A Fast Computational Technique to Assess Total Transfer Capability Using Broyden – Shamanski Method

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Abstracts - In the deregulated power system assessment of Total Transfer Capability (TTC) is a complex task which has to be done at periodic intervals for each source sink pairs. Though there are many methods available to assess TTC, the most accurate method is Repeated Power Flow using Newton Rapson (RPFNR). This method suffers from the drawback of high computational time due to the presence of multiple Jacobian inverses. In this paper a novel method, Repeated Power Flow using Broyden – Shamanski method with Sherman – Morrison formula (RPFBSS) is employed which eliminates the drawback of RPFNR method without compromising accuracy. The proposed approach is tested with WSCC 9 bus, New England 39 bus and IEEE 118 bus test system and the results are compared with the conventional RPFNR method.

Keywords: Electric power deregulation, Total Transfer Capability, Repeated Power Flow, Broyden – Shamanski, Newton Rapson.

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Keywords : Electric power deregulation, Total Transfer Capability, Repeated Power Flow, Broyden–Shamanski, Newton Rap son.

I. INTRODUCTION

According to NERC report [1], Total Transfer Capability (TTC) is defined as the amount of electric power that can be transferred over the interconnected transmission network in a reliable manner while meeting all defined pre and post contingencies. Available Transfer Capability (ATC) is a measure of transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses. Therefore ATC = TTC – Committed Uses and Committed Uses = TRM + Existing Transmission Commitments (including CBM) where TRM is Transmission Reliability Margin and CBM is Capacity Benefit Margin. Determination of TTC is the key component in ATC calculation. There are number of methods reported till date in literature to compute TTC. DC load flow [2] for transfer capability calculation is faster but does not consider losses in the network, voltage limits etc. Methods which use DC Power Transfer Distribution Factors [3] or AC Power Transfer Distribution Factors [4–5] can fetch accurate results only to those cases which are too close to the base case from which those distribution factors are derived. Artificial Neural Network [6–7] and Fuzzy logic [8] based methods are much faster in computing TTC but it requires a clear understanding of the complex network topology so that these intelligent systems can be trained for accurate results. Optimal Power Flow based methods [9–12] assess TTC with other factors such as Economic Dispatch, can get better results but at the cost of higher computational time. Two other well known methods which solve full AC load flow repeatedly to find TTC is, Continuation Power Flow (CPF) method [13] and Repeated Power Flow method (RPF) [14]. CPF method involves predictor, step length control, corrector and parameterization which make the procedure much complicated. When compared to CPF, RPF implementation is much easier and it also provides a part of P-V curve if voltage stability has to be taken into account.

From the literature available it is understood that the effectiveness of majority of methods in assessment of TTC has been proved by comparing it with the benchmark method – RPF [8–9], [15] since the results obtained using this method is very much accurate. RPF method normally uses Newton Raphson (RPFNR) for power flow which suffers from the drawback of high computational time. Computational time plays a vital role in TTC assessment since a typical assessment frequency [16] is

- Hourly TTC for the next 168 Hours : Once per day
- Daily TTC for the next 30 days : Once per week
-onthly TTC for months 2 through 13 : Once per month

In RPFNR the primary task is to find the loading factor for TTC assessment in which major portion of CPU time is spent in functional evaluations (computation of Jacobian elements) and arithmetic operations (inverting the Jacobian matrix) which is a common procedure in Newton Raphson method.

In this paper a novel method RPF using Broyden–Shamanski method with Sherman Morrison...
formula (RPFBSS) [17-19] is used for computation of TTC which eliminates the drawback of RPFNR method. This method is basically a generalization of the Secant method for solving Non-Linear equations hence it reduces the number of functional evaluations when compared to NR method. Also presence of Sherman – Morrison formula helps us to reduce the number of arithmetic operations by taking inverse of Jacobian only once for a given topology of network and for the remaining iterations a rank one update is done to compute the inverse (an approximate Jacobian inverse) irrespective of many transfer directions and source/sink pairs which reduces the time required for computing TTC.

II. MATHEMATICAL FORMULATION OF TTC

TTC deals with the transfer of maximum possible power flow in a transmission network subject to the satisfaction of certain constraints like network thermal limits, voltage limits, generation limits etc.

The mathematical formulation for TTC [20] is:

\[ P_{G_i} - P_{D_i} - \sum_{j=1}^{n} P_{loss_{ij}} = 0 \]  \hspace{1cm} (1)
\[ Q_{G_i} - Q_{D_i} - \sum_{j=1}^{n} Q_{loss_{ij}} = 0 \]  \hspace{1cm} (2)

Subject to

\[ V_{i_{min}} \leq V_i \leq V_{i_{max}} \]  \hspace{1cm} (3)
\[ S_{ij} \leq S_{ij_{max}} \]  \hspace{1cm} (4)
\[ P_{G_i} \leq P_{G_{i_{max}}} \]  \hspace{1cm} (5)

Where \( P_{G_i} \) is the real power generation at bus ‘i’
\( P_{D_i} \) is the real load in bus ‘i’
\( P_{loss_{ij}} \) is the active power loss in the line ‘ij’
\( Q_{G_i} \) is the reactive power generation at bus ‘i’
\( Q_{D_i} \) is the reactive load in bus ‘i’
\( Q_{loss_{ij}} \) is the reactive power loss in the line ‘ij’
\( V_i \) is the voltage at bus ‘i’
\( V_{i_{min}} \) and \( V_{i_{max}} \) are the minimum and maximum voltage limits at bus ‘i’
\( S_{ij_{max}} \) is the thermal limit of line ‘ij’
\( P_{G_{i_{max}}} \) is the maximum real power generation available at bus ‘i’

In RPF method, power flow equations are solved repeatedly by increasing the complex load with uniform load distribution factor and power factor at every load bus in the sink area and increasing the injected real power at generator bus in the source area until limits are incurred. \( P_{G_{sink}} \) (real power in source area), \( P_{D_{sink}} \) (real power in sink area) and \( Q_{D_{sink}} \) (reactive power in sink area) are changed in the following way.

\[ P_{G_i} = P_{G_i}^0 (1 + \lambda_{ttc}) \]  \hspace{1cm} (6)
\[ P_{D_i} = P_{D_i}^0 (1 + \lambda_{ttc}) \]  \hspace{1cm} (7)
\[ Q_{D_i} = Q_{D_i}^0 (1 + \lambda_{ttc}) \]  \hspace{1cm} (8)

Where \( P_{G_i}^0 \) is the original real power generation at bus ‘i’ in source area.
\( P_{D_i}^0 \) is the original active load in bus ‘i’ in sink area.
\( Q_{D_i}^0 \) is the original reactive load in bus ‘i’ in sink area.

\( \lambda_{ttc} \) is the scalar parameter representing the increase in bus load or generation. \( \lambda_{ttc} = 0 \) correspond to no transfer (base case) and \( \lambda_{ttc} = \lambda_{ttc_{max}} \) correspond to maximum transfer.

The TTC level in (normal or contingency state) is given by:

\[ TTC = \sum_{i_{sink}} P_{D_i}(\lambda_{ttc_{max}}) \]  \hspace{1cm} (9)

And ATC neglecting TRM, ETC is given by

\[ ATC = \sum_{i_{sink}} P_{D_i}(\lambda_{ttc_{max}}) - \sum_{i_{sink}} P_{D_i}^0 \]  \hspace{1cm} (10)

Where

\[ \sum_{i_{sink}} P_{D_i}(\lambda_{ttc_{max}}) \] is the sum of load in sink area when \( \lambda = \lambda_{ttc_{max}} \).
\[ \sum_{i_{sink}} P_{D_i}^0 \] is the sum of load in sink area when \( \lambda = 0 \).

III. POWER FLOW USING BSS METHOD

In general, an NR method finds the value of ‘x’ iteratively such that

\[ F(x) = 0 \]  \hspace{1cm} (11)

In the iterative process, say in \( m^{th} \) iteration ‘x’ is updated as given below

\[ x^{m+1} = x^m - \Delta x \]  \hspace{1cm} (12)

And

\[ \Delta x = -(J^m)^{-1} F(x^m) \]  \hspace{1cm} (13)

Where \( J^m \) is the Jacobian matrix.

In the assessment of TTC the power flow equations are solved repeatedly, for every step increment of \( \lambda_{ttc} \) there are more than one iteration and for every iteration a Jacobian matrix of size \( n \times n \) is computed and then inverted. For ‘n’ non linear equations, computation of Jacobian matrix elements includes computation of \( n^2 \) partial derivatives and ‘n’ number of component functions. Therefore \( n^2 + n \) functional evaluations need to be done. Again inversion
of an \( n \times n \) Jacobian matrix using Gauss Jordan elimination method requires \( n^3 \) arithmetic operations or if sparsity technique is used to compute Jacobian inverse with some form of Gauss elimination technique then the total time taken for the inversion is \( k \times n \), where 'k' is the average non zero entries in a row or column of the sparse LU factors and 'n' is the size of the Jacobian matrix. This procedure takes more computational time.

The Quasi–Newton BSS method [17-19] belongs to the class of two step iteration which differentiates it from the conventional Broyden’s method. Let us consider the expression (11) which has to be solved iteratively using BSS method. In the first iteration \( x^0 \) is chosen as in the case of NR method, then \( w^0 \) is calculated as given below

\[
w^0 = -(J^0)^{-1} F(x^0) 
\]

Using (14) \( s^0 \) is updated as

\[
v^0 = x^0 + w^0
\]

this is the first step iteration. Using (15) \( s^0 \) is computed as

\[
s^0 = -(J^0)^{-1} F(v^0)
\]

Then with the value of \( s^0 \) and \( v^0 \), \( x^1 \) is updated using

\[
x^1 = v^0 + (M - C \| s^0 \|) s^0
\]

Which is the second step iteration. Here \( M \), \( C \) and \( a \) are the real variables defined in [17], where the role of \( M \) is to increase the rate of convergence, \( C \) and \( a \) keeps the new iteration in the convergence region.

From the second iteration the above procedure is repeated by replacing the Jacobian matrix ' \( J \) ' with an equivalent matrix ' \( A \) ' which is defined at the \( m^{th} \) iteration as given below

\[
A^m = A^{(m-1)} + [\Delta F(x) - A^{m-1} (\Delta x)] 
\]

where

\[
\Delta F(x) = F(x^m) - F(x^{m-1})
\]

This reduces the number of functional evaluations to 'n' from '\( n^2 + n \)' when compared to the case of NR method but makes the convergence of BSS as super linear when compared to quadratic convergence of NR method.

Further the \( n^3 \) arithmetic operation for computing the inverse of \( A^m \) matrix can be reduced to \( n^2 \) operations using the Sherman Morrison formula as

\[
(A^m)^{-1} = \frac{[A^{(m-1)}]^{-1} + U}{\Delta x^T [A^{m-1}]^{-1} \Delta F(x)}
\]

Where

\[
U = \{\Delta x - [A^{m-1}]^{-1} \Delta F(x)\} \{\Delta x [A^{m-1}]^{-1}\}
\]

Unlike NR method, here the Jacobian inverse is computed only once during the first iteration and for the remaining iterations a rank one update is done to compute the inverse.

In a normal power flow, the quadratic convergence and the advantage of implementing sparsity technique in NR method proves to be superior to the super linear convergence of BSS method which has Sherman Morrison formula for Jacobian inverse. When it comes to the problem of TTC assessment power flow is solved repeatedly, which involves multiple Jacobian computations and inverses, in this process computation using BSS method is faster when compared to NR method [21].

IV. ALGORITHM TO ASSESS TTC USING RPFNMR AND RPFBSS METHOD

The algorithm to assess TTC using RPFNMR and RPFBSS method differs only in the power flow technique used as given below

a) RPFNMR method

Step 1 : Read Bus, line, generator data etc.
Step 2 : Solve power flow using NR method. Check equations (3), (4), and (5), if there is limit violations go to step 4 else go to step 3.
Step 3 : Make a step increase in \( \lambda \) ttc as \( \lambda ttc = \lambda ttc + \Delta \lambda ttc \). Compute equation (6), (7) and (8). Go to step 2.
Step 4 : Compute TTC using (9) at \( \lambda ttc_{\text{max}} = \lambda ttc \).

b) RPFBSS method

Step 1 : Read Bus, line, generator data etc.
Step 2 : solve the first iteration of power flow using NR Method
Step 3 : Use (14) to (22) for second iteration which replaces NR by BSS method. Solve power flow completely. Check equations (3), (4), and (5), if there is limit violations go to step 5 else go to step 4.
Step 4 : Make a step increase in \( \lambda \) ttc as, \( \lambda ttc = \lambda ttc + \Delta \lambda ttc \). Compute equation (6), (7) and (8). Go to step 2.
Step 5 : Compute TTC using (9) at \( \lambda ttc = \lambda ttc_{\text{max}} \).

V. RESULTS AND DISCUSSION

The effectiveness of the proposed methodology is illustrated using the WSCC 9 bus, New England 39 bus and IEEE 118 bus test system. The power flow data for the test system are considered from [22-23]. Load flow programs are executed in MATLAB using modified
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MATPOWER [24] coding in INTEL core 2 Duo CPU T5500@ 1.66 GHz processor under Windows XP professional operating system.

a) WSCC 9 bus test system

WSCC 9 bus test system consists of 3 generators with 9 transmission lines. This system has been divided into two areas for TTC computation. Area 1 includes buses 3,6,8,9 and Area 2 has buses 1, 2,4,5,7. The base load in Area 1 is 125 MW and that of Area 2 is 190MW. The TTC value, MW loss and the limit condition obtained using NR and BSS are identical, hence a common entry has been made in Table 1. The TTC value for transfer of power for Area 1-2 under base case, selected line outage and with generator outage is 410.4 MW, 239.4 MW and 269.8 MW respectively as shown in Table 1. The total time required to complete the computation of TTC for Area 1-2 with and without contingencies using BSS and NR method is 2.4333 (s) and 2.5533 (s). Similarly the transfer of power for Area 2-1 with and without contingencies is also shown in Table 1. The overall time to compute both transfer directions including contingencies using BSS method is 4.1593 (s) and for NR method is 4.2299 (s) which shows that the CPU time for BSS method is 1.7 % less when compared to that of NR method.

b) New England 39 Bus test system

New England 39 bus test system has 10 generators and 46 transmission lines. For computing TTC this system has been divided into three areas. The buses in each area are as shown in Table 2. The base loads in Area 1, Area 2 and in Area 3 is 1124 MW, 2377.5 MW and 2649 MW respectively. The transfer power for all transfer directions i.e., Area 1-2, Area 2-3 Area 3-1 and vice versa is shown in Table 3. The TTC value, limit condition and MW loss for power transfers in between areas using NR and BSS methods are identical whereas the CPU time for computing these results for both these methods differs as shown in Table 3. The computational time for all transfer directions for base case, line outage and generator outage case using BSS method is 4.8329 (s), 4.1679 (s) and 2.9049 (s) respectively. Similarly using NR method, the CPU time are 6.1719 (s), 5.1944 (s) and 3.455 (s) respectively. Hence the overall time to compute TTC value with and without contingencies for all transfer directions for BSS and NR method is 11.9057 (s) and 14.8213 (s) respectively. Similarly using NR method, the CPU time are 8.7467 (s), 8.5727 (s) and 7.7738 (s) respectively. Hence the overall time to compute TTC value with and without contingencies for all transfer directions for BSS and NR method is 14.7815 (s) and 25.0932 (s) respectively which shows that the CPU time for BSS method is 69.76 % less when compared to that of NR method.

c) IEEE 118 bus test system

This test system has 54 generators and 186 transmission lines. It has been divided into three areas with the area wise classification of buses as shown in Table 2. The base loads in Area 1, Area 2 and in Area 3 is 963 MW, 1937 MW and 1342 MW respectively. The transfer power for all transfer directions are computed and furnished in Table 4. The TTC value, limit condition and MW loss for power transfers in between areas using NR and BSS methods are identical. On the other hand the CPU time for computing these results for both these methods differs as shown in Table 4. The computational time for all transfer directions for base case, selected line outage and generator outage case using BSS method is 5.0955 (s), 4.985 (s) and 4.701 (s) respectively. Similar using NR method, the CPU time are 7.4567 (s), 6.5727 (s) and 7.7738 (s) respectively. Hence the overall time to compute TTC value with and without contingencies for all transfer directions for BSS and NR method is 17.6815 (s) and 25.0932 (s) respectively which shows that the CPU time for BSS method is 69.76 % less when compared to that of NR method.

VI. Conclusion

A fast computational technique to assess TTC using BSS method is presented and tested on WSCC 9 bus, New England 39 bus and IEEE 118 bus test system. Results indicate that the computational time to assess TTC using the proposed BSS method is far less when compared to the conventional NR method without losing accuracy. Further from the results it is also evident that the percentage reduction in CPU time for the proposed approach increases with the increase in size of the power system when compared to that of conventional approach.

REFERENCES Références Referencias

16. Determination of ATC within the Western Interconnection, WECC RRO Document MOD -003-0, (2001), 4-5.
### Table 1: TTC for WSCC 9 bus test system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Area 1 – 2</th>
<th>Area 2 – 1</th>
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<tbody>
<tr>
<td><strong>Base case</strong></td>
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</tr>
<tr>
<td>TTC (MW)</td>
<td>410.4</td>
<td>190.0</td>
</tr>
<tr>
<td>MW Loss</td>
<td>10.57</td>
<td>12.29</td>
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<td>Limiting Factor</td>
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<td>$V_y$</td>
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<td>1.2715</td>
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<td><strong>Contingency (Line outage)</strong></td>
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<td>Line outage</td>
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<td>Line 7-8</td>
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<tr>
<td>TTC (MW)</td>
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<tr>
<td>MW Loss</td>
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<td>Limiting Factor</td>
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<td>$V_y$</td>
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<tr>
<td>CPU Time(s) NR BSS</td>
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<td><strong>Contingency (Generator outage)</strong></td>
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<td>Generator outage</td>
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<td>Bus 2</td>
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<td>TTC (MW)</td>
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<td>MW Loss</td>
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<td>0.4230</td>
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### Table 2: Area wise classification of Test system

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<th>IEEE 118 Bus</th>
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<td>2</td>
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<td>24, 38, 65 – 112, 116, 118</td>
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<tr>
<td>3</td>
<td>15, 16, 19–24, 28, 29, 33–36, 38</td>
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### Table 3: TTC value with area wise power transfer of New England 39 bus test system

<table>
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<td>59.91</td>
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<td>$V_a$</td>
<td>Line 6-11</td>
<td>Line 2-3</td>
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<td>TTC (MW)</td>
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<tr>
<td>CPU time(s)</td>
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### Table 4: TTC value with area wise power transfer of IEEE 118 bus test system

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<td>Line 65-68</td>
<td>$P_{G69}$</td>
<td>$P_{G69}$</td>
<td>Line 89-92</td>
<td>Line 89-92</td>
<td>$P_{G10}$</td>
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<td>TTC (MW)</td>
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<td>1213.4</td>
<td>1570.1</td>
<td>2111.3</td>
<td>1030.4</td>
<td>1637.2</td>
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<td>173.41</td>
<td>160.78</td>
<td>143.31</td>
<td>144.36</td>
<td>154.94</td>
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<td>$P_{G69}$</td>
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<td>Line 90-91</td>
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<td>TTC (MW)</td>
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<td>$P_{G69}$</td>
<td>Line 89-92</td>
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</tr>
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<td>CPU time(s)</td>
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