Teletraffic Modeling of Cdma Systems

John S.N¹ Okonigene R.E² Akinade B.A³ Ogunremi O¹

Abstract—This paper presents teletraffic modeling of Code Division Multiple Access (CDMA) systems that enabled the analysis of such systems capacity. Analytical tools aided by software model that assisted in analysis of the system performance, capacity estimation, dimensioning and design of CDMA networks were achieved. This work, therefore, focused on modeling telephone traffic for analysis of CDMA cellular network capacity. We developed an analytical expression for blocking probability and consequently that for the determination and analysis of the capacity of CDMA networks. The analyses of obtained results showed how interference determined the capacity of CDMA networks and therefore proved that the capacity was not hard limited, but depended on predetermined quality of service for the network. Also, the result showed how the capacity of the network, in terms of number of subscribers, can be estimated for CDMA networks. Graphical results generated from the blocking model showed the effect of variations in interference parameters on CDMA capacity. The Erlang capacity from the model was adapted into Erlang B formula to estimate capacity in terms of channels, and the number of subscribers a typical CDMA sector could accommodate.

Keywords—Teletraffic, mobile Traffic modeling, CDMA network, mobile network

I. INTRODUCTION

In a world of finite spectrum resources, CDMA enables many more people to share the airwaves at the same time than do alternative technologies [1]. The capacity of CDMA systems with respect to the possible number of supportable users can be utilized for radio resource management, such as call admission control (CAC) or resource allocation for ongoing calls as well as for a measure of revenue. CDMA development was mainly for capacity reasons and the success of the technology depended on huge increased in the capacity that is not hard limited but interference limited [2, 3]. The term teletraffic covers all kinds of data communication traffic and telecommunication traffic [4]. This paper discussed teletraffic modeling of CDMA systems that enabled the analysis of CDMA systems capacity, leading to analytical tools aided by a software model which assisted in performance analysis, capacity estimation, dimensioning and design of CDMA networks.

II. MATERIALS AND METHODS

We studied and analyzed CDMA Erlang capacity vis-à-vis probability of blocking and then derived blocking probability formulas later plot their graphs of the relationship for analysis. The derived formulas were instrument for capacity analysis in CDMA networks. Comparative analyses were carried out to compare the two approximation methods on the basis of determining which preference will be given to obtain best results. We applied approximation methods on the basis of determining which preference will be given to obtain best results. We applied the Erlang B probability statistics into CDMA networks to calculate capacity in terms of number of channels and subscriber capacity in a cell or sector, for network planning and dimensioning applications.

III. CDMA BLOCKING PROBABILITY MODELING

In the conventional modeling of telephone traffic, assuming $M$ for subscribers, $N$ for available lines, $\lambda$ for average rate of arriving calls (calls/sec), $T$ for average call length in seconds and $\mu$ average for departure rate (calls/sec). As the random call traffic arrives and departs, the number of lines occupied by ongoing calls can vary from 0 to $N$. The number of lines occupied is restricted to $0 < k < N$, where $k$ is the active call per time.

If calls are rejected when all $N$ lines are occupied, then $P_k$ for the case of $k = N$; $P_k$ is the probability that a call is rejected or "blocked" for $M > N$

$$B=P_N=\frac{(\lambda/\mu)^N/N!}{\sum_{k=0}^{N}(\lambda/\mu)^k/k!} = \frac{A^N/N!}{\sum_{k=0}^{N}A^k/k!}$$

where $B$ is the Erlang B formula.

The expression (1) above is for the blocking probability and known as the Erlang B formula [5]. The traffic load in Erlangs ($A$) is given as

$$A = \frac{\lambda}{\mu} \text{ (call/sec)}$$

(2)

For a finite $M$ users $A$ (the traffic load in Erlang) can also be expressed in Terms of $\rho$, the fraction of time that each user occupies a telephone line [6, 7, 8, 9]. Thus

$$A = M\rho.$$  

(3)

In a single cell, if $C$ is the carrier power, $I$ given as the interference power at the base station, $W$ the transmission bandwidth and $N_0$ the interference power spectral density, then neglecting thermal noise the interference power caused by the $(M - 1)$ interferers is given as

$$I = C*(M - 1)$$

(4)

Then, the capacity of the CDMA is found to be as given in equation (5) below

$$M \approx M - 1 = \frac{W}{R} \cdot \frac{1}{E_b/N_0}$$

(5)

Thus, the capacity of CDMA is proportional to the processing gain.

A CDMA blocking model was developed as a tool for the analysis of the capacity of CDMA cellular networks. Erlang B model can be applied directly to the reverse links of FDMA and TDMA systems. However, the number of channels in a CDMA cellular system is not fixed and therefore, the mechanism for blocking in a CDMA cellular system [9] needed to be examined before an application of the Erlang B theory for its capacity analysis. We determined the CDMA Erlang capacities using two different...
distributions for the total user interference power: Gaussian and lognormal approximations.

received at the base station can be written as expression (6) below:

\[ \sum_{i=1}^{M} a_i P_i + (N_0 W) \]

\[ (6) \]

\( M \) reverse link signals noise power

where, the \( \{ a_i \} \) is random variable representing the reverse link voice activity, which have the experimental values given as \( E\{ a_i \} = \alpha_r = 0.4 \) and \( E\{ \alpha_r^2 \} = 0.31 \)

the \( \{ P_i \} \) are the random signal power for the \( M \) active users.

the number \( \{ a_i \} \) of signals \( M \) is itself a random variable (RV), assumed to have a Poisson distribution, so that

\[ E(M) = \bar{M} = Var(M) \]

\[ Z = \sum_{i=1}^{M} a_i = \bar{M} (1 - \eta) \]

\[ (7) \]

Thus with this, we had Erlang capacity formulas for CDMA cellular system under two separate approximations for the interference statistic: Gaussian approximation, by invoking the central limit theory (CLT), and lognormal approximations, on the assumption that the sum of \( M \) lognormal random variables (RVs) is also a lognormal of random variable. The blocking probabilities are expressed as a function of the interference parameter threshold \( \eta_0 \) and then the cell loading threshold \( X_0 \).

\[ B_{CDMA} = Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) - Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) \]

\[ (11) \]

\[ B_{CDMA} = Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) - Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) \]

\[ (11) \]

\[ \text{TABLE 1 SNR requirement varied (Gaussian approximation)} \]

<table>
<thead>
<tr>
<th>ERLANG CAPACITY</th>
<th>m_dB = E_b / N_0 - 5dB</th>
<th>m_dB = E_b / N_0 = 6dB</th>
<th>m_dB = E_b / N_0 = 7dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi cell</td>
<td>Single cell</td>
<td>Multi cell</td>
<td>Single cell</td>
</tr>
<tr>
<td>BLOCKING</td>
<td>BLOCKING</td>
<td>BLOCKING</td>
<td>BLOCKING</td>
</tr>
<tr>
<td>PROBABILITY</td>
<td>PROBABILITY</td>
<td>PROBABILITY</td>
<td>PROBABILITY</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>20</td>
<td>0.0035</td>
<td>0.000</td>
<td>0.0005</td>
</tr>
<tr>
<td>30</td>
<td>0.0550</td>
<td>0.000</td>
<td>0.0923</td>
</tr>
<tr>
<td>40</td>
<td>0.0963</td>
<td>0.0002</td>
<td>0.5217</td>
</tr>
<tr>
<td>50</td>
<td>0.9878</td>
<td>0.0097</td>
<td>0.5217</td>
</tr>
</tbody>
</table>

We now considered a single, isolated CDMA cell with \( M \) active users. The total reverse link signal-plus-noise power \( P_i = E_b / I_0 \)

and \( \eta = \frac{N_0}{I_0} \) (thermal noise)

The interference due to mobiles in other cells can be accounted for by using first- and second-order frequency reuse factors \( F = 1 + \xi \) and \( F_1 = 1 + \xi' \), respectively, where the following approximation methods were considered:

\[ \text{Gaussian approximation} \]

The CDMA blocking probability for the interference statistic, was derived as

\[ B_{CDMA} = Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) - Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) \]

\[ (10) \]

In which the Erlang capacity is \( \bar{M} \).

\[ \text{Lognormal approximation} \]

In this case the CDMA blocking probability for the interference statistic, was derived as

\[ B_{CDMA} = Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) - Q \left( \frac{W}{b} x_0 - \ln(M) - \beta_m \right) \]

\[ (11) \]

\[ \text{Analysis Of Results} \]

From equation (10) we plot BCMDA versus \( \bar{M} \) for a single cell \( (\xi = \xi' = 0) \) and for multiple cells \( (\xi = \xi' = 0.55) \) using the following typical parameter values: \( \sigma_d = 2.5dB, m_dB = 7dB, W=1.2288MH, Rb=9.6kbps, X_0= 0.9, \bar{M} = 0.4, \alpha_r^2 = 0.31 \). The plots are parametric in \( m_dB = E_b / N_0 \), which takes the values 5, 6 and 7dB.
From Figure 1, we observed that the value of Eb/N0 needed for link operations affects the average number of users that can be accommodated at a given level of blocking. Raising the Eb/N0 requirement increases the blocking probability for the same value of M or decreases the capacity for the same probability. For example, when the blocking probability is chosen to be BCDMA = 10^{-2} = 0.01 = 1%, for multiple cells, the corresponding value of the Erlang capacity M is 18 for mdB = 7 dB, 24 for mdB = 6 dB, and 33 for mdB = 5 dB.

If we denote the Erlang capacity for a single cell by $M_c$ and the capacity for multiple cells by $M$, for BCDMA = 1.0% and Eb/N0 = 6 dB, we observed that $M_c = 37.5$ and $M = 24.1$. The ratio of these values is 37.5/24.1 = 1.56, a value that is approximately equal to the assumed value of the reuse factor, $F = 1 + \xi = 1.55$. This is consistent with the definition of the reuse factor.

### TABLE 2 Loading threshold varied (Gaussian approximation)

<table>
<thead>
<tr>
<th>$m_{dB} = E_b/N_0$</th>
<th>$X_0 = 0.66$</th>
<th>$X_0 = 0.75$</th>
<th>$X_0 = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ERLANG</strong></td>
<td><strong>MULTI</strong></td>
<td><strong>SINGLE</strong></td>
<td><strong>MULTI</strong></td>
</tr>
<tr>
<td><strong>CAPACITY</strong></td>
<td><strong>CELL</strong></td>
<td><strong>CELL</strong></td>
<td><strong>CELL</strong></td>
</tr>
<tr>
<td><strong>BLOCKING</strong></td>
<td><strong>PROBABILITY</strong></td>
<td><strong>BLOCKING</strong></td>
<td><strong>PROBABILITY</strong></td>
</tr>
<tr>
<td>10</td>
<td>0.0021</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>20</td>
<td>0.0763</td>
<td>0.0003</td>
<td>0.0141</td>
</tr>
<tr>
<td>30</td>
<td>0.5095</td>
<td>0.0483</td>
<td>0.3413</td>
</tr>
<tr>
<td>40</td>
<td>0.9271</td>
<td>0.3169</td>
<td>0.8007</td>
</tr>
<tr>
<td>50</td>
<td>0.9978</td>
<td>0.6681</td>
<td>0.9661</td>
</tr>
</tbody>
</table>

$\overline{\alpha_r} = 0.4$, $\overline{a_r^2} = 0.31$, $E_b/N_0 = 6dB$, $W/R_b = 128$
When the total user loading threshold varied. Assuming that $m_{dB} = E_b/N_0 = 6$ dB, the effect of varying the loading threshold $X_0$ on $B_{CDMA}$ and $\bar{M}$ is illustrated in Figure 2, in which $X_0$ takes the values $0.66$, $0.75$, and $0.9$. These values correspond to the multiple access interference power being twice, three times, and nine times as strong as the thermal noise. Raising the loading threshold has the effect of relaxing the system requirements, and is seen in Figure 2 to result in either a decrease in the blocking probability for the same value of $\bar{M}$, or an increase in $\bar{M}$ for the same value of $B_{CDMA}$. If we substitute specific numerical parameter values into the general equation (10), such as $\sigma_{dB} = 2.5$ dB, $m_{dB} = 7$ dB, $W = 1.2288$ MHz, $R_b = 9.6$ kbps, $X_0 = 0.9$, $\bar{x}_0 = 0.4$, $\bar{x}_\bar{M} = 0.31$, and we obtain

$$B_{CDMA} = Q\left(\frac{115.2 - 2.37(1 + \xi)\bar{M}}{3.89\sqrt{(1 + \xi)\bar{M}}}\right)$$

which gives the below form

$$B_{CDMA} = Q\left(\frac{a - b\bar{M}}{\sqrt{c\bar{M}}}\right)$$

(13)

Because $Q(0) = 0.5$, we infer from (13) that the blocking probability is 50% when $\bar{M} = a/b$. This high blocking probability is of course unacceptable, so we know that an acceptable value of blocking probability is realized only when $\bar{M}$ is much less than $a/b$. It is interesting therefore to note by comparing (10) and (13) that the upper limit on $\bar{M}$ based on having a small blocking probability is

$$\bar{M} < \frac{a}{b} = \frac{\frac{w}{R_b}X_0}{\bar{x}_0P_{med}e^{\frac{\beta_2\sigma_{dB}}{2}}} DB (1 + \xi)$$

(15)

The ideal CDMA capacity was shown as

$$M = \frac{PG}{E_b N_0} \cdot \frac{1}{\alpha_r F} = \frac{PG}{E_b N_0} \cdot \frac{1}{\alpha_r} \cdot F e$$

(15)

which was valid under perfect power control and omnidirectional cell antenna assumptions. Note that under these conditions of perfect power control ($\sigma_{dB} = 0$ dB) and 100% cell loading in the ideal situation ($X_0 = 1$), then the Erlang capacity bound in (14) is equal to the ideal capacity in (15).

IV. CONCLUSION

The stochastic nature of call arrivals and departures were characterized using statistical means. The interference contributed by each user was modeled as a Poisson random variable that summed up to a statistical random variable with Gaussian and lognormal characteristics. In this paper we focused on the analysis of Gaussian approximation, which yielded a simpler result and therefore preferred.

Blocking occurred when the reverse link multiple access interference power reached a predetermined level that is set to maintain acceptable signal quality. When the total user interference at a base station receiver exceeded the set threshold, the system blocked the next user attempting to place a call.

The number of users for which the CDMA blocking probability equaled 1% was taken to be the Erlang capacity of the network. Thus, a new CDMA blocking probability model was developed that enabled the estimation and analysis of Erlang capacity of CDMA networks. Comparative capacity analysis showed that CDMA has a huge capacity advantage over TDMA and FDMA.

V. REFERENCES

