Estimating the United States Dollar Index Returns' Value at Risk: Empirical Evidence from RiskMetrics and Simultaneous Bootstrap Quantile Regression Methods

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I. Introduction

Financial data are almost always leptokurtic or fat-tailed, meaning that they do not lie on a bell shape curve. This feature leads estimates obtained from methods based on the assumption of normality (OLS, GARCH family models, RiskMetrics, etc.) to suffer from violation of normality due to extreme returns caused by unpredictable firm-specific events (Longin and Solnik, 2001; Ang and Chen, 2002; Patton, 2006; Brooks, 2008; Alexandra, 2014). Data transformations such as outliers' removal, Box-Cox transformations are often applied. All these transformations would consequently lead to spurious regressions and distort or bias results or inferences, thus, underestimate the riskiness of the investment or portfolio of investments (Box and Cox, 1964; Sakia, 1992; Szilárd and Imre, 2000).

Indeed, as the nature of the risk has changed over time, measuring methods must adapt to recent experiences(Szilárd and Imre, 2000; Robert and Simon, 2004). Value at Risk (VaR) is the most used in the financial industry, as it shows risk in terms of money. To compute the VaR, investors use different methods. Unlike the RiskMetrics method based on the assumption of normality, quantile regression allows making

inferences with distribution-free. However, in previous papers, quantile regression to VaR calculation was used on financial securities different from the United States dollar index (Kupiec, 1995; Manganelli and Engle 2001; Pierre, 2003; Wagner et al., 2008; Nieto and Ruiz 2008; Olli, 2009; Navneet, 2011; Alex, 2012; Schaumburg, 2012).

This paper differs from others as it backtests and compares RiskMetrics and simultaneous bootstrap quantile regression methods to establish which one is a better estimate of the U.S. dollar index returns' VaR. The research will answer to the following question: *"After controlling for marginal effects of the optimal hedge ratio of the U.S. dollar index futures and returns volatility in both spot and futures markets, what is the better VaR estimate method of the U.S. dollar index returns?"*

II. Theoretical Framework

a) Risk and risk basis in hedging

Instabilities characterize the financial markets. The future price of a security is source of risk in markets because first, we are uncertain about what that price will be in the future. Second, it affects utility. Indeed, if the price is high and we are investing, we can consume more from the proceeds of selling the security and increase the utility. Alternatively, we will consume less from the proceeds of buying the security (Baz and Charcko, 2003).

Knowing the spot price, basis reflects the relationship between cash or spot price (S) and futures price (F).[1] It can be over or under as S is higher or lower, respectively, than F. It is still generally less volatile than either the cash or futures price. Thus basis is less risky than both spot and futures price. Note that basis $(S_0 - F_0)$ is known by investors on the initiation day, but unknown on the day the hedge will be lifted because $S_1 - F_1$ is a random variable, and the hedger faces *"basis risk,"* which is the chance that the basis will be strengthened (an increase) or weakened (a decrease) from the time the hedge has been implemented to the

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[1] The today's price in the spot market is the underlying product in futures trading.

time when it will be removed (Benhamou, 2005; Hecht, 2017).

Figure 2-1 shows the change of the spot and futures prices over time. Both prices tend to converge with the time passing by. At time t_1, $S_1 - F_1$ is higher than at time t_2.
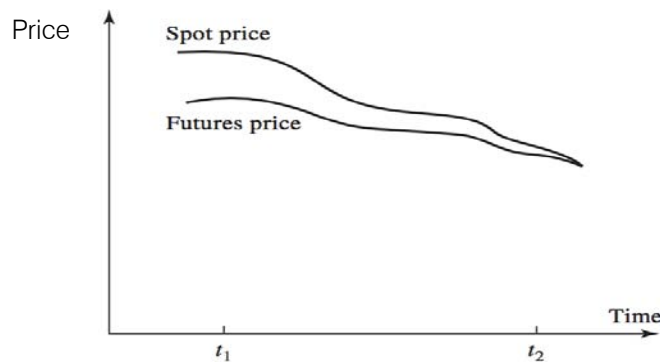
Source: Baz, J. & Chacko, G. (2009)

Figure 2-1: Basis over time

For the reasons above, hedging ambiguously cannot eliminate risk in the underlying asset. To this purpose, investors have to determine and compare the hedge ratio, which is "*the ratio of the size of the portfolio taken in futures contracts to the size of the exposure*" (Asim, 1993; Hull, 2003). Over the years, methods of calculating the optimal hedge ratio written on a variety of assets (indices, commodities, stocks, and foreign exchange) have been developed: regression method, family GARCH models, which neglect the relationships between returns across quantiles (Sheng-Syan et al., 2003; Bartoszand Zhang, 2010; Mária and Michal, 2016). Thus the risk is still real and will be taken into account, and recent financial disasters have emphasized the need for accurate risk measures. But, as the nature of the risk has changed over time, these methods must adapt to recent experience. Value at Risk is the most recent risk measure (Szilárd and Imre, 2000; Robert and Simon, 2004).

b) Value at Risk

Value at Risk model reports the maximum loss that can be expected by an investor, at a significance level, over an a given trading horizon. Indeed, VaR was developed in the 1990s to provide senior management with a single number that could quickly and easily incorporate information about the risk of a portfolio (Piazza, 2009). Nevertheless, examining different works, its origins lie further back 1990s, traced back as far as around 1922 to capital requirements for the U.S securities firms of the early 20th century. Harry Markowitz (1952) and others developed VaR measure in portfolio theory, where investors have to optimize rewards for a given risk (Glyn Holton, 2002). Nowadays, VaR is part of every risk manager's toolbox because it characterizes risk *in terms of money* while most measures (standard deviation, Sharpe, etc.) show risk as a percentage (Hull and Alan, 1998).

Due to the numerous methods used, which often sound complicated, VaR sounds like a difficult method to evaluate risk. Those models differ in many aspects with common structure summarized in: (1) The portfolio is marked-to-market daily; (2) the distribution of the portfolio returns is estimated, and (3) the VaR is computed (Robert and Simone, 2004).[2] Basing on the second aspect, we can classify the VaR methods into two broad categories: factor models, such as RiskMetrics, and portfolio models, such as historical quantiles.

i. *RiskMetrics method to VaR estimation*

Denoting x_t as the daily loss random variable and the information set available at time $t-1$ by $\mathcal{F}_{t-1}$, RiskMetrics, a benchmark for measuring risk, assumes that $x_t | \mathcal{F}_{t-1} \sim N(0, \sigma_t^2)$; where σ_t^2 is the conditional variance of x_t, and it evolves according to the model: $\sigma^2 = \alpha\sigma^2 + (1-\alpha) x^2; \ 0 < \alpha < 1$. By this way, RiskRetrics ignores the fat-tails in the distribution function (Mina and Xiao, 2001). To account *volatility clustering* of the time series, the RiskMetrics method incorporates an autoregressive moving average process to model the price; and avoid the problems related to the uniformly weighted moving averages by using the exponentially weighted moving average (EWMA) method (Szilárd and Imre, 2000; J. P Morgan, 1996; Pafka, 2001; Tsay 2011).

Based on previous studies of Giot and Laurent (2003), Khan (2007), Emilija and Dorié(2011), Askar

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[2] Three approaches have been used with variations within each. (1) The historical method uses statistical analysis of past losses to determine a loss. (2) The variance method utilizes statistical analysis assuming that returns lie on a bell shape curve based on the average returns and standard deviation. (3) Monte Carlo method involves developing a model and using to predict future investment prices and then using that in statistical analysis to determine the worst-case loss on the investment or portfolio.

(2013), Deepika (2014), VaR can be calculated through basic GARCH models (GARCH and RiskMetrics), asymmetric models (EGARCH, IGARCH, and GJR-GARCH) and FIGARCH. However, while the sum of parameters in GARCH models is almost close to unity, in the IGARCH model that sum is considered equal to one, which means that the return series is not covariance stationery, and there is a unit root in the GARCH process (Jensen and Lange, 2007). Thus, IGARCH models are unit-root GARCH models. Furthermore, Jensen and Lange argued that "*the*

conditional variance of the GARCH model converges in probability to the true unobserved volatility process even when the model is misspecified, and the IGARCH effect is due to the mathematical structure of the GARCH model and not a property of the true data-generating mechanism."Similar to ARIMA models, in IGARCH models, the impact of past squared shocks $\eta = a^2{}_{t-1} - \sigma^2{}_{t-1}$ for t> 0 ona^2 is persistent. Therefore, IGARCH (1,1) model, proposed by Engle and Bollerslev (1986), is:

$$a_t = \sigma_t \varepsilon_t, \sigma^2{}_t = \alpha_0 + \beta_1 \sigma^2{}_{t-1} + (1 - \beta_1)\alpha^2{}_{t-1} + \varepsilon_t \text{Equation} \qquad (2.1)$$

Where $\{\varepsilon_t\}$ is greater than 0, and $1 > \beta_1 > 0$.

ii. *Simultaneous bootstrap quantile regression method to VaR estimation*

Quantile regression involves constructing a set of regression curves each for different quantiles of the conditional distribution of the dependent variable. In finance, quantile regression can be used to assess risk management (Koenker and Bassett, 1978; Keming Yu et al., 2003).

Indeed, as Koenker and Bassett(1978) extended the notion of quantile to a linear regression model, then VaR is estimated as it includes the least absolute deviation (LAD) model, robust than OLS

estimators (Roger and Kevin, 2001). Later, analysis of linear regression quantile models has been extended to cases with heteroscedastic (Koenker and Bassett1982) and non-stationery dependent errors (Portnoy 1991), time series models (Bloomfield and Steiger 1983), simultaneous quations models (Amemiya 1982; Powell 1983), censored regression models (Powell 1986; Buchinsky and Hahn 1998).

While the OLS model estimates the conditional mean of a given $Y = \beta X_t + \varepsilon$ as $E(Y|X = x = \beta x_t$, the quantile estimate however is as specified in equation 2.2 (Huixia, Wang, 2007).

$$Q_\tau = (Y|X = x) = x^T \beta(\tau), 0 < \tau^* < 1 \qquad \text{Equation} \qquad (2.2)$$

Where $\beta(\tau)$ is the marginal change in the τth quantile due to the marginal change in x.

Similarly, if R_t denotes the return of investment at time t, and τ^* denotes a (pre-determined) significance interval, the respective VaR is implicit: $\Pr[R_t < VaR|\mathcal{F}_{t-1} = \tau^*$; where $\mathcal{F}_{t-1}$is the information set available at time$t - 1$. Thus, VaR is the τth conditional quantile of R_t; in other words, the one-step-ahead forecast of the τth quantile of R_t based on the information available up to period$t - 1$. From Equation 2.2, finding a VaR is equivalent to finding the conditional quantile of R_t. Following Christoffersen et al. (2001), Engle and Patton (2001), volatility can be treated as a regressor. Since the *simultaneous* analysis can borrow information across cases and offers tighter inference, the *bootstrap method* through the *"XY-pair method"* can be used to know the returns distribution, construct confidence intervals, and approximate coefficients standard errors.

For these reasons, simultaneous quantile regression can be used to compute the U.S. dollar index returns' VaR after the control of the marginal effects of the hedging strategy and returns volatility. This way, differences the research from empirical studies of Robert and Simone (2004), Taylor (2008), Piazza et al. (2009), Schaumburg (2012), Huang (2012), etc.

III. METHODOLOGY

a) *Data design*

The dataset comes from the global financial portals Investing.com (Madrid, Kingdom of Spain) and Quandl (Toronto, Canada) from 20 November 1985 to 15 February 2017. This period covers the first futures contract on the U.S dollar index as well as the international financial crisis. After the gathering process, data were arranged and analyzed using the R-Software version 3.3.1 (2016-06-21) and Excel. First of all, the daily closing prices of the U.S. dollar index and the nearby contract settlement prices of its futures contract were converted to continuously compounded returns by taking the first difference of the logarithm prices as given in equation (3.1).

$$R_{i,t} = d\left(log(\frac{P_{i,t}}{P_{i,t-1}})\right) \text{Equation} \qquad (3.1)$$

Where: $R_{i,t}$ the continuously compounded daily return at day t; $P_{i,t}$ the closing and settle prices at day t; $P_{i,t-1}$ the closing and settle prices at day $t - 1$ and d the first difference.

Then, returns volatility was extracted from the same data using the GARCH (1,1) model and used to bootstrap the simultaneous quantile regression with 200 replications with the purpose of capturing market risk

(actual negative returns). From the Box and Whisker Plots and the formal test following Seo (2002), all outliers detected in the dataset were deleted to eliminate their effects for RiskMetrics method since quantile regression is robust to outliers (John, 1995; Jason and Amy, 2004, Irad, 2005; Hans-Peter et al., 2010, Williams, 2016).At the end of all these analyses, two final samples were obtained: (1) a sample of 7891 observations (representing 99% of the entire dataset) without outliers and (2) another of 7950 days (representing 100% of the dataset) with outliers. After running the two models, ANOVA test and backtest at 95% confidence level were conducted to evaluate marginal effects and the accuracy of VaR.

b) Modeling approach

Following Christoffersen et al. (2001), the expanded quantile regression to VaR calculation is:

$$R_{s,t,\tau} = \beta_0 + \beta_1 * R_{ft} + \beta_2 * R_{sv,t-1} + \beta_3 * R_{fv,t-1} + \varepsilon_t \text{Equation} \tag{3.2}$$

Where:$R_{s,t,\tau}$ the τth conditional quantile of the U.S. dollar index returns at time t; R_{ft} the U.S. dollar index futures return at time t; $R_{sv,t-1}$ the U.S. dollar index returns volatility at time t-1; $R_{fv,t-1}$the U.S. dollar index futures return volatility at time t-1 and ε the Error term.

From the RiskMetrics perspective, VaR for the U.S. dollar index returns with a simple Integrated-GARCH (1,1) model is:

$$\sigma^2{}_t = \beta_1\sigma^2{}_{t-1} + (1 - \beta_1) * R_{s,t}{}^2{}_{t-1} \text{Equation} \tag{3.3}$$

Where $R_{s,t}$: the U.S. dollar index returns at time t (returns in the spot market).Basing on the equation 3.3, for the next trading day, VaR is the one-step volatility times the tail probability.

c) Definition of variables

i. Regressand: The U.S. dollar index returns

The U.S dollar index (DXY) is a geometrically averaged of six currencies (the Canadian Dollar, the Euro, the British Pound, the Japanese Yen, the Swedish Krona, and the Swiss Franc) against the U.S dollar. Originally, the DYX was developed by the U.S Federal Reserve in 1973 to provide an external bilateral trade-weighted average value of the U.S. dollar as it freely floated against global currencies. In fact, since the inception of futures trading on the DXY in1985, the Intercontinental Exchange (ICE) futures U.S compiles, maintains, determines, and weights the components of the DXY (Redfield, 1986; Hanan et al., 1998).

Furthermore, before the introduction of the Euro to the financial community, the DXY contained ten currencies: the ones that the index includes plus the West German Mark, the French Franc, the Italian Lira, the Dutch Guilder, and the Belgium Franc. Figure 3-1 presents the major currencies included in the index and their corresponded weights. With its 19 countries, the Euro makes up a big chunk of the index (>50%). It is followed by Japanese yen. This position would make sense since Japan has one of the biggest economies in the world. And others with about 25.2%.
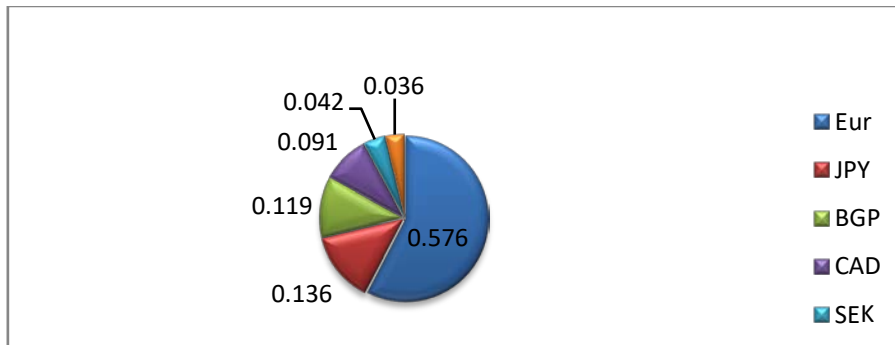
Source: Musolongo (2017)

Figure 3-1: Currencies included in the U.S. dollar index and their weights

Following Eytan and al.(1998), the index price traded as a futures contract on the ICE is:

$$U.S.Dollar\ index = 50.14348112 * EURUSD^{-0.576} * USDJPY^{0.136} * GBPUSD^{-0.119} * USDCAD^{0.091} * USDSEK^{0.042} * USDCHF^{0.036} \quad \text{Equation} \tag{3.4}$$

Where:

EURUSD — The exchange rate between the EURO and the U.S. dollar

USDJPY — The exchange rate between the Japanese Yen and the U.S. dollar

GBPUSD — The exchange rate between the British Pound and the U.S. dollar

USDCAD — The exchange rate between the Canadian Dollar and the U.S. dollar

USDSEK   The exchange rate between the Swedish Krona and the U.S. dollar

USDCHF   The exchange rate between the Swiss Franc and the U.S. dollar

From equation (3.4), when the U.S. dollar is the base currency, the index is positive; and negative when it is the quote currency. The price used for the index is the mid-point between the top of the book bid/offer in the component currencies. And at last, the prices of the

U.S. dollar index futures contract set by the market reflect the interest rate differentials between the respective currencies and the U.S. dollar (ICE futures U.S., 2015).
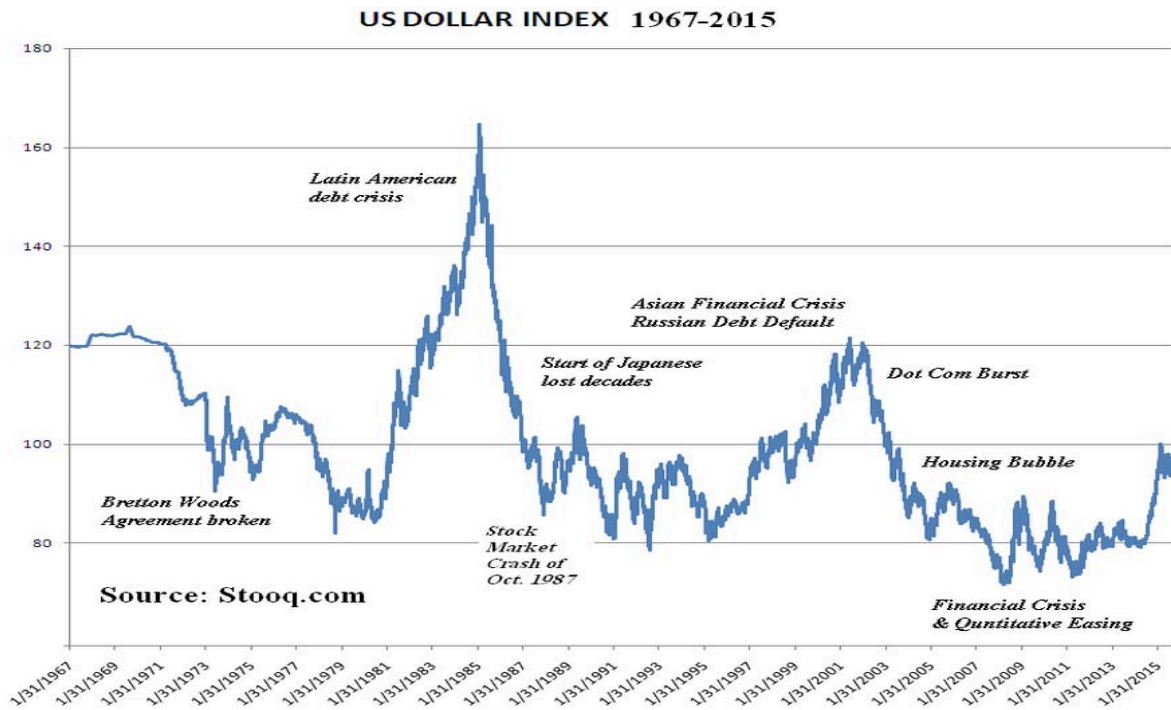

Figure 3-2: The U.S. dollar index and the financial events

Note: The U.S dollar index reached its high point in February 1985 and its low in March 2008. Compare to June 2016, the dollar has depreciated versus the basket of currencies. The U.S macroeconomic factors, as well as of the six countries' currencies included in the index impact.

ii. *Regressors*

- *The U.S. dollar index futures returns: the hedge ratio*
The U.S. dollar index futures contract began trading on 20 November 1985, on the Financial Instruments Exchange, a part of the New York Board of Trade (NYBOT). Two factors influence the pricing process of contract on this index: (i) the fact that the index is a geometric average, rather than an arithmetic, of the constituent currencies. Therefore, the divergence between the geometric and arithmetic averages depends on both volatilities of the individual currencies and their co-movements; (ii) the FOREX rates in the index (U.S. dollars per foreign exchange rate) are in the

denominator, implying that a dollar appreciation leads to a higher index level or a lower level in case of depreciation (Hanan, 1988).

- *The U.S. dollar index and futures returns volatilities*
At the end of the day in both spot and futures markets, we can use the price fluctuations between the high and low prices to measure the returns volatility (Gilroy, 2014). Thus, the U.S. dollar index and futures returns volatilities are simply the movements in the first difference of the logarithm of the U.S. dollar index and futures returns. We applied the GARCH (1,1) model to extract both the U.S. dollar index and futures returns volatilities, as written in the following equations.

$$\begin{cases} R_{st} = \alpha_1 R_{st-1} + a_t \\ \sigma^2_t = \alpha_0 + \alpha_1 a^2_{t-1} + \beta_1 \sigma^2_{t-1} \end{cases} \qquad \text{Equation} \qquad (3.5)$$

$$\begin{cases} R_{ft} = \alpha_1 R_{ft-1} + a_t \\ \sigma^2_t = \alpha_0 + \alpha_1 a^2_{t-1} + \beta_1 \sigma^2_{t-1} \end{cases} \qquad \text{Equation} \qquad (3.6)$$

Where $a_t = \sigma_t \varepsilon_t$, $R_{st}, R_{st-1}, R_{ft} and R_{ft-1}$

From equations (3.5) and (3.6), the estimation was done in two parts. First, the GARCH (1,1) model was used for the U.S. dollar index returns volatility extraction and then GARCH(1,1) model produces the U.S. dollar index futures return volatility.

IV. RESULTS AND DISCUSSION

a) Marginal effects: optimal hedge ratio estimation from quantile regression

i. Quantile regression and OLS estimates

In table 4-2 results from simultaneous bootstrap quantile regression across four quantiles (25, 50, 75, and 95% quantiles), and the OLS coefficients are summarized. Indeed, all parameters for the U.S. dollar index futures returns, which represent the optimal hedge ratios, are statistically significant (probability = 0) across quantiles. These estimates are not only significant at the confidence level, but are also big than the OLS estimates. The result is consistent with figure 4-1 in which estimated optimal hedge ratios across quantiles are above the OLS coefficients, meaning that quantile regression estimated optimal hedge ratios are different from the unique OLS hedge ratio coefficient (at the mean). It implies that the use of OLS, in this case, would underestimate parameters. Therefore the costs of the hedging strategy since quantile regression does not consider only the mean of the U.S. dollar index returns but across quantiles.

ii. ANOVA test

Table 4-1: Quantile regression analysis of deviance table

Model: U.S._dollar_index_returns~ U.S._dollar_index_futures_returns + U.S._dollar_index_returns_volatility + U.S._dollar_index_futures_returns_volatility Joint Test of Equality of Slopes: tau in {0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95}	
F value	Pr(>F)
22.192	< 2.2e-16 ***

Note: The Analysis of Variance (ANOVA) test is applied on coefficients across quantiles in table 4-1. With the probability of 0 percent, which is less than 5 percent of the significance level, parameters obtained from simultaneous quantile regression with 200 replications, with different taus, are high statistically different across quantiles. The result is consistent with the result in figure 4-1.

Table 4-2: Quantile regression and OLS estimates

U.S._dollar_index_returns						
Coef. Std. Err. t P>|t| [95% Conf. Interval] q25 |		Bootstrap				
U.S._dollar_index_futures_returns |	0.9504	0.0021	447.3800	0.000*+	-0.9463	0.9546
U.S._dollar_index_returns_volatility |	0.1636	0.0956	1.7100	0.0870	-0.0238	0.3511
U.S._dollar_index_futures_returns_volatility|	-0.2414	0.0887	-2.7200	0.007 *+	-0.4153	-0.0675
Intercept |	0.0001	0.0000	2.0000	0.045*	0.0000	0.0002
q50 |						
U.S._dollar_index_futures_returns |	0.9596	0.0017	558.8700	0.000*+	0.9563	0.9630
U.S._dollar_index_returns_volatility |	-0.0078	0.0531	-0.1500	0.8830	-0.1119	0.0963
U.S._dollar_index_futures_returns_volatility |	0.0317	0.0494	0.6400	0.5210	-0.0651	0.1286
Intercept |	-0.0001	309.0000	-3.1600	0.002 *	-0.0002	0.0000
q75 |						

```
-------------------------------------------------------------------------------------------------------
U.S._dollar_index_futures_returns |            0.9537    0.0025    389.2500    0.000*+    0.9489    0.9585
      U.S._dollar_index_returns_volatility |  -0.1853   0.0755    -2.4500     0.014*+   -0.3334   -0.0372
U.S._dollar_index_futures_returns_volatility| 0.3046    0.0712    4.2800      0.000*     0.1651    0.4442
Intercept  |                                  -0.0003   0.0000    -5.5500     0.000*+   -0.0004   -0.0002
-------------------------------------------------------------------------------------------------------

q95 |
-------------------------------------------------------------------------------------------------------
U.S._dollar_index_futures_returns |            0.9023    0.0086    104.8500    0.000*+    0.8855    0.9192

      U.S._dollar_index_returns_volatility |   0.6596    0.3020    2.1800      0.029*+    0.0676    0.1845

U.S._dollar_index_futures_returns_volatility| -0.2171   0.2728    -0.8000     0.4260    -0.7520    0.6479

Intercept|                                    -0.0007   0.0002    -3.8200     0.000*+   -0.0010   -0.0003
-------------------------------------------------------------------------------------------------------

OLS        |
-------------------------------------------------------------------------------------------------------
U.S._dollar_index_futures_returns |            0. 9020   0.00546   165.1202    0.000 *

      U.S._dollar_index_returns_volatility |  -0.0232   0.181     -0.1279     0.8982

U.S._dollar_index_futures_returns_volatility | 0.0306   0.168     0.1823      0.8554

 Intercept |                                  -0.00005  0.00010   -0.5025     0.6153
-------------------------------------------------------------------------------------------------------
```

Note: *: Coefficients significantly different from zero at 5% of the significance level. **+:* Significantly different from the OLS coefficients.
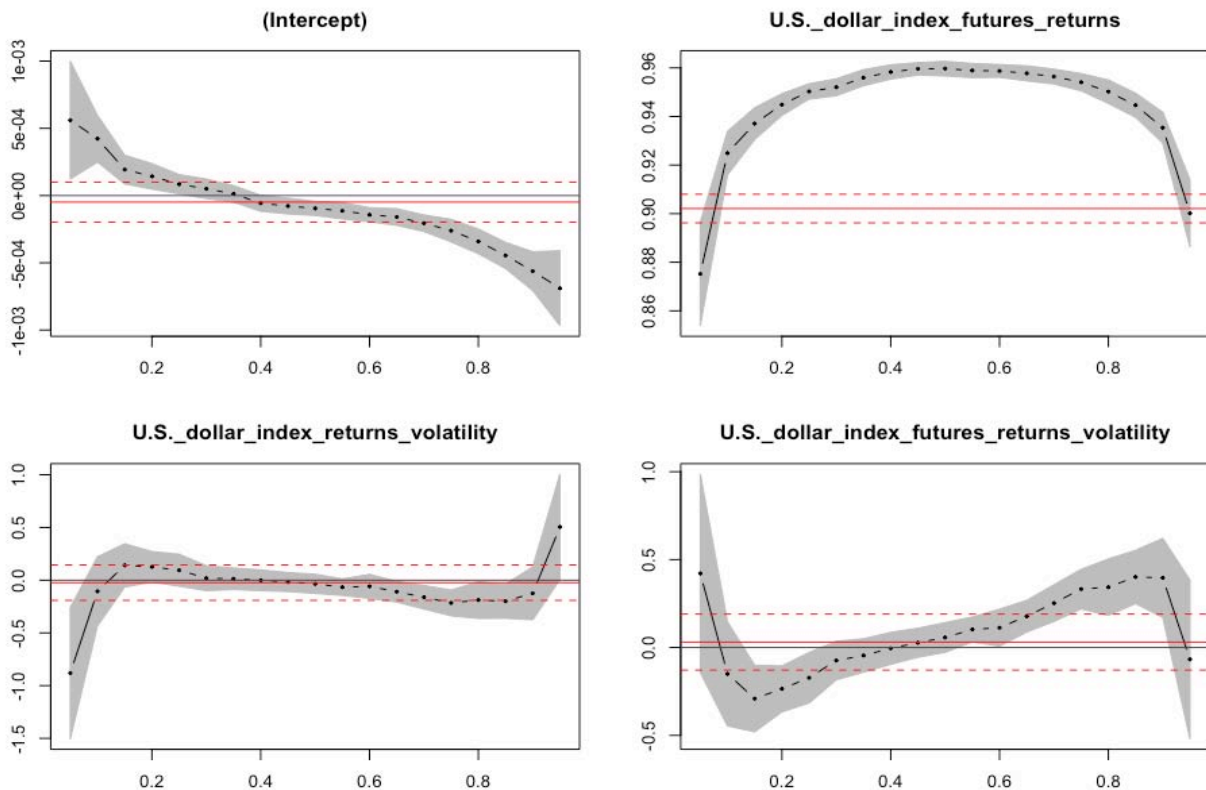
Figure 4-1: Quantile regression and OLS coefficients

Figure 4-1 illustrates first of all how coefficients change across quantiles, and secondly, how the magnitude of the marginal effects in various quantiles differs considerably from the OLS estimators, even in terms of the confidence intervals around each parameter. Moreover, for the OLS method, volatilities' coefficients and the intercept include most zero in their confidence intervals, meaning that these parameters do not have any marginal effects on the U.S. dollar index returns.

However, the intercept differs from zero in lower and higher quantiles; the U.S. dollar index and futures returns volatilities do affect the U.S. dollar index returns across quantiles and depending on the quantile. In some quantiles, the marginal effects are positive and negative elsewhere, implying that both returns volatilities have to be taken into account when estimating VaR. Furthermore, the U.S. dollar index futures return confidence intervals do not include the OLS coefficient, meaning that hedge ratios from quantile regression are all different from the OLS hedge ratio.

b) *VaR estimation*

i. *VaR from RiskMetrics method*

The RiskMetrics method with the Integrated GARCH (1, 1) was conducted to VaR calculation.

Table 4-3: Value at Risk

Probability	VaR (with outliers)	VaR (without outliers)
0.950	0.0067	0.0060
0.990	0.0095	0.0084
0.999	0.0127	0.0112

Note: In table 4-3 the maximum amount to be lost during downturn currency markets is a positive function of the confidence interval, meaning that Value at Risk increases with the increase of confidence level. At 95 percent of the confidence level, VaR, the maximum negative U.S. dollar index returns, is about 0.0067 dollars one-step-ahead, which means that if the market price goes down, investors who trade the U.S. dollar index will allow to lose up to 0.0067 US$ the following trading day. It seems a small amount for a small investment. However, this loss is highly significant for an investment of millions of dollars. Moreover, the action of removing outliers has a strong effect on the Value at Risk. As showed in table 4-3, VaR has decreased from 0.0067 to 0.0060, that is 11.7%. Meaning that the fat-tail in the dataset distorts the calculation and leads to a pseudo-VaR by overestimating it. For this reason, the VaR without was considered. The significance of this loss was compared through the backtesting process with VaR from quantile regression.

ii. *VaR from simultaneous bootstrap quantile regression method*
From the simultaneous bootstrap quantile regression method, VaR at the 95% quantile is:

$$VaR_{0.95} = -0.0007 + (0.006738994 * 0.9023) + (0.005103587 * 0.6596) + (0.00518553 * -0.2171)$$

$$VaR_{0.95} = 0.0076$$

At 95% of the confidence level, the maximum loss is about 0.0076 U.S dollars for any investment in the U.S. dollar index for the following trading day. It is important to mention that this VaR is the expected loss after controlling for the marginal effects of the U.S. dollar index futures returns and volatilities in both markets. Thus, investors will allow losing money up to 0.0076 dollars.

Results point out that quantile regression VaR estimated is big than the RiskMetrics estimate (after removing outliers), a difference of 0.0016 dollars.

c) *Robust test: Backtest to the accuracy of VaR*
How many losses exceed the VaR?

Table 4-4: Backtesting report

	RiskMetrics method		Quantile regression method At 95%
	IGARCH (1,1) with outliers	IGARCH (1,1) without outliers	
95% VaR (1)	0.0067	0.0060	0.0076
Number of exceptions (2)	369	397	398
Number of actual losses (3)	713	904	572
Violation Ratio = (3)/(2)	1.9	2.3	1.4

Note: with the backtesting process, the number of violations is much more in the IGARCH (1,1) than in the quantile regression (904 against 572 trading days) meaning that RiskMetrics 'VaR is allowing more number of negative U.S. dollar index returns. At 5% of the significance level, losses for investors using this method go beyond the VaR 507 times. The violation ratios are 2.3 for the RiskMetrics and 1.4 in quantile regression.

Furthermore, VaR from the RiskMetrics method is lower even using data with outliers (0.0067 and 0.0060 dollars) than the one from simultaneous bootstrap quantile regression (0.0076 dollars), a difference about 0.0016US$ for one unit of US dollar hedged with futures contract over the next day. This difference is understandable because volatilities and futures return marginal effects on the US$ index return are controlled. And globally, these variables are highly significant at 5% of the significance level.

V. CONCLUSION

Two methods, namely simultaneous bootstrap quantile regression and RiskMetrics with the IGARCH (1,1),were backtested and compared to establish the better Value at Risk (VaR) estimate for the United States dollar index returns.

Indeed, from the ANOVA test results, all coefficients were significantly different from each other across quantiles. While the OLS intercept and volatilities coefficients were not significant at 5% of the significance level, simultaneous bootstrap quantile regression results pointed out that these coefficients were different from zero in the lower and higher quantiles. Marginal effects along the distribution of the U.S dollar index returns were controlled, and hedge ratios across quantiles were also different from the unique OLS hedge ratio, meaning that the cost of the hedging strategy varied across quantiles and was higher than in the OLS method.

In a backtesting process, the RiskMetrics method did underestimate the maximum loss (VaR). The number of violations (2.3) was much more in the IGARCH (1,1) than in the quantile regression (1.4), which means that the RiskMetrics' VaR allowed more negative or loss U.S. dollar index returns. Furthermore, VaR from the RiskMetrics method was lower (0.0060$) than the one from simultaneous bootstrap quantile regression (0.0076$), about 0.0016US$ of difference for one US$ hedged with futures contract over the next trading day.

Thereby, quantile regression is the best VaR measure because: (1) it allows the hedge ratio calculation, which gives ideas to investors about the value of a position protected through the use of a hedging strategy, (2)the RiskMetrics model ignores the presence of fat-tails in the distribution function, which is a feature of financial data.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Annastiina, S. & Timo, T. (2008). "Multivariate GARCH models". SSE/EFI Working Pape Series in Economics and Finance No. 669, available at http://swopec.hhs.se/hastef/papers/hastef0669.pdf
2. Baz, J. & Chacko, G. (2009).*Financial derivatives Pricing, Applications, and Mathematics*. First edition, Cambridge University Press, New York
3. Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity". Journal of Econometrics, pp. 307–27.
4. Bollerslev, T. and Jeffrey M. (1992). "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances". Econometric Reviews, pp. 143–72.
5. Brooks, C. (2008). *Introductory Econometrics for Finance.* Second edition, Cambridge University Press, New York
6. Chen, J. (2001). "Investigations on Quantile Regression: Theories and Applications for Time Series Models". National Chung-Cheng University, mimeo.
7. Chen, M. and Chen, J. (2002). "Application of Quantile Regression to Estimation of Value at Risk". National Chung-Cheng University, mimeo.
8. Clifford, W. (1989). "Market Volatility: Causes and Consequences". 74 Cornell Law Review, pp. 953-962.
9. Constantinides, G. (2015).*Financial derivatives Futures, Forwards, Swap, Options, Corporate securities, and Credit Default Swaps.* World

Scientific Lecture Notes in Economics- Vol.1, Singapore

10. Davidson, A. & Hinkley, D. (1997). "Bootstrap Methods and their Application". Cambridge University Press

11. Deepika, C. (2014). "Evaluating Value at Risk Models: an application to the Johannesburg stock exchange". Master's Thesis, University of Cape Town

12. Engel, R. (2001). "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics". Journal of Economic Perspectives-Volume 15, Number 4, pp. 157–168.

13. Engle, R. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation". Econometrica, pp. 987–1007.

14. Engle, R. and Manganelli, S. (2004). "CA Via R: Conditional Autoregressive Value at Risk by Regression Quantiles". Journal of Business and Economic Statistics, Vol. 22, No. 4

15. Eytan, T. Hanan, Giora Harpaz, Steven Krull (1988). "The Pricing of Dollar Index Futures Contracts" Journal of Futures Markets, Volume 8, Number 2, pages 127-139.

16. Franses, H., Dijk, D. &, Opschoor, A. (2014).*Time series models for business and economic forecasting*. Second edition, Cambridge University Press, New York

17. James, W. et al. (2008). "Using Exponentially Weighted Quantile Regression to Estimate Value at Risk and Expected Shortfall". Journal of Financial Econometrics, University of Oxford, 2008, Vol". 6, pp. 382-406.

18. Joost, M. & Matthew, T. (1997). "Hedging Efficiency: A Futures Exchange Management Approach". The Journal of Futures Markets, Vol. 17, No. 5, 599–615.

19. Jui-Cheng, H. et al. (2006). "Hedging with zero-value at risk hedge ratio". Routledge, Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK

20. Keming, Y. et al. (2003). "Quantile regression: applications and current research areas". The Statistician, Part 3, pp. 331–350.

21. Koenker, R. (2005). "Quantile regression". Cambridge University Press.

22. Koenker, R. and Bassett, G. (1978). "Regression Quantiles". Econometrica 46, pp. 33-50.

23. Koenker, R. and Zhao, Q. (1996). "Conditional quantile estimation and inference for ARCH models". Econometric Theory. 12, pp. 793-813.

24. Koenker, R., et al. (2004). "Pessimistic portfolio allocation and choquet expected utility". Journal of Financial Econometrics 2, 477-492.

25. Meyers, G. et al. (1999). "Coherent Measures of Risk". Mathematical Finance 9, pp. 203-228.

26. Musolongo, C. (2017). "Value at Risk for the United States Dollar index returns: Empirical Evidence from Simultaneous Bootstrap Quantile Regression and RiskMetrics Methods". Master's thesis, Jiangxi University of Finance and Economics, China/

27. Nelson, B. (1991). "Conditional Heteroscedasticity in Asset Returns: A New Approach". Econometrica, pp. 347–70.

28. Olli, N. (2009). "Backtesting Value-at-Risk Models". Master's Thesis in Economics, Helsinki school of economics

29. Redfield, Corey B. (1986). "A Theoretical Analysis of the Volatility Premium in the Dollar Index Contract" .Journal of Futures Markets, Volume 6, Number 4, pages 619-627

30. Sakia, R. (1992). "The Box-Cox transformation technique: a review". The Statistician 41, Sokoine University of Agriculture, Department of Crop Science and Production Box 3005, Morogoro, Tanzania, pp. 169-178.

31. Schulze, N., (2004). "Applied Quantile Regression: Micro econometric, Financial, and Environmental Analyses". Inaugural-Dissertation zur Erlangung des Doktorgrades der Wirtschaftswissenschaftlichen Fakultät der Eberhard-Karls-Universität Tübingen, Tübingen, available at: https://www.deutsche-digitale-bibliothek.de/binary/TNSXMBNRYOGPWY FXOIFCBCWSSQUDMXI4/full/1.pdf

32. Shawkat, H. & Michael, M. (2013). "Risk management and financial derivatives: An overview". North American Journal of Economics and Finance, pp.109-115.

33. Theodossiou, A. &Theodossiou, P. (2014). "Stock return outliers and beta estimation: The case of U.S. pharmaceutical companies". Journal of International Financial Markets, Institutions and Money, Volume 30, May 2014, pp. 153-171.

34. Tsay, R. (2011).*Analysis of financial time series.* Third edition, Wily Interscience, Canada

35. Wang, H., & He, X. (2007). "Detecting differential expressions in Gene Chip microarray studies: a quantile approach". Journal of American Statistical Association, Vol. 102, No. 477, pp. 104-112.

36. Wu, G. & Xiao, Z. (2002). "An Analysis of Risk Measures". Journal of Risk 4, pp. 53-75.

37. Yihou, A. (2012). "Value at risk estimation by quantile regression and kernel estimator". Springer Science Business Media, LLC 2012.

38. Zhiguang, C. et al. (2008). "Hedging and Value at Risk: A Semi-Parametric Approach". Xfi Centre for Finance and Investment, University of Exeter, United Kingdom, available at https://business-school.ex eter.ac.uk/documents/papers/finance/2008/0808.pd