodness of fit and generates the most suitable stress test scenarios. Our approach could be an important component of sound risk management for the Reserve Bank of New Zealand. The financial institutions are expected to continue to develop their stress testing frameworks, and to use the results in our paper to inform their capital management and risk appetite setting processes.

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I. INTRODUCTION

The New Zealand banking system was about NZD 500 billion in June 2016, around twice New Zealand annual GDP (IMF, 2017). In New Zealand, registered banks are required to conduct stress tests and report the results as part of their Internal Capital Adequacy Assessment Process (ICAAP) (Reserve Bank of New Zealand, 2015). Stress testing a bank's portfolio of assets (such as loans) involves evaluating what would happen to the portfolio in the face of various adverse events. Applied across a bank, a stress test can be described as "the evaluation of a bank's financial position under a severe but plausible scenario." Stress testing is an important risk management tool for banks in evaluating their vulnerability to various types of risk across their balance sheet. The Basel II international capital framework has a specific role for stress testing to help ensure banks have sufficient capital to absorb unanticipated losses in severe economic downturns (BCBS, 2011).

A stress test may examine the consequences for banks of a full macroeconomic scenario that affects multiple aspects of the banks' business, or concentrate on a downturn in a particular sector. According the Reserve Bank of New Zealand (2015), a combination of risk factors determines the full macroeconomic scenario. In this paper, we are particularly interested in the risk factor of the stock market in New Zealand, since this risk factor is the most important factor in determining market risk of banks.

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To design stress test scenarios of the stock market in New Zealand, we consider the NZX 50 Index (NZ50). NZ50 is the main stock market index in New Zealand. It comprises the 50 biggest stocks by free-float market capitalisation trading on the New Zealand Stock Market (NZSX). The calculation of the free-float capitalisation excludes blocks of shares greater than 20% and blocks between 5% and 20% that are considered strategic. The index was introduced as the NZSX 50 Index in March 2003 and replaced the NZSE 40 Index as the headline index. It was renamed the NZX 50 Index in late 2005. The NZSE 40 Capital Index replaced the Barclays index in 1992, although the Barclays index is still compiled by the NZX but not made widely available.

To quantitatively design stress test scenarios of the stock market, we model daily returns of NZ50. As early as 1963, Mandelbrot recognized the heavy-tailed and highly peaked nature of certain financial time series. Guo (2017) creatively introduced several widely-used heavy-tailed distributions to fit the Standard&Poor's 500 index returns, and showed the Skewed *t* distribution has the best goodness of fit. Guo further successfully quantified scenarios of the stock market index return in the United States. With the insights from Guo (2017), this paper considers the widely-used heavy-tailed distributions discussed in Guo, and shows the Skewed *t* distribution also provides the best goodness of fit among these heavy-tailed distributions in modeling the stock market returns in New Zealand.

The aftermath of the recent Financial Crisis emphasized the importance of stress testing practice. Stress tests offer useful information to the Reserve Bank in its role as a prudential regulator. Our results are crucial for the Reserve Bank to determine the ability of banks or financial institutions to deal with a financial crisis.

a) Literature Review

Guo (2017) compared five widely-used statistical distributions in fitting the Standard&Poor's 500 index returns: normal, Student's *t*, Skewed *t*, normal inverse Gaussian (NIG), and generalized hyperbolic (GH) distributions. Guo showed the Skewed *t* distribution has the best goodness of fit and generates suitable stress test scenarios. The paper adopted the Skewed *t* distribution in Hansen (1994). There are some

13

other types of asymmetric Student's t distribution as in Zhu and Galbraith (2012). In this paper, we also consider the Skewed t distribution as in Hansen (1994) for its simplicity and empirical performance. Since Barndorff-Nielsen (1977) introduced generalized hyperbolic distributions into the equity market, the GH distribution has gained increasing attentions in financial econometrics. In this paper, we also consider the normal inverse Gaussian distribution since the NIG distribution is one of the most popular subclass of the GH distribution, in financial modeling (see Figueroa-Lopez, et al., 2011, for a survey). In this paper, we reconsider these five distributions but focus on the financial market in New Zealand.

Recently, researchers and practitioners have gained rapid increasing interests in design of stress test scenarios in the financial industry. There are two main strands of the interests. The first strand is on the Mahalanobis distance based method to construct stress scenarios for risk factors. Breuer, Jandacka, Rheinberger and Summer (2009) proposed a suitable region of plausibility in terms of the risk-factor distribution and search systematically for the scenario with the worst portfolio loss over this region. Darne, Levy-Rueff and Pop (2013) suggested a rigorous and flexible methodological framework to select and calibrate initial shocks to be used in bank stress test scenarios. The second strand focuses on the nonparametric likelihood based method to construct stress scenarios for risk factors. Glasserman, Kang and Kang (2014) presented a method for selecting and analyzing stress scenarios for financial risk assessment, with particular emphasis on identifying sensible combinations of stresses to multiple factors. Flood and Korenko (2014) introduced a technique for selecting multidimensional shock scenarios for use in financial stress testing. Finally, Kapinos and Mitnik (2016) developed a simple, parsimonious, and easily implementable method for stress-testing scenarios

choices using a top-down approach that incorporates the heterogeneous impact of shocks to macroeconomic variables on banks' asset valuation. In this paper, we focus on parametric methods and consider a single risk factor as a starting point.

There are many other researches on the stock market returns in New Zealand, such as in Firth (1997) and Choi, Pang and Fu (2009). However, there is no research on the topic of designs of stress test scenarios. There are some researches on stress test of New Zealand's banking system as in Jang and Kataoka (2013), but not specifically focusing on the financial market. The remainder of the paper is organized as follows. In Section 2, we introduce the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Finally, we conclude in Section 5.

II. The Heavy-Tailed Distributions

Similar as in Guo (2017), we consider four types of widely-used heavy-tailed distribution in addition to the normal distribution: (i) the Student's t distribution; (ii) the Skewed t distribution; (iii) the normal inverse Gaussian distribution (NIG); and (iv) the generalized hyperbolic distribution (GH). All the distributions have been standardized to ensure mean and standard deviation equal to zero and one respectively. Their probability density functions are given as follows.

i. Student's t distribution

$$f(e_t \mid \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}}\left(1+\frac{e_t^2}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \quad (1)$$

where ν indicates degrees of freedom and e_t is daily equity market index return in New Zealand.

ii. Skewed t distribution

$$f(e_t \mid \nu, \beta) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{be_t+a}{1-\beta}\right)^2\right)^{-(\nu+1)/2} & e_t < -a/b \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{be_t+a}{1+\beta}\right)^2\right)^{-(\nu+1)/2} & e_t \geq -a/b \end{cases} \quad (2)$$

where e_t is the standardized log return, and the constants a, b and c are given by $a = 4\beta c\left(\frac{\nu-2}{\nu-1}\right)$,

$b^2 = 1 + 3\beta^2 - a^2$, and $c = \dfrac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The density function has a mode of $-a/b$, a mean of zero, and a

unit variance. The density function is skewed to the right when $\beta > 0$, and vice-versa when $\beta < 0$. The Skewed t distribution specializes to the standard Student's t distribution by setting the parameter $\beta = 0$.

iii. Normal inverse Gaussian distribution (NIG)

$$f(e_t \mid \mu, \alpha, \beta, \delta) = \frac{\alpha \delta K_1(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\pi\sqrt{\delta^2 + (e_t - \mu)^2}} \exp(\delta\sqrt{\alpha^2 - \beta^2} + \beta(e_t - \mu)) \tag{3}$$

where $K_\lambda(\cdot)$ is the modified Bessel function of the third kind and index $\lambda = 0$ and $\alpha > 0$. The NIG distribution is

specified as in Prause (1997). The NIG distribution is normalized by setting $\mu = -\dfrac{\delta\beta}{\sqrt{\alpha^2 - \beta^2}}$ and

$\delta = \dfrac{\left(\sqrt{\alpha^2 - \beta^2}\right)^3}{\alpha^2}$, which implies $E(e_t) = 0$ and $Var(e_t) = 1$.

iv. Generalized hyperbolic distribution

$$f\left(e_t \mid p, b, g\right) = \frac{g^p}{\sqrt{2\pi}\left(b^2 + g^2\right)^{\frac{1}{2}\left(p - \frac{1}{2}\right)} d\left(p, b, g\right) K_p\left(g\right)} q\left(\frac{e_t - m\left(p, b, g\right)}{d\left(p, b, g\right)}; p, b, g\right) \tag{4}$$

where $\tilde{R}_n \square \dfrac{K_{n+p}\left(g\right)}{g^n K_p\left(g\right)}$, $\quad d\left(p, b, g\right) \square \left[\tilde{R}_1 + b^2\left\{\tilde{R}_2 - \tilde{R}_1^2\right\}\right]^{-\frac{1}{2}} \geq 0$, $\quad$ and $\quad m\left(p, b, g\right) \square -b\, d\left(p, b, g\right) \tilde{R}_1$.

p, b and g are parameters. The generalized hyperbolic distribution is a standardized version of Prause (1997).

III. DATA

We fit the heavy tailed distributions with the normalized equity market index returns of New Zealand. We choose the NZX 50 Index as it is the main stock market index in New Zealand. The index is designed to measure the performance of the 50 largest, eligible stocks listed on the Main Board (NZSX) of the NZX by float-adjusted market capitalization. Representative, liquid, and investable, it is widely considered New Zealand's preeminent benchmark index. The index is float-adjusted, covering approximately 90% of New Zealand equity market capitalization. We collected the standardized NZ 50 daily dividend-adjusted close returns from Yahoo Finance for the period from January 2, 2003 to July 6, 2017, covering all the available data in Yahoo Finance. There are in total 3560 observations. Figure 1 illustrates the dynamics of NZ 50, and the two biggest spikes were observed on October 9, 2008 and October 13, 2008. The recent financial crisis also witnessed significant volatility in the financial market.
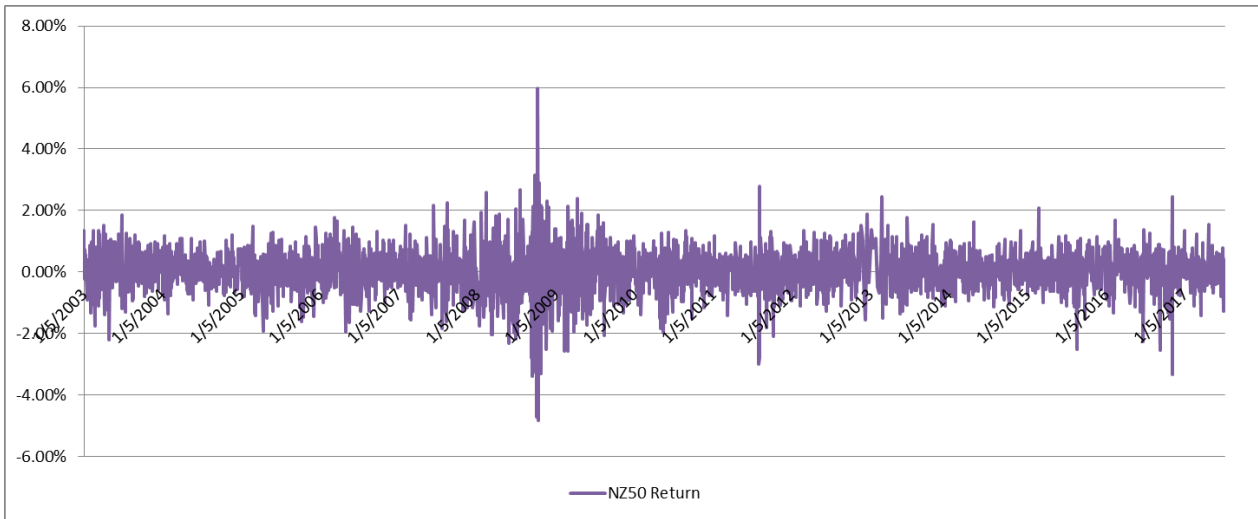
Figure 1: NZ50 returns

Table 1 exhibits basic statistics of the NZ 50 returns. The results show the NZ 50 daily returns are leptokurtotic and negatively skewed. The extreme downside move is slightly less than the extreme upside move, which is at odds with most of major stock market indexes over the world.

Table 1: Descriptive statistics

min	max	mean	std	skewness	kurtosis
-4.82%	5.99%	0.04%	0.68%	-0.35	5.02

Figure 2 is the histogram of the raw data. We fit the returns by the Gaussian distribution and the Student's *t* distribution. The upper panel in Figure 2 is fitted by the normal distribution. Clearly, the Student's *t* distribution has a much better in-sample goodness of fit.
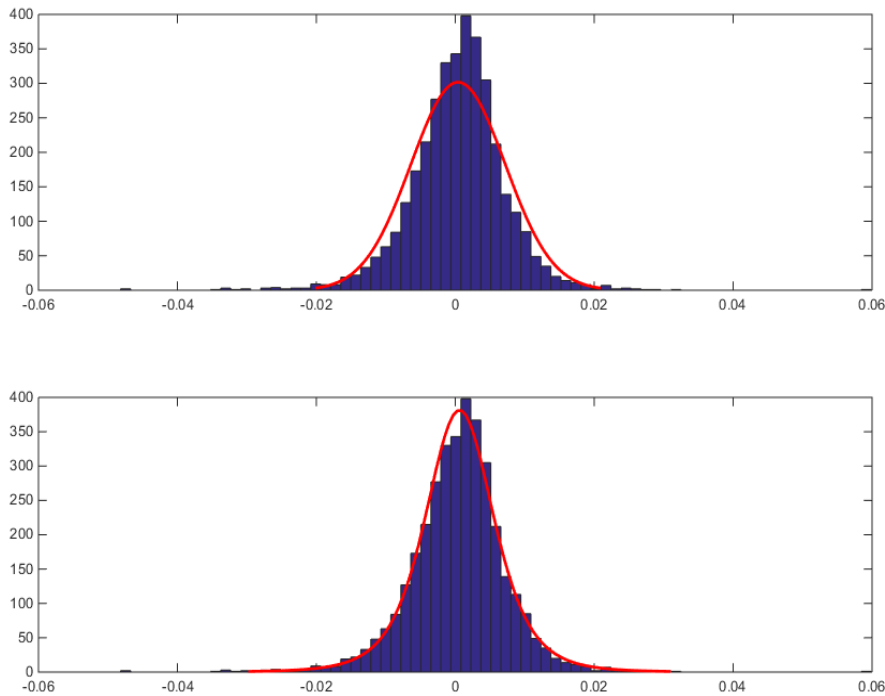

Figure 2: NZ50 returns - Normal vs. Student's *t*

IV.   EMPIRICAL RESULTS

a)   Parameters Estimation

The raw return series is normalized to allow zero mean and unit standard deviation. The series is fitted by the maximum likelihood estimation (MLE) method and the estimation results of the key parameters are given in Table 2. All the parameters are significantly different from zero at 10% significance level.

Table 2: Estimated values of key parameters

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
Symmetric	Y	Y	N	N	N
Fat-tailed	N	Y	Y	Y	Y
Estimated Parameters		Nu=4.56	Nu=4.63; beta=-0.034	alpha=1.54; beta=-0.054	p=-1.235; b=-.063; g=0.367

b)   Goodness of Fit

Based on Huber-Carol, et al. (2002) and Taeger and Kuhnt (2014), we investigate the four heavy-tailed distributions and the benchmark normal distribution in fitting the NZ50 daily returns through four different criteria: (i) Kolmogorov-Smirnov statistic; (ii) Cramer-von Mises criterion; (iii) Anderson-Darling test; and (iv) Akaike information criterion (AIC).

a. Kolmogorov-Smirnov statistic is defined as the maximum deviation between empirical CDF (cumulative distribution function) $F_n(x)$ and tested CDF $F(x)$:

$$D_n = \sup_x |F_n(x) - F(x)|, \qquad (5)$$

where $F_n(x) = \dfrac{1}{n} \sum_{i=1}^{n} I_{[-\infty, x]}(X_i)$.

b. Cramer-von Mises criterion is defined as the average squared deviation between empirical CDF and tested CDF:

$$T = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \, dF(x) = \frac{1}{12n} + \sum_{i=1}^{n} \left[\frac{2i-1}{2n} - F_n(x_i) \right]^2 \qquad (6)$$

c. Anderson-Darling test is defined as the weighted-average squared deviation between empirical CDF and tested CDF:

$$A = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1 - F(x))} \, dF(x),$$

and the formula for the test statistic A to assess if data comes from a tested distribution is given by:

$$A^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n} \left[\ln(F(x_i)) + \ln(1 - F(x_i)) \right]. \qquad (7)$$

d. Akaike information criterion (AIC) is defined as:

$$AIC = -2k - 2\ln(L), \qquad (8)$$

where L is the maximum value of the likelihood function for the model, and k is the number of estimated parameters in the model.

The comparison results are showed in Table 3, indicating the Skewed t distribution has the best goodness of fit compared with other selected types of distribution, followed by the generalized hyperbolic distribution, and the Student's t distribution.

Table 3: Comparison of selected types of distribution

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
K-S Test	0.015	0.007	0.005	0.008	0.006
Cv-M Test	0.021	0.014	0.011	0.015	0.013
A-D Test	1.43	1.21	1.13	1.19	1.15
AIC	12811.8	12201.2	11976.3	12320.4	12108.7

c) Stress Test Scenarios

We adopt Value at Risk (VaR) to calculate stress test scenarios. In quantitative risk management, VaR is defined as: for a given position, time horizon, and probability *p*, the *p* VaR is defined as a threshold loss value, such that the probability that the loss on the position over the given time horizon exceeds this value is *p*.with the estimated parameters in Section 4.1, we calculate VaRs for different confidence levels:

$$VaR_\alpha(e_t) = \inf\{e \in \Box : P(e_t > e) \le 1 - \alpha\}, \quad (9)$$

where $\alpha \in (0,1)$ is the confidence level. We select the following levels for downside moves: {99.99%, 99.95%, 99.9%, 99.5%, 99%}, and for upside moves: {0.01%, 0.05%, 0.1%, 0.5%, 1%}. From Equation (9), the stress test scenarios based on the VaR levels are given as in Table 4. Table 4 indicates that the Skewed *t* distribution has the closest VaRs to the nonparametric historical VaRs compared with other types of distributions.

Table 4: Scenarios for NZ50 shocks

Left Tail					
Confidence	99.99%	99.95%	99.90%	99.50%	99.00%
Empirical	-4.82%	-4.35%	-3.34%	-2.45%	-1.92%
Normal	-2.56%	-2.21%	-2.03%	-1.74%	-1.56%
T	-4.59%	-4.05%	-3.65%	-2.88%	-2.34%
Skewed T	-4.76%	-4.38%	-3.19%	-2.51%	-2.01%
NIG	-4.01%	-3.67%	-3.12%	-2.81%	-2.31%
GH	-4.61%	-4.13%	-3.46%	-2.67%	-2.11%

Right Tail					
Confidence	0.01%	0.05%	0.10%	0.50%	1.00%
Empirical	5.99%	3.09%	2.77%	2.09%	1.69%
Normal	2.56%	2.21%	2.03%	1.74%	1.56%
T	4.59%	4.05%	3.65%	2.88%	2.34%
Skewed T	5.26%	3.23%	2.89%	2.15%	1.75%
NIG	4.42%	3.89%	3.22%	2.75%	2.31%
GH	4.77%	3.98%	3.34%	2.71%	2.06%

V. Conclusions

Stress testing is a tool to assess the resilience of financial institutions to a hypothetical adverse event, usually a severe but plausible economic downturn. Introducing a comprehensive stress testing framework for the New Zealand banking system is a strategic priority for the Reserve Bank. In this paper, we focus on the NZX 500 index, the most important risk factor in the banking sector, and develop a methodology to construct its stress test scenarios. By comparing empirical performance of different statistical distributions, our results show the Skewed *t* distribution could generate the most suitable stress test scenarios for NZ50.

There are two directions for further research. First, one may introduce the fat-tailed distributions to the generalized autoregressive conditional heteroskedasticity (GARCH) framework and study their implications in designs of stress test scenarios. Second, one may combine the risk factor of stock market returns with other risk factors, such as unemployment and interest rates, and investigate how to extend the Skewed *t* distribution to the tail-dependence framework for designs of stress test scenarios.

References Références Referencias

1. Akaike, H. (1973). "Information theory and an extension of the maximum likelihood principle." in Petrov, B.N.; Csáki, F., *2nd International Symposium on Information Theory*, Tsahkadsor, Armenia, USSR, pp. 267–281.
2. Barndorff-Nielsen, O. (1977). "Exponentially decreasing distributions for the logarithm of particle size." *Proceedings of the Royal Society*, vol. 353, pp. 401-419.
3. BCBS (2011). "Basel III: international regulatory framework for banks." Bank for International Settlements, working paper.
4. Breuer, T., M. Jandacka, K. Rheinberger and M. Summer (2009). "How to find plausible, severe, and

useful stress scenarios." *International Journal of Central Banking*, vol. 5, pp. 205-224.

5. Choi, D., V. Fang and T.Y. Fu (2009). "Volatility spillovers between New Zealand stock market returns and exchange rate changes before and after the 1997 Asian Financial Crisis." *Asian Journal of Financial & Accounting*, vol. 1, pp. 106-117.

6. Darne, O., G. Levy-Rueff and A. Pop (2013). "Calibrating initial shocks in bank stress test scenarios: an outlier detection based approach." Banque De France, working paper.

7. Figueroa-Lopez, J., S. Lancette, K. Lee and Y. Mi (2011). "Estimation of NIG and VG models for high frequency financial data." in *Handbook of Modeling High-Frequency Data in Finance*, edited by F. Viens, M.C. Mariani and I. Florescu, John Wiley & Sons, Inc., USA.

8. Firth, M. (1997). "An analysis of the stock market performance of new issues in New Zealand." Pacific-Basin Finance Journal, vol. 5, pp. 63-85.

9. Flood, M. and G. Korenko (2014). "Systematic scenario selection: stress testing and the nature of uncertainty." *Quantitative Finance*, vol. 15, pp. 43-59.

10. Glasserman, P., C. Kang and W. Kang (2014). "Stress scenario selection by empirical likelihood." *Quantitative Finance*, vol. 15, pp. 25-41.

11. Guo, Z. (2017). "Heavy-tailed distribution and risk management of equity market tail events." *Journal of Risk & Control*, vol. 4, pp. 31-41.

12. Hansen, B. (1994). "Autoregressive conditional density estimation." *International Economic Review*, vol. 35, pp. 705-730.

13. Hargreaves, D. and G. Williamson (2011). "Stress testing New Zealand Banks' dairy portfolios." *Reserve Bank of New Zealand Bulletin*, vol. 74, pp. 15-25.

14. Huber-Carol, C., N. Balakrishnan, M. Nikulin and M. Mesbah (2002). *Goodness-of-Fit Tests and Model Validity*, Springer.

15. IMF (2017). "Stress testing the banking sector and systemic risk analysis on New Zealand – technical note." International Monetary Fund, *IMF Country Report* No. 17/119.

16. Jang, B. K. and M. Kataoka (2013). "New Zealand Banks' vulnerabilities and capital adequacy." IMF working paper, WP/13/7.

17. Kapinos, P. and O. Mitnik (2016). "A top-down approach to stress-testing banks." *Journal of Financial Services Research*, vol. 49, pp. 229-264.

18. Mandelbrot, B. (1963). "New methods in statistical economics." *Journal of Political Economy*, vol. 71, pp. 421-440.

19. Prause, K. (1999). "The generalized hyperbolic model: estimation, financial derivatives, and risk measures." Ph.D. Dissertation.

20. Reserve Bank of New Zealand (2015). "Stress testing methodology for New Zealand incorporated banks." *Reserve Bank of New Zealand Discussion Document*, Ref: #6548977.

21. Taeger, D. and S. Kuhnt (2014). "Goodness-of-fit tests." *Statistical Hypothesis Testing with SAS and R*, Wiley Online Library.

22. Zhu, D. and J. Galbraith (2012). "A generalized asymmetric Student-t distribution with application to financial econometrics." *Journal of Econometrics*, vol. 157, pp. 297-305.

20

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