Global Journal of Researches in Engineering
Volume 11 Issue 1 Version 1.0 February 2011
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
ISSN: 0975-5861

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Classification: GJRE-A Classification (FOR): 091399

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Strictly as per the compliance and regulations of:

Spreadsheet Modeling of Thermal Insulation in Deep Water Flow Lines

D.P.S Abam[1], V. Adukwu[2]

Abstract-A model has been proposed to calculate the optimal thermal insulation layer thickness of flow lines in deep water. The developed model is used in excel spreadsheet to simplify the procedures necessary for the calculations. The hydrate and wax formation temperatures is first known and critical radius of insulation is calculated to obtained the required thickness of insulation in order to mitigate cool down of the fluid. The study presents three analytical heat transfer solutions of deepwater flow lines for the determination of an appropriate insulation layer thickness. The model has been used to determine the optimal insulation thickness for a deep water flow line using three different insulation materials; Polyethylene, Polypropylene and Polyurethane.

I. Introduction

Heat transfer across the insulation of flow lines presents a unique problem affecting flow efficiency. Although sophisticated computer packages are available for predicting fluid temperature, their accuracies suffer from numeric treatments because long pipe segments have to be used to save computing time. This is especially true for transient fluid flow analysis in which a very large number of numerical iterations are performed.

The thermal performance of subsea production system is controlled by the hydraulic behavior of fluid in the flow line; conversely, it also impacts the hydraulic design indirectly through the influence of temperature on fluid properties such as gas-oil ratio (GOR), density, and viscosity. Thermal design, which predicts the temperature profile along the flow line, is one of the most important parts in the flow line design; and this information is required for pipeline analyses including expansion analysis, upheaval or lateral buckling, corrosion protection, hydrate prediction and wax deposition analysis. In most cases, the solids managements (hydrate, wax, asphaltenes, and scales) determine the requirements of hydraulic and thermal designs. In order to maintain a minimum temperature of fluid to prevent hydrate and wax deposition in the flow line, insulation layers may be added to the flow line.

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Thermal design includes both steady state and transient heat transfer analyses. In steady state operation, the production fluid temperature decreases as it flows along the flow line due to the heat transfer through pipe wall to the surrounding environment. The temperature profile in the whole pipeline system should be higher than the requirements for prevention of hydrate and wax formation during normal operation and is determined from steady-state flow and heat transfer calculations. If the steady flow conditions are interrupted due to a shut-in or restarted again during operation, the transient heat transfer analysis for the system is required to make sure the temperature of fluid is out of the solid formation range within the required time. It is necessary to consider both steady state and transient analyses in order to ensure that the performance of the insulation coatings will be adequate in all operational scenarios.

The most severe operational hazards of offshore pipelines are the risks associated with the transportation of multiphase fluids (Boyun and others, 2005). When water, oil and gas are flowing simultaneously inside the pipeline, there are quite a few potential problems that can occur: water and hydrocarbon fluids can form hydrate and block the pipeline; wax and asphaltene can deposit on the wall and may eventually block the pipeline; with high enough water cut, corrosion may occur; with pressure and temperature changes along the pipeline and/or with incompatible water mixing, scales may form and deposit inside the pipeline and restrict the flow; and severe slugging may form inside the pipeline and cause operational problems to downstream processing facilities. The challenge that engineers will face is, thus, how to design the pipeline and subsea system to assure that multiphase fluids will be safely and economically transported from the bottom of the wells all the way to the downstream processing plant. The practice of identifying, quantifying, and mitigating of all the flow risks associated with offshore pipelines and subsea systems is called flow assurance.

Flow assurance is critical for deepwater pipeline and system operations. In deepwater, the seawater temperature is usually much colder than the surface air temperature. When pipeline is submersed in the deep water, if there is no thermal insulation layer surrounding

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the pipe wall, the fluid heat can be quickly lost to the water. This is especially true if the water current around the pipeline is strong. With an un-insulated pipeline, the heat transfer coefficient at the outer pipe wall can be significant due to the forced convection by the seawater movement (Yong and Qiang, 2005). If the fluid temperature inside the pipeline becomes too low due to the heat loss, water and hydrocarbon (oil and gas) may form hydrate and block the flow. Furthermore, if the fluid temperature is low enough, wax may start to precipitate and deposit on the pipe wall. Thus, effective preservation of fluid heat is one of the most important design parameters for offshore pipeline.

In deep water, the pipeline is normally followed by a production riser which goes from the sea bottom to the surface processing facilities (topsides). The deeper the water is, the longer the production riser is. With a long riser, the pipeline operating pressure will be higher due to the hydrostatic head in the riser. For the same fluid temperature, with higher operating pressure, it is easier for the fluids to form hydrate. With pipeline and riser production system, if the flow conditions are such that severe slugging occurs, the slugs will be proportional to the riser length (Boyun and others, 2005). The longer the riser, the longer the severe slugs.

Effective management of the system thermal properties is crucial to the success of a deep water field development.

To ensure fit-for-purpose design, all available technologies are considered and, in general, for less stringent requirements, wet insulation on rigid pipeline, or insulated flexible flow lines can be used. However, for more stringent specifications a dry environment will be necessary to provide the required insulation performance.

As new developments are moving progressively into deeper water, where the ambient temperature at the seabed becomes even lower, successful operation becomes more heavily dependent on the thermal management strategy employed.

Thermal management strategy and insulation generally include the following; Overall Heat Transfer Coefficient, Steady State Heat Transfer and Transient Heat Transfer.

II. Problem Formulation and Solution

Oil field flow lines are insulated mainly to conserve heat. The need to keep the product in flow line at temperature higher than the ambient temperature could exist for reasons including the followings:

- Preventing formation of gas hydrate
- Preventing formation of gas hydrate, wax or asphaltenes
- Enhancing product flow properties
- Increasing cool down time after shutting down

In liquefied-gas pipelines, such as those for liquefied natural gas, insulation is required to maintain the cold temperature of the gas to keep it in liquid state.

Polypropylene, Polyethylene and Polyurethane are three base materials widely used in the petroleum industry for pipeline insulation. Depending on applications, these base materials are used in different forms, resulting to different overall conductivities. A three layer polypropylene applied to pipe surface has conductivity of 0,225 W/(m-°C), while a four layer polypropylene has conductivity of 0.173 W/(m-°C). Solid polypropylene has higher conductivity than polypropylene foam. Polymer syntactic polyurethane has conductivity of 0.121 W/(m-°C) while glass syntactic polyurethane has conductivity of 0.156 W/(m-°C). These materials have lower conductivities in dry conditions such as in pipe in pipe (PIP) applications.

Because of their low thermal conductivities, more and more polyurethane foam is used in deep water flow lines applications. Physical properties of polyurethane foam include density, compressive strength, thermal conductivity, closed-cell content, leachable halides, flammability, tensile strength, tensile modulus and water absorption. Typical values of these properties are available from literature (Guo and others, 2005).

The requirements for flow lines insulation vary from field to field. Flow assurance analysis need to be perform to determine the minimum requirement for a given field. These analyses include the following:

- Flash analysis of the production fluid to determine the hydrate formation temperature in the range of operating pressure.
- Global thermal hydraulic analysis to determine the required overall heat transfer coefficient at each location in the flow line.
- Local heat transfer analysis to determine the type and thickness of insulation to be used at the location.
- Local transient heat transfer analysis at special location along the flow line to develop cool down curves and time to the critical minimum allowable temperature at each location.

Formulation of the governing equations and solution to the heat transfer problem in deepwater flow lines under different conditions are given in the Appendix. The resultant equations are summarized in this section.

The internal temperature profile under steady fluid flow condition is expressed as:

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+C)}\right] \quad (1.10)$$

Where the constant groups are defined as:

$$\alpha = \frac{2\pi Rk}{v\rho C_p sA} \quad (1.20)$$

$$\beta = \alpha G cos\theta \quad (1.30)$$

$$\gamma = -\alpha T_{f,0} \quad (1.40)$$

$$C = -\frac{1}{\alpha} ln(\beta - \alpha^2 T_{f,s} - \alpha\gamma) \quad (1.50)$$

where T_f is the temperature inside the pipe, L is the longitudinal distance from the fluid entry point, R is inner radius of insulation layer, k is the thermal conductivity of the insulation material, v is the average flow velocity of fluid in the pipe, ρ is the fluid density, C_p is the heat capacity of fluid at constant pressure, s is the thickness of insulation layer, A is the inner cross sectional area of pipe, G is the principal thermal gradient outside the insulation, θ is the angle between the principal thermal gradient and the pipe orientation, $T_{f,0}$ is the temperature of outer medium at the fluid entry location and $T_{f,s}$ is the temperature of the fluid at fluid entry point.

The rate of heat transfer across the insulation layer over the whole length of the flow line is expressed as:

$$q = \frac{2\pi Rk}{s}\left(T_{f,0}L - \frac{Gcos\theta}{2}L^2 - \frac{1}{\alpha^2}\left\{(\beta - \alpha\gamma)L - \frac{\alpha\beta}{2}L^2 + \frac{1}{\alpha}\left[e^{-\alpha(L+C)} - e^{-\alpha C}\right]\right\}\right) \quad (1.60)$$

Where q is the rate of heat transfer (heat loss).
The internal temperature profile after starting up a fluid flow is expressed as follows:

$$T_f = \frac{1}{\alpha^2}\{\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha[L+f(L-vt)]}\} \quad (1.70)$$

Where the f is given by:

$$f(L-vt) = -(L-vt) - \frac{1}{\alpha} ln\{\beta - \alpha\beta(L-vt) - \alpha\gamma - \alpha^2[T_{f,s} - Gcos\theta(L-vt)]\} \quad (1.80)$$

And t is time.
Suppose after increasing or decreasing the flow rate, the fluid has a new velocity v' in the flow line. The internal temperature profile is expressed as follows:

$$T_f = \frac{1}{\alpha'^2}\left\{\beta' - \alpha'\beta' L - \alpha'\gamma' - e^{-\alpha'\left[L+f\left(L-v't\right)\right]}\right\} \quad (1.90)$$

Where

$$\alpha = \frac{2\pi Rk}{v'\rho C_p sA} \quad (2.00)$$

$$\beta' = \alpha' G cos\theta \quad (2.10)$$

$$\gamma' = -\alpha' T_{f,0} \quad (2.20)$$

And the function f is given by:

$$f(L-v't) = -(L-v't) - \frac{1}{\alpha'} ln\left\{\beta' - \alpha'\beta'^{(L-v't)} - \alpha'\gamma' - \left(\frac{\alpha'}{\alpha}\right)^2\left[\beta - \alpha\beta(L-v't) - \alpha\gamma - e^{-\alpha\{(L-v't)+C\}}\right]\right\} \quad (2.30)$$

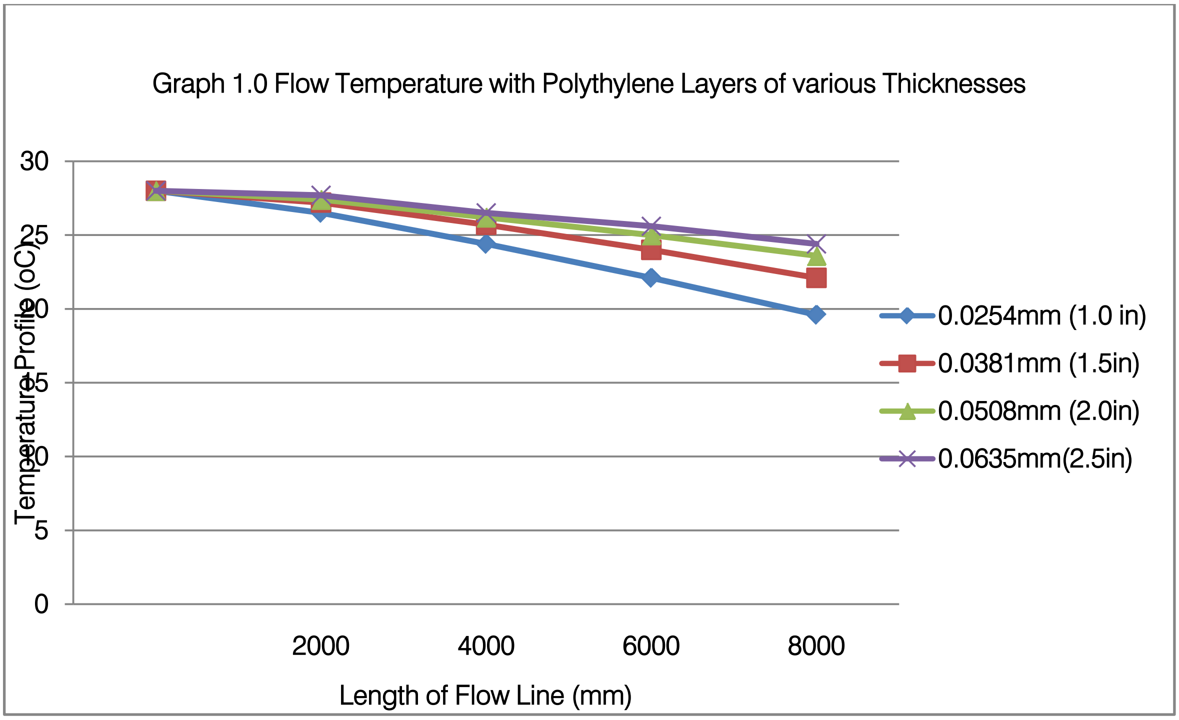
III. Result and Discussion

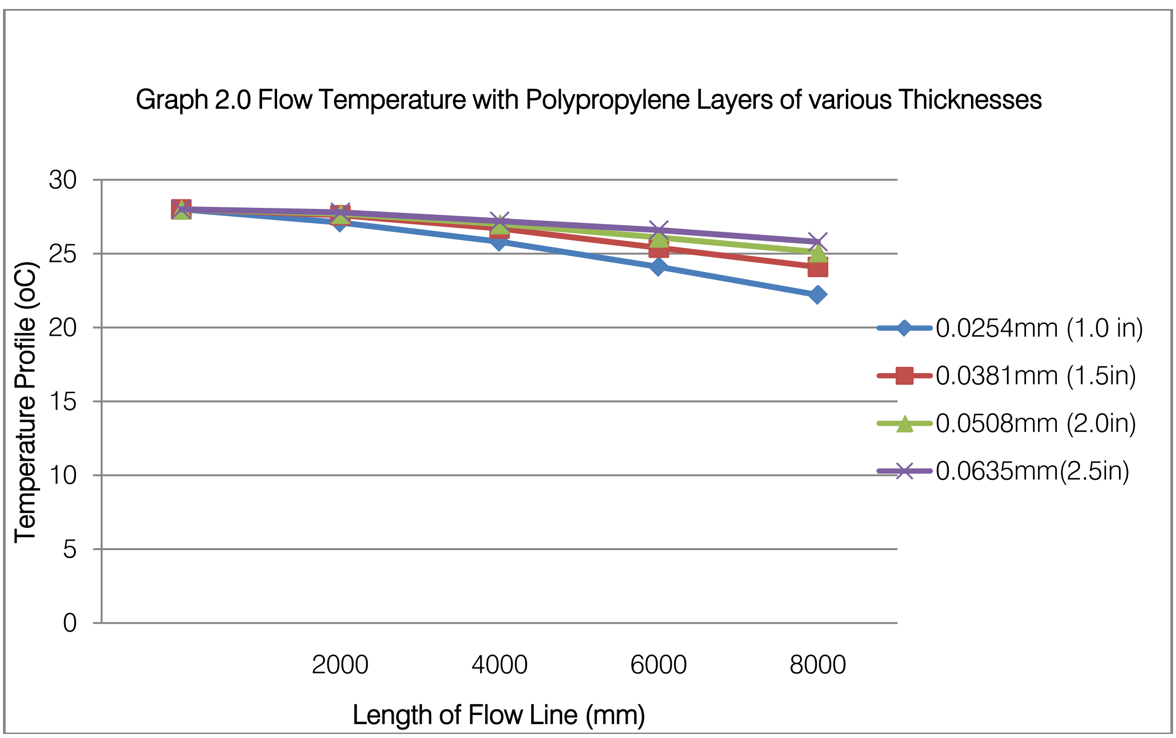
The mathematical model presented in chapter two is used to design flow line deep water. The main goal of the analysis was to select an appropriate insulation layer thickness and material. Design basis for the flow line is presented in Appendix B. The design criterion is to ensure that the temperature at any point on the flow line does not drop to below 25°C, as required by flow assurance. Insulation materials considered for this design are Polyethylene, Polypropylene and Polyurethane.

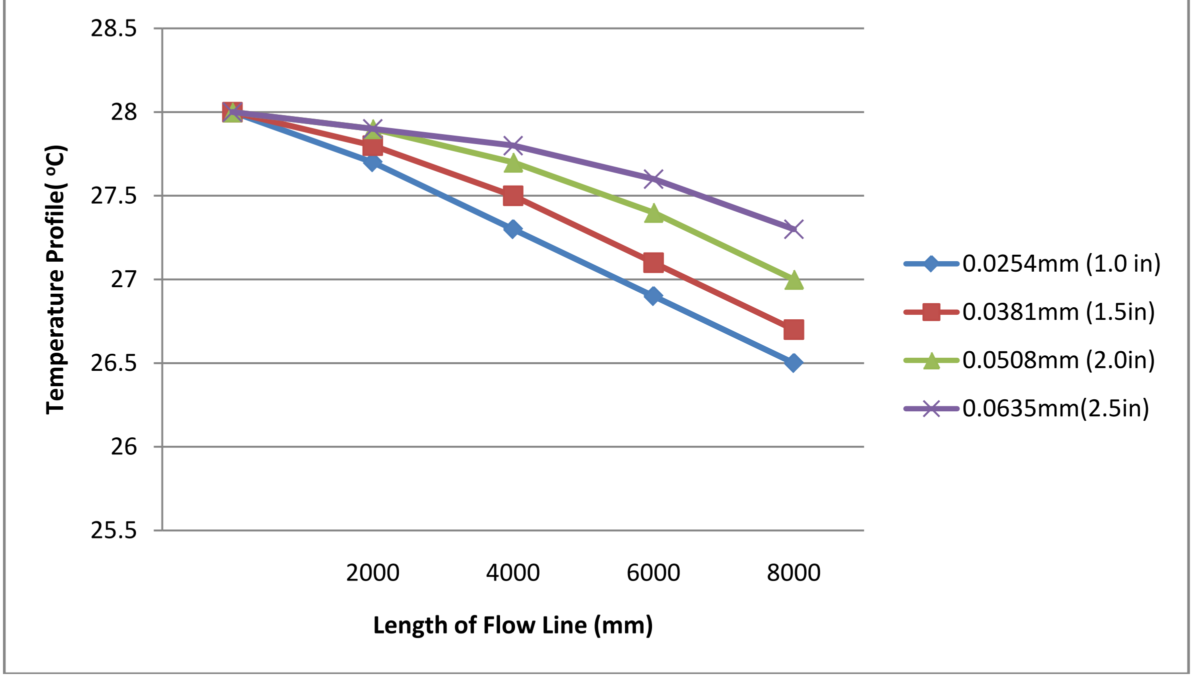
A Polyethylene layer of 0.0254mm (1 in) was first considered and later increased to 0.0381mm (1.5 in), 0.0508mm (2.0 in) and 0.0635mm (2.5 in). Graph 1 present's steady state flow temperature profile calculated using equation 1.10 with four insulation thicknesses. It shows that even a Polyethylene of 0.0635mm (2.5 in) thick will give a flow line temperature lower than 25°C, therefore, Polyethylene should not be considered for this design.

A Polypropylene layer of 0.0254mm (1 in) was then considered and later increased to 0.0381mm (1.5 in), 0.0508mm (2.0 in) and 0.0635mm (2.5 in). Graph 2 present's steady state flow temperature profile calculated using equation 1.10. It shows that a Polypropylene layer of 0.0508mm (2.0 in) thick and above will give a flow line temperature higher than 25°C.

Graph 3 present's steady state flow temperature profile calculated using Polyurethane layer of four thicknesses. It shows that a Polyurethane layer of 0.0381mm (1.5 in) thick is required to keep the flow line temperature higher than 25°C under normal operating conditions. Therefore, either a Polypropylene layer of 0.0508mm (2.0 in) thick and above or Polyurethane layer of 0.0381mm (1.5 in) thick should be chosen for insulation of the flow line. Cost analyses can justify one of the options, which is beyond the scope of this work.


Graph 1.0 Flow Temperature with Polythylene Layers of various Thicknesses
Temperature Profile (oC)
30
25
20
15
10
5
0
2000
4000
6000
8000
Length of Flow Line (mm)
0.0254mm (1.0 in)
0.0381mm (1.5in)
0.0508mm (2.0in)
0.0635mm(2.5in)


Graph 2.0 Flow Temperature with Polypropylene Layers of various Thicknesses
Temperature Profile (oC)
30
25
20
15
10
5
0
2000
4000
6000
8000
Length of Flow Line (mm)
0.0254mm (1.0 in)
0.0381mm (1.5in)
0.0508mm (2.0in)
0.0635mm(2.5in)



Graph 3.0 Flow Temperature with Polyurethane Layers of various Thicknesses

IV. Conclusion

Thermal insulation is a critical element in the design and operation of flow lines in deep waters due to a combination of low temperatures and high pressure, as result of these; stringent requirement should be taken for optimal insulation. A spreadsheet model has been proposed for the design of deep water flow lines insulation thickness. For optimal insulation thickness to be achieved in the design of deep water flow line, comparison should be made among different insulation materials with various thicknesses. The temperature profile should be plotted against the flow line length to know where there is possibility of hydrate formation or wax appearance region.

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APPENDIX A

Steady and Transient Solutions for Flow Line Temperature

1. Assumptions

The following assumptions are made in model formulation:

1) friction-induced heat is negligible
2) Heat transfer in the radial direction is fully controlled by the insulation fluid
3) Specific heat of fluid is constant

2. Governing Equation

Consider the heat flow during a time period of Δt_f. Heat balance is given by

$$q_{in} - q_{out} - q_R = q_{acc} \quad (A.1)$$

where

q_{in} = heat energy brought into the pipe element by fluid due to convection, J
q_{out} = heat energy carried away from the pipe element by fluid due to convection, J
q_R = heat energy transferred through the insulation layer due to conduction, J
q_{acc} = heat energy accumulation in the pipe element, J

These terms can be further formulated as

$$q_{in} = \rho_f C_p v A_f T_{f,L} \Delta t_f \quad (A.2)$$
$$q_{out} = \rho_f C_p v A_f T_{f,L+\Delta L} \Delta t_f \quad (A.3)$$
$$q_R = 2\pi R_n k_n \Delta L \frac{\partial T_f}{\partial r} \Delta t_f \quad (A.4)$$
$$q_{acc} = \rho_f C_p A_f \Delta L \Delta \bar{T}_f \quad (A.5)$$

where

ρ_f = fluid density, Kg/m^3
C_p = specific heat at constant pressure, J/kg-°C
v = the average flow velocity of fluid in the pipe, m/s
A_f = cross-sectional area of pipe open for fluid flow, m^2
$T_{f,L}$ = temperature of the flowing-in fluid, °C
Δt_f = flow time, s
$T_{f,L+\Delta L}$ = temperature of the flowing-out fluid, °C
s = insulation layer thickness
R_n = inner-radius of insulation layer, m
k_n = thermal conductivity of the insulation layer, W/m- °C
ΔL = length of the pipe segment, m
$\frac{\partial T_f}{\partial r}$ = radial-temperature gradient in the insulation layer, °C/m
$\Delta \bar{T}_f$ = the average temperature increase of fluid in the pipe segment, °C

Substituting Equations (A.2) through (A.5) into Equation (A.1) gives

Substituting Equations (A.2) through (A.5) into Equation (A.1) gives

$$\rho_f C_p v A_f T_{f,L} \Delta t_f - \rho_f C_p v A_f T_{f,L+\Delta L} \Delta t_f - 2\pi R_n k_n \Delta L \frac{\partial T_f}{\partial r} \Delta t_f = \rho_f C_p A_f \Delta L \Delta \bar{T}_f \quad (A.6)$$

$$\rho_f C_p v A_f \Delta t_f \left(T_{f,L} - T_{f,L+\Delta L}\right) - 2\pi R_n k_n \Delta L \frac{\partial T_f}{\partial r} \Delta t_f = \rho_f C_p A_f \Delta L \Delta \bar{T}_f \quad (A.7)$$

Dividing all the terms of this equation by $\Delta L \Delta t_f$ yields

$$\rho_f C_p v A_f \frac{\left(T_{f,L} - T_{f,L+\Delta L}\right)}{\Delta L} - 2\pi R_n k_n \frac{\partial T_f}{\partial r} = \rho_f C_p A_f \frac{\Delta \bar{T}_f}{\Delta t_f} \quad (A.8)$$

For infinitesimal of ΔL and Δt_f this equation becomes

$$v \frac{\partial T_f}{\partial L} + \frac{\partial \bar{T}_f}{\partial t_f} = -\frac{2\pi R_n k_n}{\rho_f C_p A_f} \frac{\partial T_f}{\partial r} \quad (A.9)$$

The radial-temperature gradient in the insulation layer can be formulated as

$$\frac{\partial T_f}{\partial r} = \frac{T_{f,L} - \left(T_{f,0} - G\cos(\theta) L\right)}{s} \quad (A.10)$$

where

$T_{f,0}$ = Temperature of the medium outside the insulation layer at $L = 0$, ℃
G = Geothermal gradient, $℃/m$
θ = Inclination time, degree
s = Thickness of the insulation layer, m

Substituting Equation (A.10) into Equation (A.9) yields

$$v \frac{\partial T_f}{\partial L} + \frac{\partial \bar{T}_f}{\partial t_f} = aT_f + bL + c \quad (A.11)$$

where

$$a = -\frac{2\pi R_n k_n}{\rho_f C_p s A_f} \quad (A.12)$$
$$b = aG\cos(\theta) \quad (A.13)$$
$$c = -aT_{f,0} \quad (A.14)$$

3. Solutions

Three solutions are sought in this study: Solution A: Steady flow; Solution B: Transient flow with static fluid as the initial condition; and Solution C: Transient flow with steady flow as the initial condition

Solution A gives temperature profile during normal operation conditions; Solution B simulates temperature change during a start-up process; and Solution C yields temperature trend during a shut-down process.

4. Steady Heat Transfer

If the mass flow rate is maintained for a significantly long time, a steady heat transfer condition between the system and its surroundings is expected. Under steady flow conditions, the temperature at any point in the system is time-independent. Therefore, Equation (A.11) becomes

$$v\frac{\partial T_f}{\partial L} = aT_f + bL \qquad (A.15)$$

This equation can be solved with boundary condition

$$T = T_{f,s}\ at\ L \qquad (A.16)$$

To simplify the solution, Equation (A.15) is rearranged to be

$$\frac{\partial T_f}{\partial L} + \alpha T_f + \beta L + \gamma = 0 \qquad (A.17)$$

where

$$\alpha = -\frac{a}{v} \qquad (A.18)$$

$$\beta = -\frac{b}{v} \qquad (A.19)$$

$$\gamma = -\frac{c}{v} \qquad (A.20)$$

Let

$$u = \alpha T_f + \beta L + \gamma \qquad (A.21)$$

Then

$$T_f = \frac{u - \beta L - \gamma}{\alpha} \qquad (A.22)$$

and

$$\frac{dT_f}{dL} = \frac{1}{\alpha}\frac{du}{dL} - \frac{\beta}{\alpha} \qquad (A.23)$$

Substituting Equations (A.22) and (A.23) into Equation (A.17) gives

$$\frac{1}{\alpha}\frac{du}{dL} - \frac{\beta}{\alpha} + u = 0 \qquad (A.24)$$

Integration of this equation with the method of separation of variables yields

$$-\frac{1}{\alpha}\ln(\beta - \alpha u) = L + C \qquad (A.25)$$

where C is a constant of integration. Substituting Equation (A.21) into Equation (A.25) and rearranging the latter result in

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+C)}\right] \quad (A.26)$$

Applying boundary condition (A.16) to Equation (A.26) gives the expression for the integration constant

$$C = -\frac{1}{\alpha}\ln\left(\beta - \alpha^2 T_{f,s} - \alpha\gamma\right) \qquad (A.27)$$

5. Transient Heat Transfer during Starting-Up

The temperature profile along the flow line during the starting-up process can be obtained by solving Equation (A.11) with the method of characteristics, subject to the initial condition

$$T_f = T_{f,0} - G\cos(\theta)L \quad at\ L = 0 \qquad (A.28)$$

Consider a family of curves defined by the equation

$$dt_f = \frac{dL}{v} = \frac{dT_f}{aT + bL + c} \qquad (A.29)$$

The characteristics are

$$L = vt_f + K \qquad (A.30)$$

We also have from Equation (A.29)

$$\frac{dT_f}{dL} = \frac{aT_f + bL + c}{v} \qquad (A.31)$$

Using notations (A.18), (A.19), and (A.20), Equation (A.31) becomes

$$\frac{dT_f}{dL} + \alpha T_f + \beta L + \gamma = 0 \qquad (A.32)$$

which is exactly Equation (A.17). Its solution is the same as Equation (A.26), i.e.,

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+A)}\right] \qquad (A.33)$$

where A is an arbitrary constant of integration. This constant is different on each characteristic curve. Further, each characteristic curve has a different value of K. Hence, as K varies, A varies and we may write $A = f(K)$, where f is an arbitrary function to be determined. Writing $A = f(K)$ in Equation (A.33) yields

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+f(K))}\right] \qquad (A.34)$$

Eliminating K using Equation (A.30), gives:

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha\left[L+f(L-vt_f)\right]}\right] \quad (A.35)$$

Now applying the initial condition (A.28) gives

$$T_{f,0} - G\cos(\theta)L \quad = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha[L+f(L)]}\right](A.36)$$

which gives

$$f(L) = -L - \frac{1}{\alpha}ln\left[\beta - \alpha\beta L - \alpha\gamma - \alpha^2\left(T_{f,0} - G\cos(\theta)L\right)\right](A.37)$$

Therefore,

$$f(L - vt_f) = -(L - vt_f) - \frac{1}{\alpha}ln\left[\beta - \alpha\beta(L - vt_f) - \alpha\gamma - \alpha^2\left(T_0 - G\cos(\theta)(L - vt_f)\right)\right] \quad (A.38)$$

Substituting Equation (A.38) into Equation (A.35) results in the solution to Equation (A.11) subject to the initial condition (A.28). This solution is valid for $L - vt_f > 0$. For points at which $L - vt_f < 0$, $L - vt_f = 0$ should be used.

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6. Transient Heat Transfer during a Flow Rate Change

The temperature trend along the flow line during a flow rate change (shutting-down is a special case) process can be obtained by solving Equation (A.11) with a new velocity v' ' corresponding to a new flow rate. The general solution is still given by Equation (A.35) with new parameters corresponding to the low velocity, i.e.

$$T_f = \frac{1}{\alpha'^2}\left[\beta' - \alpha'\beta' L - \alpha'\gamma' - e^{-\alpha'[L+f(L-v't_f)]}\right] \quad (A.39)$$

Where

$$\alpha' = -\frac{a}{v'} \quad (A.40)$$

$$\beta' = -\frac{b}{v'} \quad (A.41)$$

$$\gamma' = -\frac{c}{v'} \quad (A.42)$$

The initial condition is defined by Equation (A.26), i.e.,

$$T_f = \frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+C)}\right] \; at \; t_f = 0 \quad (A.43)$$

where the constant C is given by Equation (A.27).

Now applying the initial condition (A.43) to Equation (A.39) gives

$$\frac{1}{\alpha^2}\left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+C)}\right] = \frac{1}{\alpha'^2}\left[\beta' - \alpha'\beta' L - \alpha'\gamma' - e^{-\alpha'[L+f(L)]}\right] \quad (A.44)$$

which yields

$$f(L) = -L - \frac{1}{\alpha'} ln\left\{\beta' - \alpha'\beta' L - \alpha'\gamma' - \left(\frac{\alpha'}{\alpha}\right)^2 \left[\beta - \alpha\beta L - \alpha\gamma - e^{-\alpha(L+C)}\right]\right\} (A.45)$$

Therefore,

$$f(L - vt_f) = -(L - vt_f) - \frac{1}{\alpha'} ln\left\{\beta' - \alpha'\beta'(L - vt_f) - \alpha'\gamma' - \left(\frac{\alpha'}{\alpha}\right)^2 \left[\beta - \alpha\beta(L - vt_f) - \alpha\gamma - e^{-\alpha[(L-vt_f)+C]}\right]\right\} \quad (A.45)$$

Substituting Equation (A.46) into Equation (A.39) results in the solution to Equation (A.11) subject to the initial condition (A.43).

APPENDIX B

DESIGN INSULATION OF A DEEP WATER FLOW LINE			
DESIGN EXAMPLE: GAS FLOW LINE			
User Input			
Description	Symbol	Value	Units
Longitudinal distance from the fluid entry poin	L	8047	M
Outer diameter of pipe	D_o	0.2032	M
Wall thickness	t	0.00635	M
Fluid density	ρ_f	881	kg/m^3
Fluid specific heat at constant pressure	Cp	2012	J/kg-°C
Average external temperature	T_o	10	°C
Fluid temperature at entry point	T_s	28	°C
Fluid flow rate	Q	62000000	m^3/d
Inner radius of insulation	R	0.1016	M
Thermal conductivity of insulation material	k	0.35	W/m-°C
Average flow velocity of fluid	v	3.228304	m/s
Inner cross sectional area of pipe	A	0.032429	m^2
Calculated Steady State Flow Temperature Profile with a Polyethylene layers of various thicknesses			

Calculated Temperature Profile with a Polyethylene layer of 0.0254m								
T	28	26.6	24.4	22.1	19.6			
L	0	2000	4000	6000	8000			
Calculated Temperature Profile with a Polyethylene layer of 0.0381m								
T	28	27.2	25.7	24	22.1			
L	0	2000	4000	6000	8000			
Calculated Temperature Profile with a Polyethylene layer of 0.0508m								

T	28	27.4	26.2	25	23.6				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0635m									
T	28	27.7	26.5	25.6	24.4				
L	0	2000	4000	6000	8000				
Calculated Steady State Flow Temperature Profile with a Polypropylene layers of various thicknesses									
Calculated Temperature Profile with a Polyethylene layer of 0.0254m									
T	28	27.1	25.8	24.1	22.2				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0381m									
T	28	27.6	26.7	25.4	24.1				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0508m									
T	28	27.7	27	26.1	25.1				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0635m									
T	28	27.8	27.2	26.6	25.8				
L	0	2000	4000	6000	8000				
Calculated Steady State Flow Temperature Profile with a Polyurethane layer of various thicknesses									
Calculated Temperature Profile with a Polyethylene layer of 0.0254m									
T	28	27.7	27.3	26.9	26.5				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0381m									
T	28	27.8	27.5	27.1	26.7				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0508m									
T	28	27.9	27.7	27.4	27.0				
L	0	2000	4000	6000	8000				
Calculated Temperature Profile with a Polyethylene layer of 0.0635m									
T	28	27.9	27.8	27.6	27.3				
L	0	2000	4000	6000	8000				

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