Experimental and numerical studies on the behavior of cylindrical and conical shells with varying thickness along the length subjected to axial compression

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Abstract-The behavior of aluminum cylindrical and conical shells of thickness variation along their lengths subjected to quasi-static axial compression was examined. The study was conducted using experimental and FE simulation approaches. The cylinders had mean radius of 25 mm, while the cones had smaller and larger radii of 15 mm and 35 mm respectively. Their thicknesses were increased linearly from top end to bottom end with the thickness ratio (TR) of 0.0, 0.1 and 0.2. The mean thicknesses of specimens were 1.0 mm, 1.5 mm and 2.0 mm. The numerical simulations on those experiments were also conducted using the finite element code, ABAQUS. The results from experiment and FEA simulation were compared and good agreement was achieved. It was found that the energy absorption capacity of cylinder with thickness varying along the length is increasing as the value of TR increases. In contrast, the cone with thickness varying along the length seems to lose its energy absorption capacity as the value of TR increases. In addition, their deformation histories and load-displacement curves were presented and discussed.

Keywords: Cylindrical shells; Conical shells; Energy absorption; Axial compression

I. Introduction

In the view of impact-prevention engineering, various shell structures are usually selected since they offer high energy absorption capacity compared to their own weight. Among several kinds of shell structures, cylindrical and conical shells have gained high interest since they are considered as the most effective energy absorber elements. Many attempts have been made to understand the collapse mechanism of those structures, and link to their energy absorption characteristics.In case of cylindrical shells, Pugsley and Macaulay [1] was one of the pioneer who studied the progressive collapse of cylindrical shells. The three and four-lobe modes were analyzed and assumed that no residual elastic strain energy was stored in the tube. The expression for the mean compressive load was derived. In the same year Alexander [2] also performed an analysis for the deformation of cylinders but focused on concertina mode. His model has been developed by many researchers such as Abramowicz and Jones [3, 4], Wierzbicki et al [5] and later by Singace et al [6, 7]. Experimental study on the collapse mode of cylindrical shells under axial compression was made by Singace and El-Sobky [8]. Effect of end-constraints on the collapse behavior was discussed. The progressive folding of cylindrical tube was investigated by Karagiozova et al [9, 10] using simulation technique. Their studies focused on the influence of inertia effect and impact velocity. They suggested that cylinder can absorb more energy by increasing the impact velocity and reducing the impact mass. Recently, Gupta and Venkatesh [11] performed a study on the crush characteristic of cylindrical shells by experiment and numerical simulation. The influence of diameter to thickness ratio (*D/t*) on the mode of collapse and energy absorption capacity was discussed as well as the static and dynamic responses. Focusing on conical shells, experimental and analytical studies on its' crush behaviors have also been developed for few decades. Some examples of these pioneers are Seid [12], Singer [13], Weingarten et al [14] and Esslinger and Geier [15]. Their studies focused on the stability of conical shell using different approaches. The energy absorption capacity and collapse mechanics of conical shells were investigated comprehensively by Mamalis et al [16, 17, 18, and 19]. Some analytical models for estimating the energy absorbed by the structure were proposed. Xue et al [20] used a plastic hinge line technique to calculate energy dissipating in a flat-topped cone during collapse process. The result provided an upper bound for the load displacement curve of the specimen. El-Sobky et al [21] also investigate the energy absorption capacity and collapse mode of cones, but focused on the effect of end condition. Modes of collapse were categorized in groups and were found to depend on geometric ratio parameters, i.e. radius-to-thickness ratio and length-to-thickness ratio. Recently Gupta et al [22, 23] conducted experiment and FE simulation on the crush characteristic of conical shells. Effect of geometrical parameters on the mode of collapse, load-deformation curve and energy absorption capacity were discussed. The comparison between the experimental and simulation result was made and good agreement was achieved. Although the behavior of cylindrical and conical shells subjected to axial compression has been studied extensively for several decades, the influence of thickness variation on the crush characteristic has not gained sufficiently attention. It is usually assumed that the thickness of shell is uniform and

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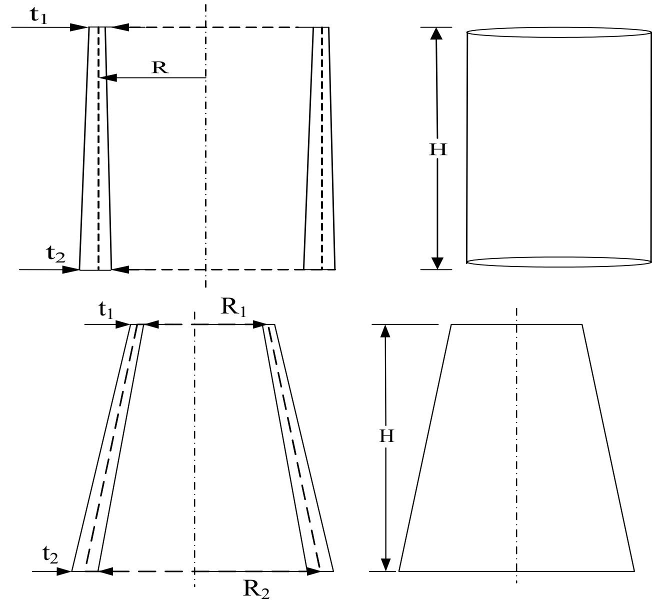
constant over the whole surface in order to simplify the problem. In fact, the thickness of shell is rarely constant. In addition, it may be necessary to use variable thickness shell for some applications such as joining two unequal thickness tubes. In this concern, there are some reports revealed the influence of the variable thickness on the buckling behavior of shells. Koiter et al [24] studied the stability of variable thickness cylinder and found that the buckling resistance capacity of cylinder is reduced as the thickness variable parameter increases. The effect of thickness variation and initial imperfection on the buckling behavior of composite cylinder was studied by Li et al [25, 26]. It was concluded that the load carrying capacity of composite cylinder is sensitive to thickness variation and especially sensitive to initial geometry imperfection. Recently, Aghajari et al [27] studied the buckling and post-buckling of cylindrical shells with varying thickness under external pressure. The study was conducted by experiment and finite element model. The result from experiment agreed well with the simulation. It also found that thickness variation has direct influence on the location of final waves in general buckling stage.

The present study was aimed to investigate the crush characteristic of cylindrical and conical shells with thickness variation along the length, subjected to axial compression. The study was conducted by experiment and finite element simulation. Their collapse progression, load-displacement curves and energy absorption capacities were presented.

II. Experimental Setup and Finite Element Modeling

The specimens were made from aluminum, fabricated by using EDM-wire cutting process in order to achieve precise thickness variation. Figure 1 illustrates the diagram of linearly variable thickness cylinder and cone used in the present study. The mean radius (R) of cylindrical shell was 25 mm while the top and bottom radii of conical shell were 15 mm and 35 mm respectively. The height of cone and cylinder were 90 mm. The mean thickness of shell was varied from 1.0, 1.5 and 2.0 mm.



The local thickness of shell wall was increased linearly from minimum thickness (t_1) at the top end to maximum thickness (t_2) at the bottom end. The degree of thickness increasing gradient is called thickness ratio (TR) and is defined by equation (1).

$$TR = \frac{(t_2 - t_1)}{(t_2 + t_1)} \qquad (1)$$

The value of thickness ratio (TR) was varied from $TR = 0.0$ (constant thickness), $TR = 0.1$ and $TR = 0.2$. The geometric parameters of the cylindrical and conical shells are tabulated in Tables 1.

1 Dimensions of cylindrical and conical shells with thickness varying along the length

Minimum thickness (t_1, mm)	Maximum thickness (t_2, mm)	Mean thickness (mm)	Thickness ratio (ε)	Height (H, mm)
1.0	1.0	1.0	0.00	90
1.5	1.5	1.5	0.00	90
2.0	2.0	2.0	0.00	90
0.9	1.1	1.0	0.1	90
1.3	1.7	1.5	0.1	90
1.8	2.2	2.0	0.1	90
0.8	1.2	1.0	0.2	90
1.2	1.8	1.5	0.2	90
1.6	2.4	2.0	0.2	90

The specimens were quasi-statically crushed in axial direction using the universal testing machine.The crush speed was 5 mm/min and the specimens were tested under simply supported end condition. The progressive collapse of specimens was observed. Their load and displacement were recorded and plotted.

The general-purpose finite element program called ABAQUS was used to perform the simulation in this study. The three dimensional cylindrical and conical shells were modeled using S4R shell elements. The mechanical properties of specimens were determined by standard tensile test. The stress-strain behavior and other properties of the tested specimens were assigned in the FE model. The models were simulated to be crushed between two platens under the same condition as the experiment.

III. Result and Discussion

a) Progressive collapse observation

It is well known that the failure modes of structures provide useful information to understand their crush characteristics. Therefore, the deformation histories of cylindrical and conical shells were observed and the results are presented and discussed in this section. Figures 2-3 show the experimental and numerical results of the progressive folding of cylindrical shells under axial compression. It should be noted that the cylinder is generally failed in concertina mode or in multi-lobes mode, depending on their *D/t* and *L/D* ratios. In the present study, the geometric parameters of all specimens were in the rage to fail in concertina mode, followed Andrews's suggestion [28]. Therefore, all of them were collapsed in axisymmetric concertina mode, as expected. An example of progressive collapse of a cylinder with constant thickness ($TR = 0$) is shown in Figure 2.

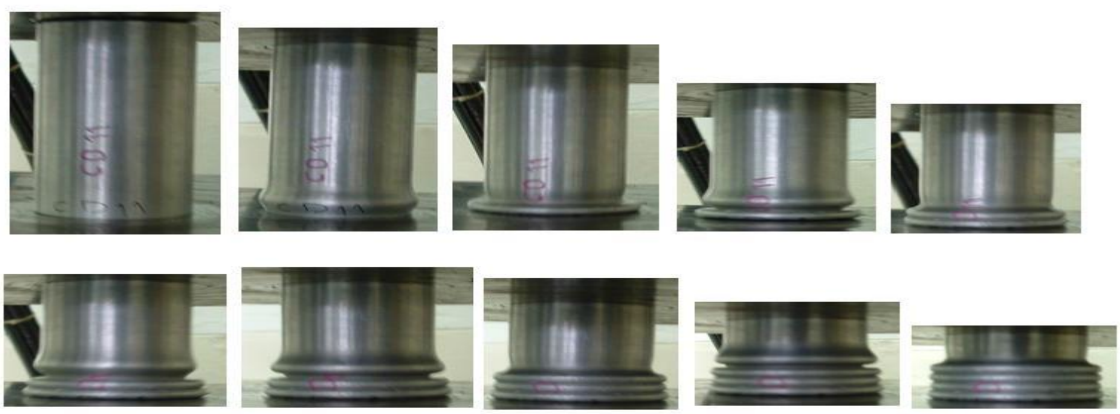


Fig.2.Typical deformation history of cylindrical shells under axial compression

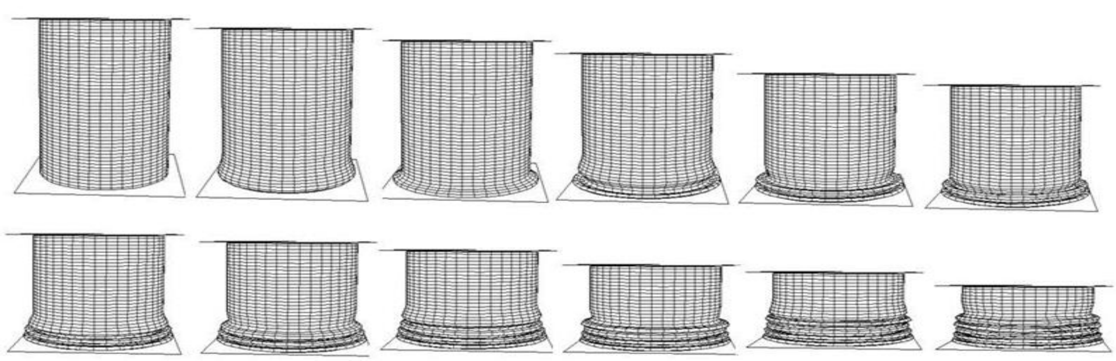


Fig. 3. The FE model of deformation history of cylindrical shells under axial compression.

Fig. 4. Typical deformed specimens (a) cylindrical shell with varying thickness along the length (b) conical shells with varying thickness along the length.

It is observed that the cylindrical shell starts to collapse by forming a first expanded axisymmetric ring near bottom end. As the first ring is completed, the cylinder continues to form other rings throughout the collapse process and finally ends up in a regular concertina mode. In cases of the cylindrical shells with thickness variation along the length ($TR \neq 0$), the collapse starts from the top end, where the wall is thinner, and progresses to bottom end. The collapse process is happened in similar manner as the progressive collapse of constant thickness cylinder, but the later rings are bigger than the former rings as can be observed from Figure 4 (a). This is because the wall thickness that is increasing as the collapse progresses downward along the length. The FE simulation of collapse history of the corresponding cylinder is shown in Figure 3. It is obviously seen that the collapse histories observed from experiment and from FE simulation are very similar. Good agreement between experiment and FE simulation was found for all tested cylindrical shells.

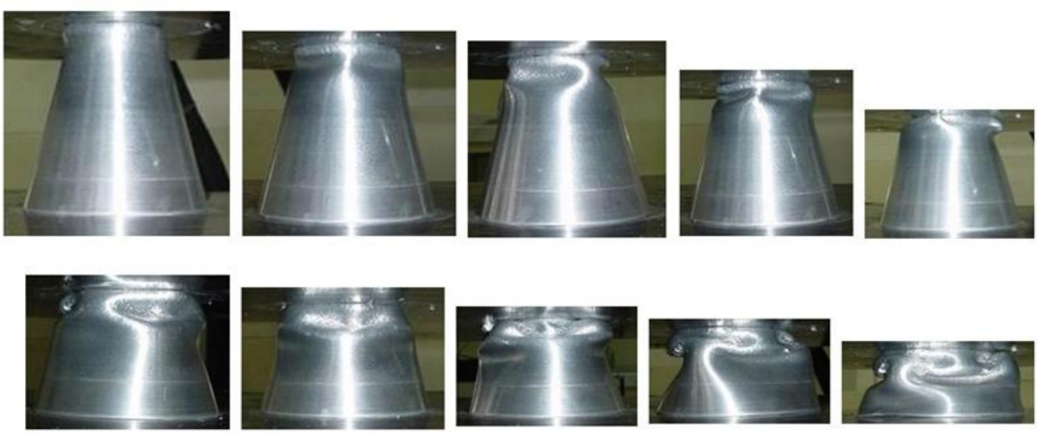


Fig. 5. Typical deformation history of conical shells under axial compression.

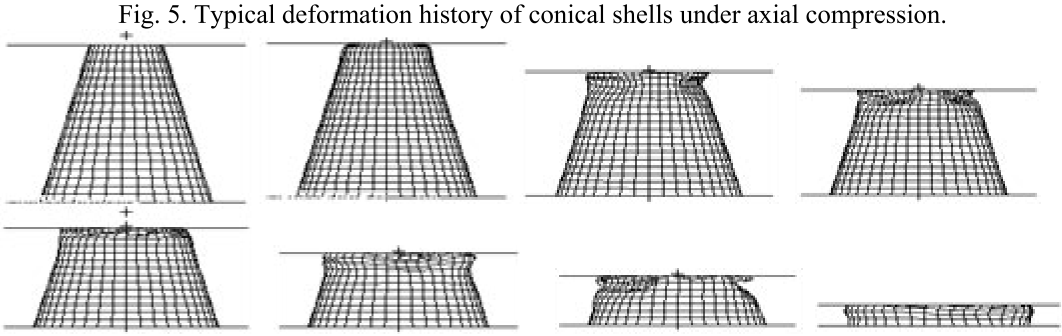


Fig. 6. The FE model of deformation history of conical shells under axial compression.

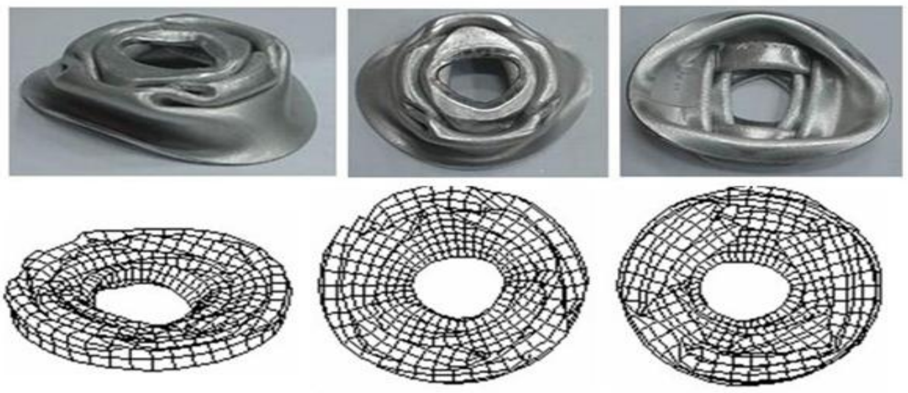


Fig. 7. The comparison of final collapse of conical shells achieved from experiment and FE model

An example of the folding process of constant thickness conical shell($TR = 0$) is presented in Figure 5. It is observed that the cone starts to collapse from the narrower end by bending the top edge inwards, forming an expanded ring near the top. As the collapse progresses downward, the cross section of cone is increasing and the ring transfers to be a series of 3-lobes mode which piles on the following layers. The FE simulation also provides the same progressive collapse for conical shells, as shown in Figure 6. Figure 7 shows the comparison of terminal mode of collapse for cone achieved from experiment and FE simulation. Good agreement between them can be observed. In case of the conical shells with thickness variation along the length ($TR \neq 0$), the collapse is also processed in similar manner as the progressive collapse of constant thickness cone. An example of the final collapse of conical shells with thickness varying along the length is shown in Figure 4 (b) and it is found similar to the collapse mode of constant thickness cone but with the larger size of lobe

b) Load-displacement curves observations

As the conical and cylindrical shells were being crushed, the response force and corresponding displacement were recorded and then plotted. The typical experimental load-displacement curves of cylindrical and conical shells with thickness varying along their length are presented in Figure 8 and 9, respectively

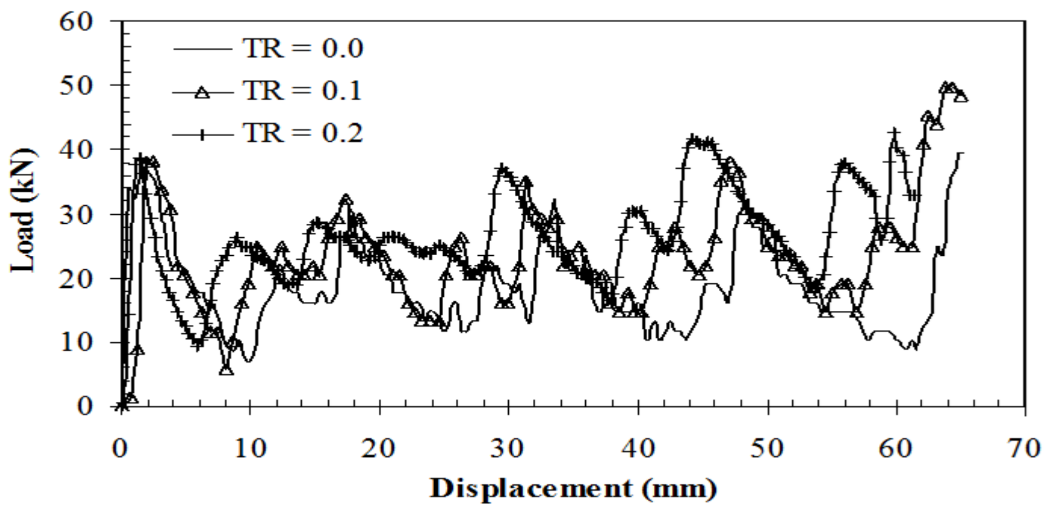


Fig. 8. The comparison of load-displacement curves of cylindrical shells having thickness variation with different values of tr

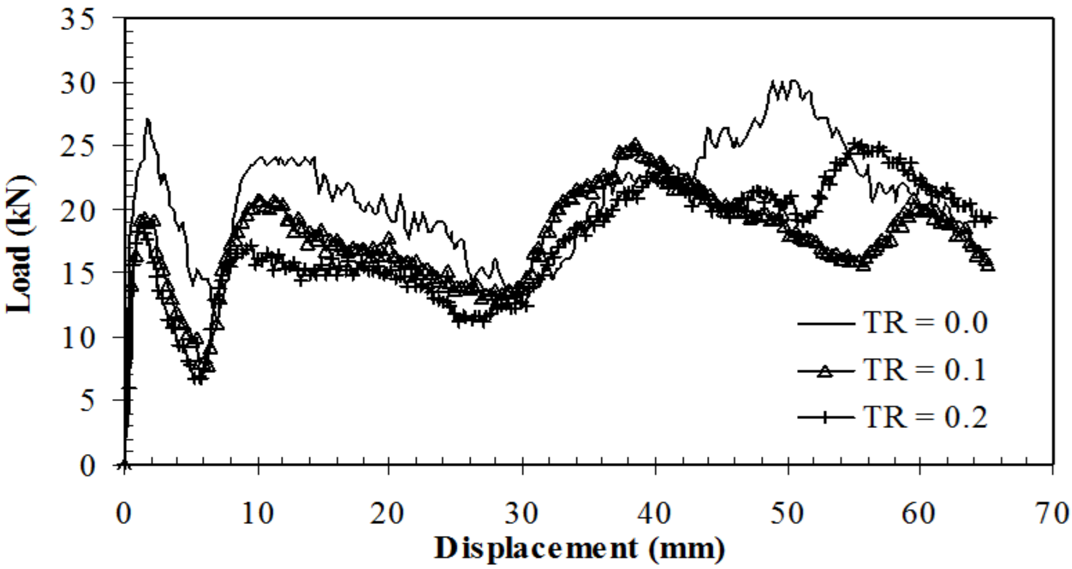


Fig. 9. The comparison of load-displacement curves of conical shells having thickness variation with different values of TR.

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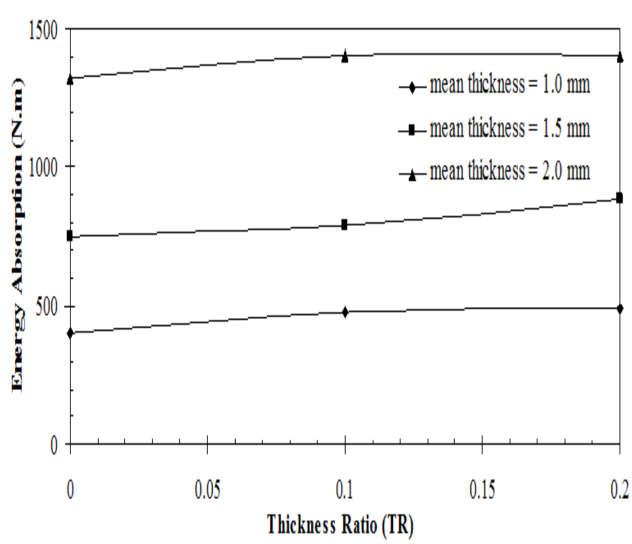
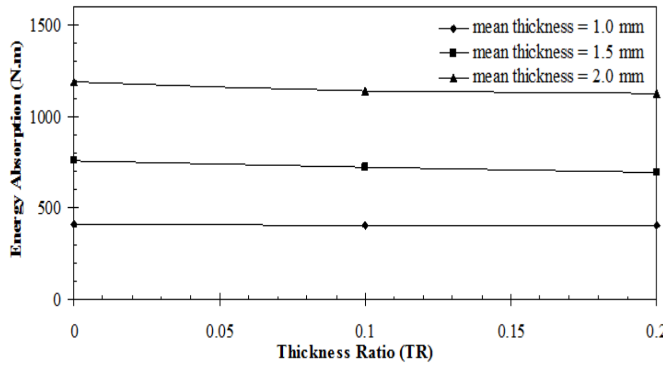
As can be seen from Figure 8 that the load-displacement curves of the cylindrical shells fluctuate in wavy shapes. Each wave in the curve is corresponding to each axisymmetric ring that formed in the collapse process. For cylindrical shells, they usually produce the first peak which is highest in the whole curve. The first peak corresponds to the onset of the first ring when the shell starts buckling. After the first peak, the load-displacement curve of constant thickness cylinder ($TR = 0$) fluctuates up and dawn in almost the same values. In contrast, the cylinders with varying thickness along the length ($TR = 0.1$ and $TR = 0.2$), they produce the load-displacement curves which are fluctuating higher as the collapse progresses. This implies that the cylindrical shells with thickness varying along the length may be able to absorb more energy as their failures progress. In addition, it is also observed that the load-displacement curve of the cylindrical shell with $TR = 0.2$ tends to lie above the curves of cylindrical shell with $TR = 0.1$ and above that of the cylindrical shell with $TR = 0.0$ (constant thickness), respectively. This may suggests that the cylindrical shells with thickness varying along the length may offer higher energy absorption capacity compared to constant thickness cylinders.In case of conical shells, see Figure 9, their load-displacement curves are also fluctuating in wavy shape but with less numbers of wave compared to the cylindrical shells. The first peak of the constant thickness conical shell ($TR = 0$) is highest compared to those of the conical shells of thickness variation with $TR = 0.1$ and $TR = 0.2$ respectively. This indicates that as the value of *TR* increases the first peak in the load-displacement curve of conical shells tends to be decreased. In addition, the load-displacement curve of the conical shell with $TR = 0.2$ tends to lie below the curves of conical shell with $TR = 0.1$ and that of the conical shell with $TR = 0.0$ (constant thickness), respectively. This is in contrast with the cylindrical shells, and implies that the conical shells with thickness varying along the length lose their energy absorption capacities as the value of *TR* increases

c) Energy absorption characteristic

The energy absorptions (E_a) of the cylindrical and conical shells were approximated by the area under their load-displacement curves. Therefore, the different patterns of the load-displacement curves yield different energy absorption values. In this study the energy absorption of each specimen was calculated and summarized in Table 2, compared to the results from FE simulations.



(a) Cylindrical shells with varying thickness along the length



b) Conical shells with varying thickness along the length

as can be seen from the comparison in Table 2 that, in general, the FE simulation provides the prediction for energy absorption of each specimen with acceptable degree of error. In order to observe the tendency of the energy absorption capacities of the cylindrical and conical shells with thickness variation along the length, their experimental energy absorptions are plotted as shown in Figures 10 (a) and (b), respectively.

Table 2 Energy absorption values of cylindrical and conical shells with thickness varying along the length achieved from experiment compared to FE model.

Mean thickness	Thickness rati	Cylindrical shells			Conical shells		
(mm)	(*TR*)	FE Model	Experiment	Error (%)	FE Model	Experiment	Error (%)
1.00	0.00	405.24	401.23	0.99	377.40	412.34	-9.26
1.50	0.00	707.19	750.12	-6.07	775.19	760.56	1.89
2.00	0.00	1,226.10	1,321.55	-7.78	1,216.60	1,186.13	2.50
1.00	0.10	416.34	476.78	-14.52	368.19	401.24	-8.98
1.50	0.10	781.95	789.02	-0.90	774.90	720.34	7.04
2.00	0.10	1,353.65	1,401.23	-3.51	1,176.19	1,137.35	3.30
1.00	0.20	444.86	489.01	-9.92	358.79	401.20	-11.82
1.50	0.20	807.06	884.51	-9.60	752.82	695.98	7.55
2.00	0.20	1,385.22	1,404.31	-1.38	1,117.45	1,125.12	-0.69

Table 3Energy absorption increments of cylindrical and conical shells with thickness varying along the length.

Mean thickness (mm)	Thickness ratio parameter (*TR*)	Energy absorption increment (%)	
		Cylindrical shells	Conical shells
1.0	0.1	18.83	-2.69
1.0	0.2	21.88	-2.70
1.5	0.1	5.19	-5.29
1.5	0.2	17.92	-8.49
2.0	0.1	6.03	-4.11
2.0	0.2	6.26	-5.14

In case of cylindrical shells with varying thickness along the length (see Figure 10 (a)), their energy absorption capacities tend to increase as the value of *TR* is increasing. The increment of energy absorption of cylindrical shell with varying thickness along the length ($TR = 0.1$ and $TR = 0.2$ compared to the constant thickness cylindrical shell($TR = 0.0$) is shown in Table 3. It is observed that the cylindrical shells of 1 mm mean thickness can absorb more energy by 18.83% and 21.88% when their thicknesses are varied along the length with the value of *TR*=0.1 and *TR*=0.2, respectively. For the cylindrical shells with mean thickness of 1.5 mm, their energy absorptions are increased by 5.19% and 17.92% when the thicknesses are varied with *TR*=0.1 and

TR=0.2, respectively. For thicker cylindrical shells, mean thickness = 2 mm, the increments of energy absorptions due to thickness variation are smaller, which are about 6.03% and 6.26% with *TR*=0.1 and *TR*=0.2, respectively.The relationship between the energy absorption capacities and the thickness ratio parameters (*TR*) of conical shells with varying thickness along the length is shown in Figure 10 (b). In contrast with the cylinder, it is observed that the energy absorption capacity of cone is decreased as the value of *TR* is increasing. The data in Table 3 indicates that the conical shells of 1 mm mean thickness with varying thickness of *TR* = 0.1 and 0.2 lose their energy absorption capacity by about 2.7%. For the conical shells with mean thickness of 1.5 mm, their energy absorptions are reduced by 5.29% and 8.49% when the thickness is varied with *TR*=0.1 and *TR*=0.2, respectively. In case of conical shells with the mean thickness = 2 mm, their energy absorptions are reduced by 4.11% and 5.14% with *TR*=0.1 and *TR*=0.2, respectively.

IV. Conclusions and Remarks

This study has investigated the behavior of cylindrical and conical shells with thickness varying along the length. Their patterns of collapse were observed and found that the later folds are in slightly larger size than the previous folds. This is due to the thickness of wall that is increased downward along the length. In addition, the load-displacement curves of them were also investigated. It was observed that the load-displacement curves of cylindrical shells with varying thickness along the length tend to increase as the collapse progress. This indicates that the energy absorption capacity of varying thickness cylinder may be increased as the collapse progresses. Considering the energy absorption capacity, it was found that the cylindrical shells with thickness varying along their length can absorb more energy compared to constant thickness cylinder. In addition, their energy absorption capacities were observed to increase as the value to *TR* increased. However, this was in contrast for the conical shells. The energy absorption capacity of conical shells was found to decrease as the thickness varying along the length.

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VI. Refferences

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