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Dynamic Programming and Taguchi Method Optimization of Water-Treatment-Plant Design

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I. Introduction

The water Industry is one of the principal industries in every country. The total expenditure for water supply, including operation and maintenance, accounts to nearly 6 billion dollars per year in the U.S alone, of this approximately 15% is spent on actual water treatment (Wiesner, 1985). This massive investment requirement calls for more economical system design to allow for an efficient use of public funds. This objective can be achieved using "Optimization". Because in many cases pilot scale studies are conducted prior to full-scale plant construction, the use of an optimization model can greatly reduce the time and money spent on the pilot scale study.

A water treatment system is a combination of several unit processes. Design of such a system is a difficult task and the least cost design is most difficult of all. The principal unit process in a conventional water-treatment systemincludes coagulation-flocculation, sedimentation, filtration and disinfection. The performance of each treatment unit affects the efficacy of the subsequent units. Ideally, therefore, the design decisions should be made with regard to the interaction between various unit operations. So for very few works have been done on the optimization of a water treatment system. Letterman & Iyer (1977) analyzed the effects of selected process decision variables on overall system cost and performance. The authors used simplified process models (mostly empirical relationships). One comprehensive work is that of Wiesner et. al(1978)which is an important step towards the optimal design of water treatment system. The authors have provided a number of optimization models for integral analysis and economic optimization of the components of a water treatment plant. Mhiasalkar(1993) has developed a mathematical model incorporating the Performance relationships and cost functions for the component units of conventional water treatment system, and an algorithm using dynamic programming has described by them for functional and minimal cost design of the system. The last and one of the comprehensive models in this ground belongs to Dharmappa et. al (1994). The authors believe the incorporation of particle size distribution (PSD) is necessary for optimal process design and selection. In this context this works includes not only all three levels of system design but also process design and selection using PSD and the Algorithm they have used was a GP and NLP programming.

The major disadvantages of GP and NLP algorithms are that, global optima is not assured and they can not be directly used for system optimization because of the presence of discrete decision variables and requirement high computer storage. Using Dynamic Programming solely for large problems requires very high computer storage. Accordingly, this paper addresses it self to the development of a model for minimal cost and Energy design of a conventional water treatment plant using dynamic programming and Taguchi Design Of Experiments methods optimization model. The scope of this research was restricted to the economic optimization of conventional water treatment system for turbidity removal comprising four water treatment processes, which are listed in table 1.

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Table 1 : Conventional water treatment processes considered in this study

1.	Rapid Mixing (back Mix)
2.	Flocculation (Horizontal)
3.	Sedimentation (Rectangular)
4.	Granular filtration (Downflow & constant Rate)
	(Note: Specific type considered is given in parenthesis)

The serial and interactive nature of the various unit processes favors application of Dynamic Programming for minimal cost design. Dynamic Programming is a mathematical technique often useful for making a sequence of interrelated decisions. It provides a systematic procedure for determining the combination of decisions that maximizes overall effectiveness [Hillier 1967]. Deterministic dynamic programming can be described diagrammatically, as shown in Fig. 1

Fig. 1: The basics structure for deterministic dynamic programming.

Thus at stage n the process will be in some state S_n. Making policy decision X_n then moves the process to some state S_{n+1} at stage (n+1). From that point onward the objective function value for the optimal policy has been previously calculated to be $f^*_{n+1}(S_{n+1})$. Conventional water Treatment systems can be considered as a multistage process, with the stages represented by various unit processes. The states represented by the levels of water quality parameters such as turbidity and suspended solids . The decision variables are design parameters depending on the type of unit.

Traditionally, measuring nonspecific parameters like turbidity units (Tu) suspended solids (SS); etc assesses the performance of water treatment plants. Though their parameters describe the Important aspects regarding the water quality of the effluent form the treatment processes (AWWA: water 1990). Insofar as turbidity is used as a measure of the efficiency of the removal of particulate matter throughout the purification process of water, low turbidity in the finished water is an indication of effectiveness of unit processes (Guidelines 1984). Because mathematical models of water treatment describe the behavior of systems in relation to the removal of mass concentration of suspended solids. And we can in some ways correlate turbidity measurements with suspended solids data, suspended solids has been used as a state parameter to indicate the effectiveness of the system at various stages of treatment. Fig. 2 Show the sequence of stages and the stage return for each input-decision combination.

II. Taguchi Method

Taguchi approach to parameter design provides the design engineer with a systematic and efficient method for determining near optimum design parameters for performance and cost (Kackar, 1985: phadke, 1989; Taguchi, 1986). In this research we have used orthogonal arrays from design of Experiments theory to study a large number of variables with a Small number of experiments. Whereupon the time of DP model analysis reduced considerably.

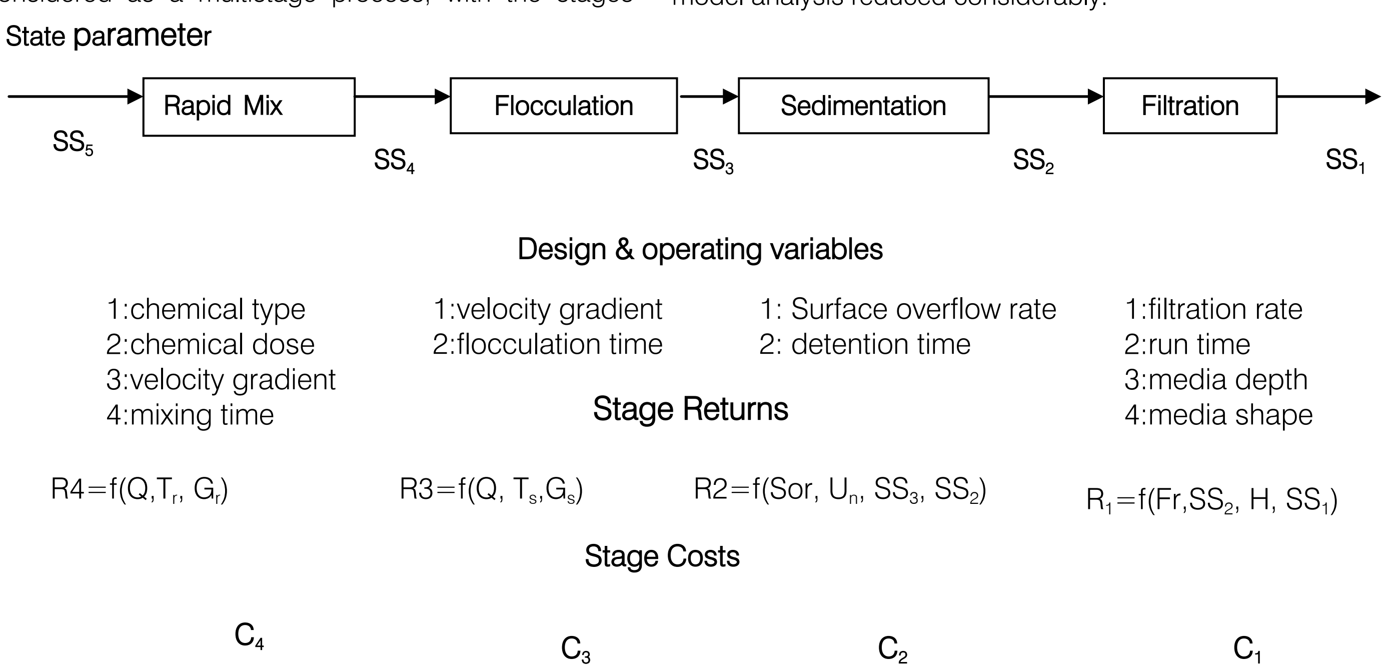

Fig.2 : Unit sequences & Return of Conventional Water Treatment system

a) Functional relationships of units

Functional relationships for each unit process, linking input and output states parameters with a characteristic loading parameter, have been established based on the mechanics of unit process. These relationships are largely empirical models. Table 2 shows these relationships and key assumptions for each of the process models.

III. Cost Functions

Since the present study seeks to incorporate multiple design criteria in the optimization, there are three design criteria to be evaluated from the cost functions:

1. Annualized Capital Cost (ACC)
2. Operation & Maintenance cost (O & M)
3. Energy requirement (ER)

There is a substantial literature on cost functions for water treatment plant component (Hinomoto, 1977; Clark, 1982). The annualized capital and operation and maintenance cost functions are most taken from Clark (1982). Some changes have been made by Dharmappa (1994) so as to make them more generals. Capital cost functions as presented by Clark (1982) were based on the annualization factor 0.102 (interest 8% and amortization 20 years). There were modified to incorporate a general annualization factor, of, which can be calculated by (assuming zero salvation value:

$$a_f = \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right.$$

Where a_f = annualization factor;
i= interest rate, Fraction; and
n= amortization period, years.

Table 2 : Relationships and keys Assumption for each of the process models.

Unit	Functional Relationships Unit processes	
Rapid Mix	$SS_5 + K. A = SS_4$	A :alum dose K : Constant
Flocculation	$SS_4 = SS_3$ $d_f = 26.88 G_s^{-0.91}$ $T_s G_s^{2.8} = \frac{44 \times 10^5}{A}$	Ts: detention time Gs: applied velocity gradient d_f: average diameter of the floc
Sedimentation	$Vs = \frac{g}{18}(S_s - 1)\frac{d_f{}^2}{\nu}$ $\frac{ss_3 - ss_2}{ss_3} = 1 - [1 + n\frac{Vs}{Q/A}]^{-1/n}$	Q/A: surface overflow rate n: coefficient basinperformance Vs: surface overflow rate for ideal basin
Rapid sand filter	$\frac{ss_2 - ss_1}{ss_2} = Pc$ $Y = 0/725.Fr^{0/29} d^{0/62} t$ $R = \frac{4/55.d^{2/5} H}{Fr^{1/2} ss_2}$ $\log(\frac{U}{13/3L}) = -0/208 + 1/950 \log[\frac{Y}{(13/3L)^{1/2}}] - 0/645[\log\frac{Y}{(13/3L)^{1/2}}]^2$ $\log(\frac{R}{13/3L}) = -3/250 + 1/013 \log[\frac{Y}{(13/3L)^{1/2}}] - 0/036[\log\frac{Y}{(13/3L)^{1/2}}]^2$ $Pc = \sum_{j=0}^{(t/2-1)} \frac{e^{(-u/2)}(u/2)^j}{J!}$	SS_1:influent suspension concentration H: increase in head loss at the end of t d : diameter of sand grain t: filter run time Fr: filtration rate

The relationships for energy requirement are developed from Gumerman et al. (1979). Cost functions for unit processes (extracted and compiled from Clark, 1982, with Dharmappa (1994) modifications are listed in Table 3.

In this table PR; DHR, PPI are: Power cost, Direct Hourly wages Rate and Producer Price Index respectively.

Table 3 : Cost function for various unit processes

Unit processes	Annual Construction Cost(ACC) Operation and Maintenance Cost (O & M)
Rapid mix	$ACC = 0.08507(Q \times Tr)^{0.79}(CCI)^{0.988}(1.0007)^{Gr}(a_f)$ $OM = 1.56947(Q \times Tr)^{0.799}(PR)^{0.717}(DHR)^{0.181}(1.00251)^{Gr}$
Alum dry stock	$ACC = 0.0197(A \times Q)^{0.656}(CCI)^{0.994}$ $OM = 3.30366 \times 10^{-4}(A \times Q)^{0.849}(PR)^{0.1847}(PPI)^{0.0259}(DHR)^{0.743}$
Flocculation	$ACC = 642.48(Q \times Ts)^{0.6916}(CCI)^{0.992}(1.00383)^{Gs}(a_f)$ $OM = 0.040581(Q \times Ts)^{0.785}(PR)^{0.357}(PPI)^{0.399}(1.0081)^{Gs}$
Clarifier	$ACC = 1872.527(A_c)^{0.701}(CCI)^{0.993}(U_n)^{1.00047}(a_f)$ $OM = 53.487(A_c)^{0.969}(PPI)^{0.181}(DHR)^{0.756}(U_n)^{1.006}$
Filter structure	$ACC = 13317.11(A_f)^{0.671}(CCI)^{0.989}(a_f)$ $OM = 913.377(A_f)^{0.549}(PR)^{0.147}(PPI)^{0.183}(DHR)^{0.61}$
Filter Media(single)	$ACC = 71.3413(A_f)^{0.9336}(CCI)^{0.996}(a_f)$ $OM = 0$
Filter backwash	$ACC = 108.127(6.1A_f)^{0.59}(ACC)^{0.966}(a_f)$ $OM = 747.398(6.1A_f)^{0.65}(PR)^{0.543}(PPI)^{0.219}(DHR)^{0.137}$
Filter surface wash	$ACC = 568.4533(A_f)^{0.801}(CCI)^{0.982}(a_f)$ $OM = 204.732(A_f)^{0.7146}(PR)^{0.526}(DHR)^{0.315}$

IV. Model Optimization

There are a number of designs that satisfy water quality standards. The objective, therefore, is to minimize the system cost satisfying all constrains. The bounds used in the optimizations are given in table 4. The general form of objective function is expressed as.

$$\text{Opt} . \sum_{i=1}^{4} [R_i]$$

Design variables

Where: R= Stage Returns

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Table 4 : bounds on variables used in optimizations

Variable	Units	Symbol	Lower bound	Upper bound
Velocity Gradient	s^{-1}	G_r	400	1000
Mixing Time	S	T_r	10	50
Velocity Gradient	s^{-1}	Gs	20	70
Flocculation time	min	Ts	10	60
Settling Time	hr	T_c	2	4
Settling Efficiency	-	-	0	0.95
Surface Overflow Rate	M hr^{-1}	SOR	0.9	2.0
Filter Area	m^2	A_f	1	∞
Filter run time	hr	T_f	8	100
Filtration Rate	hr^{-1}	Fr	3	15
Backwash fraction	-	B	0	0.15
Time for backwash	Hr	T_b	0.33	0.33
Terminal head loss	m	H	0	2.5
Filtrate mass Concentration	mg/L	C_{max}	0	2
Clarifier effluent Concentration	mg/L	C_e	2	20

V. RECURSIVE EQUATIONS

For deriving Recursive equations it is necessary the Return functions for each unit process be prepared with economic data. The values of parameters for determining Return functions that are used in the optimizations, as well as the assumed chemical costs are listed in Table 5.The Return functions for all unit processes are given in table 6

Table 5 : Economic data used in Return function

Item	Value
Power(PR)	0.04$/kWh
Labor Wage Rate(DHR)	11.00$/hr
Construction Cost Index/100(CCI)	3.25
Producer Price Index/100(PPI)	2.44
Alum	140$/tone
Polymer	400$/ tone
Polymer demand	$6*10^{-8}$moles/m^2 of particle
Polymer molecular weight	$5*10^4$

Table 6 : Return Equations of different unit processes

Units	Return Equation of units
Sand Filter	$R_1 = 8636.59(A_f)^{671} + 4688.94(A_f)^{546} + 46.877(A_f)^{9336}$ $293.847(A_f)^{.59} + 2006.13(A_f)^{.65} + 363.86(A_f)^{.801} + 119.195(A_f)^{.7146}$
Sedimentation Tank	$R_2 = 1223.52(A_c)^{.701}(U_n)^{1.00047} + 599.165(A_c)^{.469}(U_n)^{1.006}$
Flocculation	$R_3 = 1.45443(Q.T_s)^{.6916}(1.00383)^{G_s} + 0.0345(Q.T_s)^{.785}(1.0081)^{G_s}$
Alum feed & Rapid mix	$R_4 = .54067(Q.T_r)^{.79}(10007)^{G_r} + 0.3817(Q.T_r)^{.794}(1.00251)^{G_r} +$ $o.12662(A.Q)^{.656} + 0.00160934(A.Q)^{.849}$

Now by using there Return functions we can derive the Recursive equations for all of the unit processes. The general forms of recursive equations for unit processes are given in table 7.

Table 7 : Recursive equations of different unit processes

Units	Recursive equations of Units
Filter	$f_1^*(SS_2) = \underset{A_f, Q}{opt.}[R_1]$
Sedimentation	$f_2^*(SS_3) = \underset{A_c, Q, G_s}{opt.}[R_2 + f_1^*(SS_2)]$
Flocculation	$f_3^*(SS_4) = \underset{G_s, T_s, Q}{opt.}[R_3 + f_2^*(SS_3)]$
Rapid mix	$f_4^*(SS_5) = \underset{A, Q, G_s, T_r, G_r}{opt.}[R_4 + f_3^*(SS_4)]$

VI. OPTIMIZATION ALGORITHM

The sequential structure of serial systems can be exploited to transform the N-decision, one-state problems. This is accomplished by the procedure called dynamic programming, due to Richard Bellman (1957). The DP algorithm for functional and minimal cost design of water treatment system was developed in this research. The major inputs required are the design data on flow; raw-water suspended-solids concentration; alum dose; water temperature; and effective size, uniformity coefficient, and depth of filter sand. Data on unit costs of excavation, plain and reinforced cement concrete, steel, filter sand and appurtenances are also necessary. Data on discount rate, and chemical costs are required for economic evaluation. The algorithm involves four stages in DP programming, and the optimal function of the previous stage is implemented in the next stage. In each stage in addition to the previous optimal function, the return function concerning to the current stage is also used to optimize the existing stage.

This procedure will be followed until the last stage. The solution of this system is computed by backward form. With this parametric solution of stages, we can derive the optimal objective function in a parametric form. This parametric form includes all design parameters of the water treatment system. This objective function could be optimized by appropriate software based on the Taguchi Design of Experiments again could be calculated.

VII. CASE STUDY

The application of the optimization algorithm is illustrated for a 6250-m3/hr conventional water treatment plant. The optimal results of this model have been compared with EPA traditional water Treatment plant. Design criteria and cost calculation of EPA conventional water treatment plant exists in EPA Documents Gumerman et al. (1979). The optimal design results in saving of 9.5% in capital and Annualized cost. The comparison is presented in table 8.

Table 8 : Comparison of Total Annualized Cost with EPA and Model 6250 m^3/hr

Water treatment Plant / **Items**	EPA WaterTreatment Plant	Suggested Model Water Treatment Plant
Total Annualized Cost($)	1046570	955757
Reduction(%)	9.5	

VIII. SENSITIVITY ANALYSIS

One particularly important feature of this model is the ease with which a sensitivity analysis can be carried out. This technique analyzes the sensitivity of the optimal solution to changes in various decision variables without re-solving the problem for each new value. It is highly unlikely that cost data of sufficient accuracy will ever be available (Walski 1991). Therefore, variation in total Annualized cost was also studied. The results of this analysis have been presented in table 9.

Table 9 : Optimum cost versus 3 more sensitive variables.

Parameter	Change in parameter over base value(%)	Optimal Design Value					Change in total annualized cost (%)
		SS_2	SS_3	G_r	T_r	T_s	
Flow	+10	7.5	150	400	20	20	+6.8
	+20	7.5	150	400	20	20	+12.6
	-10	7.5	150	400	20	20	-7.5
	-20	7.5	150	400	20	20	-15.2
Alum dose	+10	7.5	150	400	20	20	+3.6
	+20	7.5	150	400	20	20	+7.1
	-10	7.5	150	400	20	20	-3.76
	-20	7.5	150	400	20	20	-7.9
Filter Headloss	+10	7.5	150	400	20	20	-1.5
	+20	7.5	150	400	20	20	-2.95
	-10	7.5	150	400	20	20	+1.6
	-20	7.5	150	400	20	20	+3.39

The results signifies the objective function is more sensitive to Design flow rate (Q), alum dose (A) and gravity sand filter head loss (H) respectively. If the design flow is varied in the range of ±20%, the total annualized cost will vary +12.6% and 15.2 %

A practical approach to analyzing either uncertain or variable situation is to test the sensitive of the optimal design to variation in key system parameters.

IX. CONCLUSION

So far very few works have been done on the optimization of a water treatment system. The scope of this research was restricted to the economic optimization of conventional water treatment systems for turbidity removal, comprising four water treatment processes (viz.-rapid mix, flocculation, sedimentation, and rapid send filters). Mathematical models describing the performance relationships and cost functions for component units of a conventional water-treatment system have been formulated. By combining these models, an algorithm using Dynamic programming and Taguchi Design of Experiments methods software was developed for functional and minimal design of the system. The proposed approach is tested with a case study. The conclusions are listed as follows:

- The computer times requirements for this model is very less than other similar model doing the same task.
- The optimal design results in savings of roughly 9.5% in capital and annualized cost compared to the conventional design.
- The sensitivity analysis results signifies the objective function is more sensitive to design flow rate (Q), alum dose (A) and gravity sand filter head loss (H), respectively.

Suspension characteristics such as particle size and distribution are of significance in the overall process of water treatment. There are some shortcomings in major requirements for the successful incorporation of PSD in the process design and selection. (Dharmappa, 1994). Future research dynamic programming & Taguchi methods optimization of water treatment plant design may be directed to test the feasibility of considering PSD and molecular particle size distribution in water and further research in the field of software development with more levels for control factors.

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