A Literature Review on Inventory Lot Sizing Problems

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***Abstract*- The present paper, discuss one of the most challenging subjects for the management namely production planning. It appears to be a hierarchical process ranging from long to medium to short term decisions. Production planning models are divided into two categories which are capacitated and uncapacitated. This paper reviews the literature on single-level single-resource lotsizing models and provides a survey of the literature dealing with inventory lot sizing problems and other concepts considered in this area. The purpose of this dissertation is to review the developments and to identify the status of existing literature in this area.**

Keywords- inventory, capacitated lotsizing, production planning, uncapacitated lotsizing.

I. Introduction

There are several ways of classifying the inventory models. Some of the attributes useful in distinguishing between various inventory models are given in this section. (Giil 1992, see Figure-1). Obtaining cost-efficient production plans balancing the trade-off between setup and inventory holding costs - lot-sizing - has been a fundamental goal of practitioners since the beginning of industrialization. The first published work in this area by Harris titled "How many parts to make at once?" dates back as far as to 1913(Harris, 1913) which is known as EOQ model. Several extensions to the basic EOQ model are discussed in Hax and Candea (1984). They cover models which allow for backlogging, lost sales and quantity discounts. Tersine and Price (1981) discuss the temporary price discounts case. Solutions to finite horizon cases where costs are time-dependent are presented by Lev and Weiss (1990) and Gascon (1995). Wilson (1934) contributed a statistical approach to find order points, thereby popularizing the EOQ formula in practice. This method determines a single point or quantity and assumes a constant demand. But, when the demand rate varies from period to period the results from the EOQ formula may be deceptive.The technique which performs optimally in a situation with variable demand was first suggested by Wagner and Whitin (1958) in their well known paper. They used dynamic programming to solve the problem, perhaps forced by the recursive nature of the computations. Their work was based on some important theorems established in their paper (Grewal, 1999). However, the Wagner-Whitin algorithm is quite limited when the finite rolling horizon is large, because a dynamic programming based algorithm requires a tremendous amount of CPU computing time. To fix the problem, Silver and Meal (Silver, Pyke & Peterson, 1998) developed a heuristic for the time-phased inventory model in 1969. There are several well-known dynamic lot size heuristics existing.

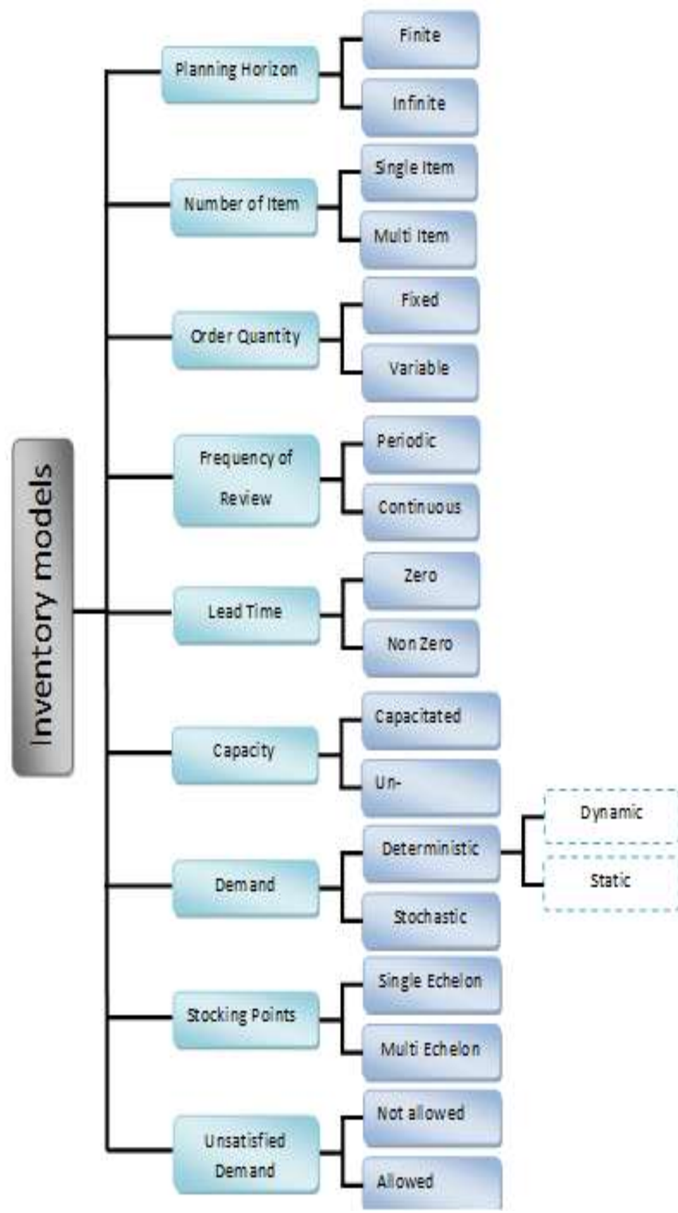

Fig.1. Classification of inventory model

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All of these heuristics use. forward programming to develop the procedures The Silver-Meal (SM) heuristic guarantees only a local minimum for current replenishment. The least unit cost (LUC) heuristic is similar to the Silver-Meal heuristic except that instead of averaging costs over periods, it averages costs over units. The Part-Period algorithm (PPA) is a heuristic lot size approach that determines order sizes by balancing ordering costs and holding costs.

II. UNCAPACITED DYNAMIC LOT SIZING MODEL

1) Single Item

Lot sizing problems have been an area of active research starting from the seminal study of Wagner & Whitin (1958) (Mohammadi et al. 2009) developed an O (T^2) dynamic programming algorithm for an uncapacitated model where the production and holding cost are linear, and the unit production costs, unit holding costs and the setup costs are the same for all periods. The algorithm can give optimal solutions in reasonable run times when the number of periods is not large enough. Veinott (1963) showed that even if production and inventory costs are general concave functions, the problem is still solvable by an O (T^2) dynamic programming algorithm. Zangwill (1969), Gupta and Brennan (1992) (among others) generalized the WW-procedure to solve the problem when backlogging is allowed. Martel and Gascon (1998) proposed an algorithm to solve the problem when inventory holding cost is a percentage of the product cost. Whitin (1958) approach has drawbacks from the practitioner's standpoint. Therefore, the natural question to ask is, "Is there a simpler approach that will capture most of the potential savings in the total replenishment and carrying cost?" A number of researchers (Diegel 1966, DeMatteis 1968, Mendoja 1968(The Part Period Balancing heuristic), Gorham 1968(The Least Unit Cost Heuristic), Silver & Meal 1973, Donaldson 1977, Groff 1979, Karni 1981, Wemmerlöv 1981, Brosseeau 1982, Freeland &Colley 1982, Boe & Yilmaz 1983, Mitra at el. 1983, Bookbinder & Tan 1984, Baker 1989, Coleman & McKnew 1990, Triantaphyllou 1992, Teng 1994, Hariga 1995, Goyal Hariga & Alyan 1996, and Zhou 1996) have suggest various decision rules, some of which have been widely used in practice(Wee 1995,Ting & Chung 1994, Bose, Goswami & Chaudhuri 1995, Hariga & Ben-Daya 1996). Federgmen and Tzur (1994) proposed a forward algorithm to solve general dynamic lot sizing problems. They assumed that the planning horizon could be divided into n periods. The proposed algorithm is a forward algorithm with sequential determination of the last setup j period and the minimum cost in the period. The procedure is similar to the classical shortest path. It showed that the proposed algorithm is about 3 times faster than the classical Wagner-Whhin's algorithm when the period is set as n. Federgmen and Tzur showed that an optimal zero-inventory ordering policy exists. The results confirm Chung's research. However, if the planning horizon can be divided into n period, instinctively, zero inventories can be achieved easily. Aggarwal & Park (1993) show that for concave cost economic lot size problems, the dynamic programming formulation of the problem gives rise to a special kind of array, called a Monge array. Then show how the structure of Monge arrays can be exploited to obtain significantly faster algorithms for these economic lot size problems. Loparic, Pochet &Wolsey (1999) examine a variant of the uncapacitated lot-sizing model of Wagner-Whitin involving sales instead of fixed demands, and lower bounds on stocks. Two extended formulations are presented, as well as a dynamic programming algorithm and a complete description of the convex hull of solutions. Richter & Sombrutzki (2000) studied the reverse Wagner-Whitin's dynamic production planning and inventory control model and some of its extensions. In such reverse (product recovery) models, used products arrive to be stored and to be remanufactured at minimum cost. For the reverse model with given demand the zero-inventory-property of optimal solutions is proved, the corresponding Wagner-Whitin algorithm is presented and the stability of optimal solutions is discussed for the case of a large quantity of low cost used products. Furthermore, the model of the alternate application of remanufacturing and manufacturing processes is analyzed. Taşgetiren & Liang (2003) find order quantities which will minimize the total ordering and holding costs of ordering decisions. A binary particle swarm optimization algorithm and a traditional genetic algorithm are coded and used to solve the test problems in order to compare them with those of optimal solutions by the Wagner and Whitin algorithm. Aksen, Altınkemer & Chand (2003) introduce a profit maximization version of the well-known Wagner–Whitin model for the deterministic uncapacitated single-item lot-sizing problem with lost sales. Costs and selling prices are assumed to be time-variant, differentiating their model from previous models with lost sales. A forward recursive dynamic programming algorithm is developed to solve the problem optimally in O (T^2) time, where T denotes the number of periods in the problem horizon. The proposed algorithm can solve problems of sizes up to 400 periods in less than a second on a 500 MHz Pentium® III processor.DeToledo & Shiguemoto (2005) propose an efficient implementation of a forward dynamic programming algorithm for problems with one single production center. Next, the authors studied the problem with a production environment composed of several production centers. For this problem two algorithms are implemented, the first one is an extension of the dynamic programming algorithm for one production center and the second one is an efficient implementation of the first algorithm. Radzi, Haron & Johari (2006) introduce neural network approach to solve the single level lot-sizing problem. Three models are developed based on three well known heuristic techniques, which are Periodic Order Quantity (POQ), Lot-For-Lot (LFL) and Silver-Meal(SM). The planning period involves in the model is period where demand in the periods are varies but deterministic. The model was developed using MatLab software. Back-propagation learning algorithm and feed-forward multilayered architecture is chosen in this project. algorithms for some special cases to solve an explicit description of the convex hull of solutions to the uncapacitated lot-sizing problem with backlogging, in its natural space of production, setup, inventory and backlogging variables. The authors identify valid inequalities that subsume all previously known valid inequalities for this problem. Conventional approaches for solving the production lot size problems are by using the differential calculus on the long-run average production-inventory cost function with the need to prove optimality first. Chiu (2008) presents a simple algebraic method to replace the use of calculus for determining the optimal lot size. Their study refers to the approach used by Grubbström & Erdem(1999) and extends it to the model examined by Chiu &Chiu (2006). This paper demonstrates that the lot size solution and the optimal production-inventory cost of an imperfect EMQ model can be derived without derivatives. Salvietti & Smith (2008) extend the ELSP to include price optimization with the objective to maximize profits. A solution approach based on column generation is provided and shown to produce very close to optimal results with short solution times on a set of test problems. Gutiérrez et al. (2008) address the dynamic lot size problem assuming time-varying storage capacities. They divided planning horizon into T periods and stock outs are not allowed. For each period, Gutiérrez et al. consider a setup cost, a holding unit cost and a production/ordering unit cost, which can vary through the planning horizon. Although this model can be solved using O (T^3) algorithms, they Show that under this cost structure an optimal solution can be obtained in O (T log T) time. They also show that when production/ordering unit costs are assumed to be constant (i.e., the Wagner–Whitin case), there exists an optimal plan satisfying the Zero Inventory Ordering (ZIO) property.Enyigit (2009) study proposes new heuristics that consider demand and purchasing price uncertainties simultaneously when all the costs are constant over time, which was the classical dynamic lot sizing problem for which the optimal solution can be obtained by the Wagner-Whitin algorithm. Purchasing decisions are made on a rolling horizon basis rather than fixed planning horizon. Well known Least Unit Cost and Silver-Meal algorithms are modified for both time varying purchasing price and rolling horizon. The proposed heuristic is basically based on a cost benefit evaluation at decision points. Gaafar, Nassef & Aly (2009) was applied simulated annealing (SA) is to find the solution of the deterministic dynamic lot-sizing problem with batch ordering and backorders. Batch ordering requires orders that are integer multiples of a fixed quantity that is larger than one. The performance of the developed SA heuristics compared to that of a genetic algorithm (GA) and a modified silver-meal (MSM) heuristic.Okhrin & Richter (2009) minimize the total inventory cost only with respect to the lot size restrictions, and not the sum of setup cost and inventory cost, as in mainstream models. They formulate the single item dynamic lot sizing problem with minimum lot size restriction and elaborate a dynamic programming algorithm for its solution. The preliminary computational results show that the algorithm is highly efficient and determines optimal solutions in negligible time.

Chandrasekaran et al. (2009) were investigated economic lot scheduling problem using time-varying lot sizes approach. The process of finding the best production sequence consists of two-phase implementation of Meta heuristics. In the first phase, they propose a GA that makes use of the proposed new lower bound to arrive at the good set of production frequencies of products for ELSP without/with backorders. In the second phase, the best sequence of part production is achieved by using the above set of frequencies and employing a GA and an ant-colony algorithm. Computational experiments reveal the effectiveness of the two-phase approach over the conventional single-phase approach.Vargas (2009) presents an algorithm for determining the optimal solution over the entire planning horizon for the dynamic lot-size model where demand is stochastic and non-stationary. Sankar (2010) investigates an EPL (Economic Production Lotsize) model in an imperfect production system in which the production facility may shift from an in-control state to an out-of-control state at any random time. The basic assumption of the classical EPL model is that 100% of produced items are perfect quality. This assumption may not be valid for most of the production environments. More specifically, they extend the article of Khouja & Mehrez (1994). The proposed model is formulated assuming that a certain percent of total product is defective (imperfect), in out-of-control state.

2) Multi Item

The main concern of this class of problems is to determine production or procurement lots for multiple products over a finite (in the case of dynamic demand) or infinite (in the case of static demand) planning horizon so as to minimize the total cost, while known demand is satisfied. The total relevant cost generally consists of setup costs, inventory holding costs, and production or procurement costs. When no capacity restrictions are imposed, the multi-item problem is relevant when joint setup/order costs exist. In the constant demand case, this is known as the Economic Order Quantity with Joint Replenishment (EOQJR) problem. This problem has the same assumptions as those of the classical Economic Order Quantity (EOQ), except for the major setup/order cost. The objective is to determine the joint frequency of production/order cycles and the frequency of producing/procuring individual items so as to minimize the total cost per unit of time. The EOQJR problem occurs, for example, when several items are purchased from the same supplier. In this case, the fixed order cost can be shared by replenishing two or more items jointly. EOQJR may also be attractive if a group of items uses the same machine. Van Eijs et al. (1992) distinguished between two types of strategies used by the algorithms proposed to solve this problem: the “indirect grouping strategy” and the “direct grouping strategy”. Both strategies assume a constant replenishment cycle (the time between two subsequent replenishments of an individual item). The items that have the same replenishment frequency form a “group” (set of items that are jointly replenished). The algorithms that use the "indirect grouping strategy" assume a constant family replenishment cycle (basic cycle). The replenishment cycle of each item is an integer multiple of this basic cycle time. The problem is then to determine the basic cycle time and the replenishment frequencies of all items simultaneously. A group is (indirectly) formed by those items that have the same replenishment frequency. An optimal enumeration procedure to solve this problem is found in Goyal (1974.a) and Van Eijs (1993). Unfortunately, the running time of those procedures grows exponentially with the number of items. Recently, Wildeman et al. (1997) proposed an efficient optimal solution method based on global optimization theory (Lipschitz optimization). The running time of this procedure grows linearly in the number of items. On the other hand, heuristic methods for the problem are discussed by Brown (1967), Shu (1971), Goyal (1973, 1974.b), Silver (1976), Kaspi and Rosenblatt (1983, 1985, 1991), Goyal and Deshmukh (1993) and Hariga (1994). The replenishment cycles of individual items in "the direct grouping strategy" are not imposed to be an integer multiple of a basic cycle. The problem is to form (directly) a predetermined number of groups that minimizes the total cost. Heuristics that use this strategy can be found in Page and Paul (1976), Chakravarty (1981) and Bastian (1986).

Based on a simulation study, Van Eijs et al. (1992) showed that the "indirect grouping strategy" slightly outperforms the "direct grouping strategy" and that it requires less computer time. Zangwill (1966) showed that there exists an optimal policy in which the schedule of each item is of Wagner and Whitin type. All the existing approaches for the LPJS in the literature make use of this property to generate solutions for the problem. The algorithms suggested by Zangwill (1966), Kao (1979), Veinott (1969) and Silver (1979) are based on different dynamic programming formulations of the problem. However, all these procedures fail to solve problems with practical dimensions due to high memory and extensive computational effort requirements. Branch and Bound procedures are proposed by Erenguc (1988), Afentakis and Gavish (1986), Kirca (1995), Robinson and Gao (1996). The lower bounds in Erenguc (1988) are computed by ignoring the major set-up costs and solving independent uncapacitated single item lot-sizing problems. In Afentakis and Gavish (1986), lower bounds are obtained by applying the Lagrangean relaxation method. By solving the linear relaxation dual of a new problem formulation, Kirca (1995) proposed an efficient way to obtain tight lower bounds. The same idea was exploited also by Robinson and Gao (1996) to obtain the lower bounds, but instead of solving the linear relaxation to optimality, the authors use a heuristic dual ascent method to solve the "condensed dual" of the relaxed problem. Different kind of heuristic methods were also proposed to solve the LPJS. See Atkins & Iyogun (1988), Chung & Mercan (1992), Federgrum & Tzur (1994), Joneja (1990) (who proposed a bounded worst case heuristic) among others. Some optimality conditions were proposed by Haseborg (1982).

III. CAPACITED DYNAMIC LOT SIZING MODEL

The dynamic lot sizing problem with constrained capacity has received considerable attention from both academics and industry during the past two decades. Specifically, the problem is that determining lot sizing for a single item when time is discretized into periods (e.g. days, weeks, months) and each time production is initiated, a setup cost is incurred. A holding cost is incurred for each unit of inventory that is carried from one period to next. The objective is to minimize the total costs, while ensuring that all demand is satisfied on time. Many optimal and heuristic techniques have been developed for variation of this problem (Baker et al. 1978, DeMatteis 1968, Florian & Klein 1971, IBM, Lambrecht 1976, Silver & Dixon 1978, Silver & Meal 1973,).Several methods have been proposed for the solution of the multi item DLSP (Dzielinski 1965, Eisenhut 1975, Lambrecht & Vanderveken 1979, Lasdon & Turjung 1971, Manne 1958, Newson 1975, VanNunen & Wessels 1978). Most of these techniques either cannot guarantee the generation of a feasible solution or are computationally prohibitive. Dixon (1979) has shown that even the two item problem with constrained capacity is NP-hard, so this class of problems is extremely difficult to solve reasonable amount of time.

a) Single Item

Love (1973) discussed the production capacity and bound storage capacity for formulating the inventory systems. The cost function is formulated as a piecewise concave function. The algorithm searches for an optimal schedule in which the bounds on production are zero or infinite. An algorithm under inventory bounds with satisfying exact requirements is also presented. It is demonstrated that these algorithms are applicable in capacity constraints. Erenguc & Aksoy (1990) develop a branch and bound algorithm for solving a deterministic single item nonconvex dynamic lot sizing problem with production and inventory capacity constraints. The production cost function is neither convex nor concave. The algorithm finds a global optimum solution for the problem after solving a finite number of linear knapsack problems with bounded variables. Sandbothe &Thompson (1990) consider the lot size model for the production and storage of a single commodity with limitations on production capacity and the possibility of not meeting demand, i.e., stockouts, at a penalty. The forward algorithm is shown in the worst case to be asymptotically linear in computational requirements, in contrast to the case for the classical lot size model which has exponential computing requirements. Two versions of the model are considered: first, in which the upper bound on production is the same for every time period; and second, in which the upper bound on production is permitted to vary each time period. Diaby et al. (1992) develop several optimal/near-optimal procedures for the Capacitated Lot-Sizing and Scheduling Problem (CLSP) with setup times, limited regular time and limited overtime. Diaby et al. formulate a mixed-integer linear programming model of the problem and solve it by Lagrangean relaxation. Their results show that large problems can be solved in reasonable computer times and within one-percent accuracy of the optimal solutions. The authors solved 99 × 8 (i.e., 99 items and 8 periods), 50 × 12 and 50 × 8 problems in 30.61, 36.25 and 12.65 seconds of CDC Cyber 730 computer time, respectively. Sandbothe &Thompson (1993) consider the lot size model for the production and storage of a single commodity with limitations on production capacity and storage capacity. There is also the possibility of not meeting demand. Chung, Flynn & Lin (1994) studied the capacitated single item dynamic lot size problem. The problem is to find an optimal production schedule that minimizes the setup, manufacturing, and inventory holding costs subject to the production capacity and the demands that need to be delivered on time. Dynamic programming and the branch and bound search procedure are used to find the solution to the problem. Although the problem is a NP problem, they claimed that the algorithm could solve moderately size problems in a reasonable time.Chen, Hearn & Lee (1994) develop a new dynamic programming method for the single item capacitated dynamic lot size model with non-negative demands and no backlogging. This approach builds the optimal value function in piecewise linear segments. It works very well on the test problems, requiring less than 0.3 seconds to solve problems with 48 periods on a VAX 8600. Problems with the time horizon up to 768 periods are solved. Empirically, the computing effort increases only at a quadratic rate relative to the number of periods in the time horizon. Similar work is done by Florian et al. (1980) and by Bitran & Yanasse (1982). An exact algorithm for this problem is discussed by Karmarkar et al. (1987). Many authors proposed polynomial algorithms to solve the constant capacity version of the problem. Florian & Klein (1971) presented an O (T^4) dynamic programming algorithm based on the shortest path method to solve the constant capacity case with concave costs. Their algorithm can deal with the backlogging situation. Jagannathan & Rao (1973) extended Florian & Klein's results to a more general production cost function which is neither concave nor convex.Fleischmann & Meyr (1997) addresses the problem of integrating lot sizing and scheduling of several products on a single, capacitated machine which is known as GLSP (General Lot sizing and Scheduling Problem). Continuous lot sizes, meeting deterministic, dynamic demands, are determined and scheduled with the objective of minimizing inventory holding costs and sequence-dependent setup costs. As the schedule is independent of predefined time periods, the GLSP generalizes known models using restricted time structures. Three variants of a local search algorithm, based on threshold accepting, are presented. Computational tests show the effectiveness of these heuristic approaches and are encouraging for further extensions of the basic model.Hoesel & Wagelmans (1997) proposed a more efficient O (T^3) dynamic programming algorithm to solve the constant capacity, concave production costs and linear holding costs case. Hill (1997) reduced the constant capacity problem. An O (2^T) dynamic programming algorithm, proposed by Baker et al. (1978), Florian et al. (1980) extended Florian and Klein's (1971) dynamic programming algorithm to the problem with arbitrary capacities. However, the required computation time becomes substantially larger. Kirca (1990) offered improvements to their algorithm. Lambert and Luss (1982) studied the problem in which the capacity limits are integer multiples of a common divisor and devised an efficient algorithm. In the case of a general cost function, Pochet (1988) proposed a procedure based on polyhedral techniques in combination with a branch and bound procedure. Chen et al. (1992.b) proposed a dynamic algorithm for the case of a piecewise linear cost function with no assumption of convexity or concavity, where arbitrary capacity restrictions on inventory and backlogging are allowed. Other contributions for restricted versions of the problem are found in Bitran and Matsuo (1986), Chen et al. (1992.a), Chung and Lin (1988) and Chung et al. (1994).Shaw & Wagelmans (1998) consider the Capacitated Economic Lot Size Problem with piecewise linear production costs and general holding costs, which is an NP-hard problem but solvable in pseudo-polynomial time. The running time of their algorithm is only linearly dependent on the magnitude of the data. This result also holds if extensions such as backlogging and startup costs are considered. Moreover, computational experiments indicate that the algorithm is capable of solving quite large problem instances within a reasonable amount of time. For example, the average time needed to solve test instances with 96 periods, 8 pieces in every production cost function, and average demand of 100 units is approximately 40 seconds on a SUN SPARC 5 workstation. Gutiérrez et al. (2003) address the dynamic lot size problem with storage capacity. As in the unconstrained dynamic lot size problem, this problem admits a reduction of the state space. New properties to obtain optimal policies are introduced. Based on these properties a new dynamic programming algorithm is devised. Superiority of the new algorithm to the existing procedure is demonstrated. Furthermore, the new algorithm runs in O (T) expected time when demands vary between zero and the storage capacity. This new approach is conceptually more understandable than the one proposed previously by Love (1973). Moreover, the computational results indicate that the algorithm introduced in this paper is almost 30 times faster than Love's procedure.

Pai (2003) studies on capacitated lot size problem and found most of them were based on the assumption that the capacity is known exactly. In most practical applications, this is seldom the case. Fuzzy number theory is ideally suited to represent this vague and uncertain future capacity. So the author was applied fuzzy sets theory to solve this capacitated lot size problem.Liu et al. (2004) formulate the single-item inventory capacitated lot size model with lost sales. They assumed that the costs are time variant. Some new properties are obtained in an optimal solution and a dynamic programming algorithm was developed to solve the problem in O (T^2) time.Enns & Suwanruji (2005) reviwe some research on lot sizing problem and found that mostly assumed single echelon systems. Even when multiple echelon systems have been used, capacity constraints are seldom considered. However, in manufacturing capacity constraints can lead to significant queuing effects. Commonly used lot sizing policies like Lot-For-Lot (LFL) and Period Order Quantity (POQ) do not take these effects into account. They compares these policies with a Fixed Order Quantity (FOQ) policy, within which lot sizes are based on minimizing estimated lot flow times at capacity-constrained machines. Simulation is used to study a small production and distribution network using time-phased planning. Results show that the FOQ policy performs better than both LFL and POQ when inventory levels and delivery performance are of concern. Song & Chan (2005) consider a single item lot-sizing problem with backlogging on a single machine at a finite production rate. The objective is to minimize the total cost of setup, stockholding and backlogging to satisfy a sequence of discrete demands. Both varying demands over a finite planning horizon and fixed demands at regular intervals over an infinite planning horizon are considered. They have characterized the structure of an optimal production schedule for both cases. As a consequence of this characterization, they proposed dynamic programming algorithm for the computation of an optimal production schedule for the varying demands case and a simpler one for the fixed demands case.Brahimi et al. (2006) state-of-the-art of a particular planning problem, the Single Item Lot Sizing Problem (SILSP), is given for its uncapacitated and capacitated versions. First classes of lot sizing problems are briefly surveyed. They reviewed various solution methods for the Uncapacitated Single Item Lot Sizing Problem (USILSP) and presented four different mathematical programming formulations of the classical problem. They discussed different extensions for real-world applications of this problem. Complexity results of the Capacitated Single Item Lot Sizing Problem (CSILSP) are given together with its different formulations and solution techniques.Heuvel & Wagelmans (2006) consider the capacitated lot-sizing problem (CLSP) with linear costs. They derive a new O (T^2) algorithm for the CLSP with non-increasing setup costs, general holding costs, non-increasing production costs and non-decreasing capacities over time, where T is the length of the model horizon. Heuvel & Wagelmans show that in every iteration they do not consider more candidate solutions than the O (T^2) algorithm proposed by Chung & Lin (1988). They also develop a variant of our algorithm that is more efficient in the case of relatively large capacities. Numerical tests show the superior performance of new algorithms compared to the algorithm of Chung & Lin (1988).Hardin, Nemhauser & Savelsbergh (2007) analyze the quality of bounds, both lower and upper, provided by a range of fast algorithms. Special attention is given to LP-based rounding algorithms.Pochet & Wolsey (2007) consider the single item lot-sizing problem with capacities that are non-decreasing over time. When the cost function is non-speculative or Wagner-Whitin and the production set-up costs are non-increasing over time. When the capacities are non-decreasing, they derive a compact mixed integer programming reformulation whose linear programming relaxation solves the lot-sizing problem to optimality when the objective function satisfies i) and ii). The formulation is based on mixing set relaxations and reduces to the (known) convex hull of solutions when the capacities are constant over time. They illustrate the use and effectiveness of this improved LP formulation on a few test instances, including instances with and without Wagner-Whitin costs, and with both non-decreasing and arbitrary capacities over time.Haugen, Olstad & Pettersen (2007a) extend the results for capacitated lot-sizing research to include pricing. Based on a few examples, the new version appears to be much easier to solve computationally. Including price can modify demand as well as production schedule. The authors found a feasible solution easily due to model assumptions (form of demand), unlike CLSP.Haugen, Olstad & Pettersen (2007b) introduce a simple heuristic for a quadratic programming sub problem within a Lagrangean relaxation heuristic for a dynamic pricing and lotsize problem. They introduce price constraints within the framework of dynamic pricing, discuss their relevance in a real world market modeling, and demonstrate their applicability within this algorithmic framework.Berk, Toy & Hazır (2008) consider the dynamic lot-sizing problem with finite capacity and possible lost sales for a process that could be kept warm at a unit variable cost for the next period t + 1 only if more than a threshold value Q_t has been produced and would be cold, otherwise. Production with a cold process incurs a fixed positive setup cost, K_t and setup time, S_t, which may be positive. Setup costs and times for a warm process are negligible. Berk, Toy & Hazır develop a dynamic programming formulation of the problem; establish theoretical results on the structure of the optimal production plan in the presence of zero and positive setup times with Wagner–Whitin-type cost structures. Chubanov, Kovalyov & Pesch (2008) study a generalization of the classical single-item capacitated economic lot-sizing problem to the case of a non-uniform resource usage for production. The general problem and several special cases are shown to be non-approximable with any polynomially computable relative error in polynomial time. An optimal dynamic programming algorithm and its approximate modification are presented for the general problem. Fully polynomial time approximation schemes are developed for two NP-hard special cases: Cost functions of total production are separable and holding and backlogging cost functions are linear with polynomially related slopes, and all holding costs are equal to zero. Wakinaga & Sawaki (2008) consider a dynamic lot size model for the case where single-item is produced and shipped by an overseas export company. They explore an optimal production scheduling with the constraint of production and shipment capacity so as to minimize the total cost over the finite planning horizon when the demands are deterministic by a dynamic programming approach. Wakinaga & Sawaki extend a dynamic lot size model to the case of incorporating shipping schedule into the model. And they deal with the model with backlogging and no backlogging, respectively. They also presented some numerical examples to illustrate optimal policies of the developed model under several demands and cost patterns.Akbalik et al. (2009) presents a new class of valid inequalities for the single-item capacitated lot sizing problem with step-wise production costs (LS-SW). They first provide a survey of different optimization methods proposed to solve LS-SW. Then, flow cover and integer flow cover inequalities derived from the single node flow set are described in order to generate the new class of valid inequalities. The single node flow set can be seen as a generalization of some valid relaxations of LS-SW. A new class of valid inequalities they call mixed flow cover, is derived from the integer flow cover inequalities by a lifting procedure. Pan, Tang & Liu (2009) address the capacitated dynamic lot sizing problem arising in closed-loop supply chain where returned products are collected from customers. The capacities of production, disposal and remanufacturing are limited, and backlogging is not allowed. It is shown that the problem with only disposal or remanufacturing can be converted into a traditional capacitated lot sizing problem and be solved by a polynomial algorithm if the capacities are constant. A pseudo-polynomial algorithm is proposed for the problem with both capacitated disposal and remanufacturing. Ng, Kovalyov & Cheng (2010) present a better solution of the first fully polynomial approximation scheme (FPTAS) for the single-item capacitated economic lot-sizing (CELS) with concave cost functions which was first developed by Hoesel & Wagelmans (2001), Chubanov et al. (2006) later presented a sophisticated FPTAS for the general case of the CELS problem with a monotone cost structure. The ideas and presentation of their FPTAS were simple and straightforward. Its running time is about n^4/ε^2 times faster than that of Chubanov et al., where n is the number of production periods and ε is the anticipated relative error of the approximate solution.Konstantaras & Skouri (2010) were considered a production-remanufacturing (used products are collected from customers and are kept at the recoverable inventory warehouse for future remanufacturing) inventory system, where the demand can be satisfied by production and remanufacturing. The cost structure consists of the EOQ-type setup costs, holding costs and shortage costs.

b) Multi Item

Barany, Roy, & Wolsey (1984) gives the convex hull of the solutions of the economic lot-sizing model is given. In addition, an alternative formulation as a simple plant location problem is examined, and here too the convex hull of solutions is obtained. It is well-known that the economic lot-sizing model is well-solved by dynamic programming. On the other hand, the standard mixed integer programming formulation of this problem leads to a very large duality gap.Thizy & Chen (1990) show that the multi-item capacitated lot-sizing problem, which consists of determining the magnitude and the timing of some operations of durable results for several items in a finite number of processing periods so as to satisfy a known demand in each period, is strongly NP-hard. They compare this approach with every alternate relaxation of the classical formulation of the problem, and show that it is the most precise in a rigorous sense. Wagelmans, Hoesel, & Kolen (1992) consider the n-period economic lot sizing problem, where the cost coefficients are not restricted in sign. In their seminal paper, H. M. Wagner and T. M. Whitin proposed an $O(n^2)$ algorithm for the special case of this problem, where the marginal production costs are equal in all periods and the unit holding costs are non-negative. It is well known that their approach can also be used to solve the general problem, without affecting the complexity of the algorithm. Wagelmans, Hoesel, & Kolen present an algorithm to solve the economic lot sizing problem in O (n log n) time, and we show how the Wagner-Whitin case can even be solved in linear time. Our algorithm can easily be explained by a geometrical interpretation and the time bounds are obtained without the use of any complicated data structure. Furthermore, they show how Wagner and Whitin's and their algorithm are related to algorithms that solve the dual of the simple plant location formulation of the economic lot sizing problem.Kirca & Kökten (1994) give a framework for a new heuristic approach for solving the single level multi-item capacitated dynamic lot sizing problem is presented. The approach uses an iterative item-by-item strategy for generating solutions to the problem. In each iteration a set of items are scheduled over the planning horizon and the procedure terminates when all items are scheduled. An algorithm that implements this approach is developed in which in each iteration a single item is selected and scheduled over the planning horizon. Each item is scheduled by the solution of a bounded single item lot sizing problem where bounds on inventory and production levels are used to ensure feasibility of the overall problem. The performance of this algorithm is compared to some well-known heuristics over a set of test problems. The computational results demonstrated that on the average their algorithm outperforms other algorithms. The suggested algorithm especially appears to outperform other algorithm for problems with many periods and few items.

DeSouza & Armentano (1994) presented a multi-item capacitated lot-sizing model includes a setup time for the production of a lot of an item. The production of items in a given period is constrained by a limited regular time and a limited overtime. Moreover, the production level of any item in a given period is also limited. This problem is tackled by a Cross decomposition based algorithm which can provide an optimal solution or a near optimal solution if computational time is restricted.Hindi (1995) addressed the problem of multi-item, single-level, dynamic lot sizing in the presence of a single capacitated resource. A model based on variable redefinition is developed leading to a solution strategy based on a branch-and-bound search with sharp low bounds. The multi-item low bound problems are solved by column generation with the capacity constraints as the linking constraints. The resulting sub problems separate into as many single-item; uncapacitated lot sizing problems as there are items. These sub problems are solved as shortest path problems. Good upper bounds are also generated by solving an appropriate minimum-cost network flow problem at each node of the branch-and-bound tree. The resulting solution scheme is very efficient in terms of computation time. Its efficiency is demonstrated by computational testing, the efficiency with which the low bound problems are solved and the frequent generation of good upper bounds; all of which leading to a high exclusion rate.Sox

(1997) describe a formulation of the dynamic lot sizing problem when demand is random and the costs are non-stationary. Assuming that the distribution of the cumulative demand is known for each period and that all unsatisfied demand is backordered, the problem can be modeled as a mixed integer nonlinear program. An optimal solution algorithm is developed that resembles the Wagner-Whitin algorithm for the deterministic problem but with some additional feasibility constraints. They derive two important properties of the optimal solution. The first increases the computational efficiency of the solution algorithm. The second property demonstrates that the lot sizes used in the rolling-horizon implementation of this algorithm is bounded below by the optimal lot sizes for a stochastic dynamic programming formulation.Although there is a significant amount of literature on the capacitated lot sizing problem, there has been insuficient consideration of planning problems in which it is possible for a lot size, or production run, to continue over consecutive time periods without incurring multiple setups. While there are papers that consider this feature, they typically restrict production to at most one product in each period. Sox & Gao (1999) present a set of mixed integer linear programs for the capacitated lot sizing problem that incorporate setup carry-over without restricting the number of products produced in each time period. Efficient reformulations are developed for finding optimal solutions, and a Lagrangian decomposition heuristic is provided that quickly generates near-optimal solutions. The computational results demonstrate that incorporating setup carry-over has a significant effect on both cost and lot sizes.Ozdamar & Bozyel (2000) consider the CLSP is extended to include overtime decisions and capacity consuming setups. The objective function consists of minimizing inventory holding and overtime costs. Setups incur costs implicitly via overtime costs, i.e., they lead to additional overtime costs when setup times contribute to the use of overtime capacity in a certain period. The resulting problem becomes more complicated than the standard CLSP and requires methods different from the ones proposed for the latter. Consequently, new heuristic approaches are developed to deal with this problem. Among the heuristic approaches are the classical HPP approach and its modifications, an iterative approach omitting binary variables in the model, a GA approach based on the transportation-like formulation of the single item production planning model with dynamic demand and a SA approach based on shifting family lot sizes among consecutive periods. Computational results demonstrate that the Simulated Annealing approach produces high quality schedules and is computationally most efficient.

Omar & Deris (2001) addresses heuristic decision rules for the situation of a deterministic linearly increasing and decreasing demand patterns with a finite input rate. They determine the timing and sizing of replenishment so as to keep the total relevant costs low as possible. They extended the Silver-Meal heuristic method and found the penalty cost is very low.Degraeve & Jans (2003) found that Dantzig-Wolfe decomposition for the Capacitated Lot Sizing Problem (CLSP), which was proposed by Manne in 1958, has an important structural deficiency. Imposing integrality constraints on the variables in the full blown master will not necessarily give the optimal IP solution as only production plans which satisfy the Wagner-Whitin condition can be selected. They propose the correct Dantzig-Wolfe decomposition reformulation separating the set up and production decisions. This formulation gives the same lower bound as Manne's reformulation and allows for branch-and-price. Column generation is speeded up by a combination of simplex and subgradient optimization for finding the dual prices. The results show that branch-and-price is computationally tractable and competitive with other approaches. Finally, they briefly discuss how this new Dantzig-Wolfe reformulation can be generalized to other mixed integer programming problems, whereas in the literature, branch-and-price algorithms are almost exclusively developed for pure integer programming problems.Karimi, Ghomi & Wilson (2003) consider single-level lot sizing problems, their variants and solution approaches. After introducing factors affecting formulation and the complexity of production planning problems, and introducing different variants of lot sizing and scheduling problems, they discuss single-level lot sizing problems, together with exact and heuristic approaches for their solution. They also conclude with some suggestions for future research.There have been recent advances in using queuing relationships to determine lot sizes that minimize mean flow times when multiple product types are being produced at capacity constrained resources. However, these relationships assume lot inter arrival times are independent, which is not the case in most manufacturing scenarios. Enns & Li (2004) examines the performance lot-sizing optimization relationships based on GI/G/1 relationships when lot inter arrival times are auto-correlated. Simulation and response surface modeling are used to experimentally determine optimal lot sizes for a sample problem. The flowtimes for "optimal" lot sizes determined analytically are found to compare poorly with the best flowtimes obtained experimentally. An approach is then developed that uses feedback during simulation to adjust parameters within queuing heuristics that support dynamic lot-size optimization. Performance using this approach compares well with the best performance obtained using the much more difficult experimental approach.Degraeve & Jans (2004) present new lower bounds for the CLSP with Setup times. They improve the lower bound obtained by the textbook Dantzig-Wolfe decomposition where the capacity constraints are the linking constraints. Dantzig-Wolfe decomposition is applied to the network reformulation of the problem. The demand constraints are the linking constraints and the problem decomposes into sub problems per period containing the capacity and set up constraints. They propose a customized branch-and-bound algorithm for solving the sub problem based on its similarities with the Linear Multiple Choice Knapsack Problem. They present a Lagrange Relaxation algorithm for finding this lower bound. This is the first time that computational results are presented for this decomposition and a comparison of their lower bound to other lower bounds.Young & Sung-soo (2005) were considered the single machine capacitated lotsizing and scheduling problem with sequence dependent setup costs and setup times (CLSPSD). The objective of the problem is minimizing the sum of production costs, inventory holding costs and setup costs while satisfying the customer demands. To handle the problem more efficiently, a conceptual model is suggested, and one of the well-known Meta heuristics, SA approach is applied. To illustrate the performance of this approach, various instances are tested and the results of this algorithm are compared with those of CLPEX. This approach generates optimal or near optimal solutions. Moreover, most of the existing researches cannot demonstrate the real world situations including sequence dependent setup costs and setup times. Federgruen & Meissner (2005) conducts a probabilistic analysis of an important class of heuristics for multi item capacitated lot sizing problems. They characterize the asymptotic performance of so-called progressive interval heuristics as T, the length of the planning horizon, goes to infinity, assuming the data are realizations of a stochastic process of the following type: the vector of cost parameters follows an arbitrary process with bounded support, while the sequence of aggregate demand and capacity pairs is generated as an independent sequence with a common general bivariate distribution, which may be of unbounded support. The authors show that important subclasses of the class of progressive interval heuristics can be designed to be asymptotically optimal with probability one, while running with a complexity bound which grows linearly with the number of items N and slightly faster than quadratically with T.Jodlbauer (2006) developed a non-time discrete approach for an integrated planning procedure, applied to a multi-item capacitated production system with dynamic demand. The objective is to minimize the total costs, which consist of holding and setup costs for one period. The model does not allow backlog. Furthermore, a production rate of zero or full capacity is the only possibility. The result is a schedule, lot-sizes and the sequences for all lots. The approach is based on a specific property of the setup cost function, which allows for replacement of the integer formulation for the number of setup activities in the model. In a situation where the requirements for the multi-item continuous rate economic order quantity, the so-called economic production lot (EPL) formula, are fulfilled, both the EPL as well as the presented model results are identical for the instances dealt with. Moreover, with the new model problems with an arbitrary demand can be solved.DeToledo & Armentano (2006) address the capacitated lot-sizing problem involving the production of multiple items on unrelated parallel machines. A production plan should be determined in order to meet the forecast demand for the items, without exceeding the capacity of the machines and minimize the sum of production, setup and inventory costs. They proposed a heuristic based on the Lagrangian relaxation of the capacity constraints and subgradient optimization. Rizk, Martel & Ramudhin (2006) study a class of multi-item lot-sizing problems with dynamic demands, as well as lower and upper bounds on a shared resource with a piecewise linear cost. The shared resource might be supply, production or transportation capacity. The model is particularly applicable to problems with joint shipping and/or purchasing cost discounts. The problem is formulated as a mixed-integer program. Lagrangean relaxation is used to decompose the problem into a set of simple sub-problems. A heuristic method based on subgradient optimization is then proposed to solve a particular case often encountered in the consumer goods wholesaling and retailing industry. Their tests show that the heuristic proposed is very efficient in solving large real-life supply planning problems.Jian-feng, Yue-xian, & Zan-dong (2006) analyzes the capacitated lot-sizing problem considering an individual machine's production capacity using a two-layer hierarchical method to minimize the sum of the dynamic inventory cost and the overtime penalty cost. The genetic algorithm, the parameter linear programming method, and a heuristic method were used in the developed method. The method uses the genetic operator to define the lot-sizing matrix (the first layer), linear programming to determine eachmachine"s schedule (the second layer) according to the lot-sizing matrix, and the heuristic method to verify the feasibility of the solutions by adjusting them to meet the constraint requirements. The scheduling of machines in a press shop demonstrates the effectiveness of the algorithm. The result shows that the algorithm is convergent.Gaafar (2006) applied genetic algorithms to the deterministic time-varying lot sizing problem with batch ordering and backorders. Batch ordering requires orders that are integer multiples of a fixed quantity that is larger than one. The developed genetic algorithm (GA) utilizes a new '012' coding scheme that is designed specifically for the batch ordering policy. The performance of the developed GA is compared to that of a modified Silver-Meal (MSM) heuristic based on the frequency of obtaining the optimum solution and the average percentage deviation from the optimum solution. In addition, the effect of five factors on the performance of the GA and the MSM is investigated in a fractional factorial experiment. Results indicate that the GA outperforms the MSM in both responses, with a more robust performance. Significant factors and interactions are identified and the best conditions for applying each approach are pointed out.Kämpf & Köchel (2006) investigate the following decision problem. A manufacturing unit has to meet a random demand for N items. At the same time only one item can be manufactured. Manufacturing times are random whereas set-up times are known positive constants but different for different items. Finished production is stored in a warehouse with finite capacity. The availability of raw material is always guaranteed. Demand that cannot be satisfied by items in the warehouse will be backordered in a queue with given capacity. Demand that meets a full backorder queue is lost. The problem now is to define such a manufacturing policy, i.e. a sequencing rule and a lot size rule, which maximizes the expected profit per time unit. Since that problem is too complex for an analytical solution we restrict our search for an optimal policy to simple structured policies, which can be described by a few parameters. To find optimal parameter values we use simulation optimization, where a simulator for the system is combined with a GA as an optimizer. The paper is finished with some numerical examples to show the applicability of the proposed approach. Parveen & Haque (2007) considers the multi-item single level capacitated dynamic lot-sizing problem which consists of scheduling N items over a horizon of T periods. The objective is to minimize the sum of setup and inventory holding costs over the horizon subject to a constraint on total capacity in each period. The current research work has been directed toward the development of a model for multi-item Dixon Silver considering the setup time and limited lot size per setup. Dixon & Silver (1981) presented a simple heuristic which will always generates a feasible solution, if one exists, and does so with a minimal amount of computational effort, but they ignore setup time and the model is hard to implement in practice because of their large computational requirements. The inclusion of setup times makes the feasibility problem Np-complete (Adenso-dfaz & Laguna 2009). Federgruen, Meissner & Tzur (2007) consider a family of N items that are produced in, or obtained from, the same production facility. Demands are deterministic for each item and each period within a given horizon of T periods. If in a given period an order is placed, setup costs are incurred. The aggregate order size is constrained by a capacity limit. The objective is to find a lot-sizing strategy that satisfies the demands for all items over the entire horizon without backlogging, and that minimizes the sum of inventory-carrying costs, fixed-order costs, and variable-order costs. All demands, cost parameters, and capacity limits may be time dependent. In the basic joint setup cost (JS) model, the setup cost of an order does not depend on the composition of the order. The joint and item dependent setup cost (JIS) model allows for item-dependent setup costs in addition to the joint setup costs. They develop and analyze a class of so-called progressive interval heuristics. A progressive interval heuristic solves a JS or JIS problem over a progressively larger time interval, always starting with period one, but fixing the setup variables of a progressively larger number of periods at their optimal values in earlier iterations. Different variants in this class of heuristics allow for different degrees of flexibility in adjusting continuous variables determined in earlier iterations of the algorithm. For the JS-model and the two basic implementations of the progressive interval heuristics, they show under some mild parameter conditions that the heuristics can be designed to be ε-optimal for any desired value of $\varepsilon > 0$ with a running time that is polynomially bounded in the size of the problem. They can also be designed to be simultaneously asymptotically optimal and polynomially bounded. A numerical study covering both the JS and JIS models shows that a progressive interval heuristic generates close-to optimal solutions with modest computational effort and that it can be effectively used to solve large-scale problems.Absi & Sidhoum (2007) address a multi-item capacitated lot-sizing problem with setup times that arises in real-world production planning contexts. Demand cannot be backlogged, but can be totally or partially lost. Safety stock is an objective to reach rather than an industrial constraint to respect. The problem is NP-hard. We propose mixed integer programming heuristics based on a planning horizon decomposition strategy to find a feasible solution. The planning horizon is partitioned into several sub-horizons over which a freezing or a relaxation strategy is applied. Some experimental results showing the effectiveness of the approach on real-world instances are presented. They also reported a sensitivity analysis on the parameters of the heuristics is reported.Hwang (2007) considers a dynamic lot-sizing model with demand time windows where n demands need to be scheduled in T production periods. For the case of backlogging allowed, an O (T^3) algorithm exists under the non-speculative cost structure. For the same model with somewhat general cost structure, the authors propose an efficient algorithm with O (max $\{T^2, nT\}$) time complexity.Absi & Sidhoum (2008) address a multi-item capacitated lot-sizing problem with setup times and shortage costs that arises in real-world production planning problems. Demand cannot be backlogged, but can be totally or partially lost. The problem is NP-hard. A mixed integer mathematical formulation is presented. They propose some classes of valid inequalities based on a generalization of Miller et al. (2003) and Marchand & Wolsey (1999) results. They also describe fast combinatorial separation algorithms for these new inequalities and use them in a branch-and-cut framework to solve the problem. They reported some experimental results showing the effectiveness of the approach.

Sidhoum & Absi (2008) address a multi-item capacitated lot-sizing problem with setup times and shortage costs and demand cannot be backlogged, but can be totally or partially lost. The problem can be modelled as a mixed integer program and it is NP-hard. In this paper, we propose some classes of valid inequalities based on a generalization of Miller et al. (2003) results. They study the polyhedral structure of the convex hull of this model which helps to prove that these inequalities induce facets of the convex hull under certain conditions.Süer, Badurdeen & Dissanayake (2008) considered order, inventory carrying, and labor costs to determine the production schedule and lot sizes that will minimize the total costs involved under capacity constraints. The fitness function for the chromosome is computed using these cost elements. Next, the chromosomes are partitioned into good and poor segments based on the individual product chromosomes. This information is later used during crossover operation and results in crossover among multiple chromosomes. Product chromosomes are grouped into three groups, group 1 (top X %), group 2 (next Y %), and group 3 (last Z %). Product chromosomes from Groups 1, 2 and 3 can only form pairs with chromosomes from group 1. Besides, different crossover and mutation probabilities are applied for each group. The results of the experimentation showed that the different strategies of the proposed approach produced much better results than the classical genetic algorithm.Levi, Lodi & Sviridenko (2008) study the classical capacitated multi-item lot-sizing problem with hard

capacities. There are N items, each of which has specified sequence of demands over a finite planning horizon of T discrete periods; the demands are known in advance but can vary from period to period. All demands must be satisfied on time. Each order incurs a time-dependent fixed ordering cost regardless of the combination of items or the number of units ordered, but the total number of units ordered cannot exceed a given capacity C. On the other hand, carrying inventory from period to period incurs holding costs. The goal is to find a feasible solution with minimum overall ordering and holding costs. They show that the problem is strongly NP-hard, and then propose a novel facility location type LP relaxation that is based on an exponentially large subset of the well-known flow-cover inequalities; the proposed LP can be solved to optimality in polynomial time via an efficient separation procedure for this subset of inequalities. Moreover, the optimal solution of the LP can be rounded to a feasible integer solution with cost that is at most twice the optimal cost; this provides a 2-approximation algorithm which is the first constant approximation algorithm for the problem. They also describe an interesting on-the-fly variant of the algorithm that does not require solving the LP a-priori with all the flow-cover inequalities. As a by-product they obtain the first theoretical proof regarding the strength of flow-cover inequalities in capacitated inventory models.Denize et al. (2008) present a proof to show the linear equivalence of the Shortest Path (SP) formulation and the Transportation Problem (TP) formulation for CLSP with setup costs and times. Their proof is based on a linear transformation from TP to SP and vice versa. In our proof, we explicitly consider the case when there is no demand for an item in a period, a case that is frequently observed in the real world and in test problems.
Ferreira, Morabito & Rangel (2009) present a mixed integer programming model that integrates production lot sizing and scheduling decisions of beverage plants with sequence dependent setup costs and times. The model considers that the industrial process produces soft drink bottles in different flavors and sizes, and it is carried out in two production stages: liquid preparation (stage I) and bottling (stage II). The model also takes into account that the production bottleneck may alternate between stages I and II, and a synchronization of the production between these stages is required. A relaxation approach and several strategies of the relax and fix heuristic are proposed to solve the model. Computational tests with instances generated based on real data from a Brazilian soft drink plant are also presented. The results show that the solution approaches are capable of producing better solutions than those used by the company.
Absi & Sidhoum (2009) address a multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages. Demand cannot be backlogged, but can be totally or partially lost. Safety stock is an objective to reach rather than an industrial constraint to respect. The problem is NP-hard. They propose a Lagrangian relaxation of the resource capacity constraints. They develop a dynamic programming algorithm to solve the induced sub-problems. An upper bound is also proposed using a Lagrangian heuristic with several smoothing algorithms. Some experimental results showing the effectiveness of the approach are reported.Anily, Tzur & Wolsey (2009) consider a multi-item lot-sizing problem with joint set-up costs and constant capacities. Apart from the usual per unit production and storage costs for each item, a set-up cost is incurred for each batch of production, where a batch consists of up to C units of any mix of the items. In addition, an upper bound on the number of batches may be imposed. Under widely applicable conditions on the storage costs, namely that the production and storage costs are nonspeculative, and for any two items the one that has a higher storage cost in one period has a higher storage cost in every period, They show that there is a tight linear program with O (mT^2) constraints and variables that solves the joint set-up multi-item lot-sizing problem, where m is the number of items and T is the number of time periods. This establishes that under the above storage cost conditions this problem is polynomially solvable. For the problem with backlogging, a similar linear programming result is described for the uncapacitated case under very restrictive conditions on the storage and backlogging costs. Computational results are presented to test the effectiveness of using these tight linear programs in strengthening the basic mixed integer programming formulations of the joint set-up problem both when the storage cost conditions are satisfied, and also when they are violated.
Madan et al. (2010) present a heuristic for the Capacitated Lot-Sizing (CLS) problem without set-up time considerations and no backordering option. The CLS problem is formulated as a mixed integer-programming problem with an underlying fixed charge transportation problem structure. This formulation is flexible enough to handle different types of production capacity such as regular time capacity, overtime capacity and subcontracting. They also present a new Lower Bound Procedure for the multi-item CLS problems. Narayanan & Robinson (2010) proposes two heuristics, for the capacitated, coordinated dynamic demand lot-size problem with deterministic but time-varying demand. In addition to inventory holding costs, the problem assumes a joint setup cost each time any member of the product family is replenished and an individual item setup cost for each item type replenished. The objective is to meet all customer demand without backorders at minimum total cost. They propose a six-phase heuristic (SPH) and a simulated annealing meta-heuristic (SAM). The SPH begins by assuming that each customer demand is met by a unique replenishment and then it seeks to iteratively maximize the net savings associated with order consolidation. Using SPH to find a starting solution, the SAM orchestrates escaping local solutions and exploring other areas of the solution state space that are randomly generated in an annealing search process. The results of extensive computational experiments document the effectiveness and efficiency of the heuristics. Over a wide range of problem parameter values, the SPH and SAM find solutions with an average optimality gap of 1.53% and 0.47% in an average time of 0.023 CPU seconds and 0.32 CPU seconds, respectively. The heuristics are strong candidates for application as standalone solvers or as an

upper bounding procedure within an optimization based algorithm. The procedures are currently being tested as a solver in the procurement software suite of a nationally recognized procurement software provider

IV. Conclusion

In the present paper, we reviewed the literature on single level single-resource lotsizing models. Although research on capacitated lotsizing started some fifty years ago, lotsizing problems are still challenging because many extensions are very difficult to solve. Finally, the interaction between modeling and algorithms will play an important role in future research. The inclusion of industrial concerns lead to larger and more complex models and consequently more complex algorithms are needed to solve them. Solution approaches for integrated models will be based on previous research on the separate models. Existing knowledge about the structure and properties of a specific subproblem can be exploited in solving integrated models. Many more opportunities are still unexplored. This research field thus remains very active.

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