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## CFD Prediction of Natural Convection in a Wavy Cavity Filled with Porous Medium

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**Abstract-** In this paper, the natural convection heat transfer and fluid flow inside a square cavity having two wavy walls has been numerically investigated. The cavity is filled with a porous medium. The two wavy vertical walls are maintained at different isothermal temperatures while the horizontal walls are kept insulated. A general non-orthogonal body-fitted coordinates system was used to transfer the considered physical space in to a computational one. The governing stream function equation was solved by using an iteration method with SOR while the energy equation with an alternate difference implicit scheme(ADI).The study was performed for Rayleigh numbers up to 1000. The effect of amplitude, Rayleigh number and number of wavy walls undulations on the flow and thermal field was studied. The obtained results showed that the number of wavy walls undulations has a significant effect on heat transfer and fluid flow. Also the results indicated that the rate of heat transfer increases as Rayleigh number increases and decreases with the increase of amplitude.

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## NOMENCLATURE

A	amplitude
g	gravitational acceleration, m/s <sup>2</sup>
H	height of the cavity wall, m
J	Jacobian of the transformation
K	permeability, m <sup>2</sup>
Nu	local Nusselt number
Nu <sub>av</sub>	average Nusselt number
Ra	Rayleigh number
T <sub>c</sub>	cold wavy wall temperature, °C
T <sub>h</sub>	hot wavy wall temperature, °C
u, v	velocity components, m/s
x, y	Cartesian coordinates, m
X Y	dimensionless Cartesian coordinates
α, β, γ, τ, σ	Transformation parameters in grid generation
ξ, η	coordinates in the transformed domain
ψ	stream function, m <sup>2</sup> /s
Ψ	dimensionless stream function
ρ	density, Kg/m <sup>3</sup>
a	thermal diffusivity, m <sup>2</sup> /s
θ	dimensionless temperature

## I. INTRODUCTION

Study of natural convection heat transfer inside an enclosure has received a great deal of attention due to its implication in most engineering applications. These applications include solar collectors, cooling of micro-electric devices and nuclear reactors. Indeed, most of the researcher's works were concentrated on rectangular or square enclosures. So few studies were found on natural convection in non-rectangular enclosures. The medium inside these enclosures was porous or non-porous. Natural convection heat transfer in wavy enclosures (i.e. non-rectangular enclosures) was motivated by the researchers in recent years. Due to its importance in most technological applications such as geophysical applications and heat exchangers design. The change of surface waviness, waviness mode besides to the location of these waviness is considered a controlling parameters for assessment the flow and thermal field characteristics. Thus the present literature tries to review the previous studies. Al-Amiri [1] investigated the momentum and energy transfer in a lid-driven cavity filled with a porous medium. He used the inertia and viscous effects through the general formulation of momentum and energy transfer. Braden et al. [2] adopted the Darcy model and Buossinesque approximation to investigate the natural convection flow in a porous medium adjacent to vertical or horizontal surface. The surface was heated and cooled sinusoidally along its length. Oothuizen and Patrick [3] studied the natural convection in an inclined square enclosure. The enclosure was differentially heated and partially filled with a porous medium. The study was focused on the average rate of heat transfer across the enclosure. A theoretical study of buoyancy-driven flow and heat transfer in an inclined trapezoidal enclosure filled with a fluid –saturated porous medium was studied by Yasin et al.[4]. The governing equations were solved numerically using a finite difference method. The study was performed for inclination trapezoidal angles ranged from 0° to 180° and Ra from 100 to 1000. Also the wall

angles was ranged from 67° to 81°. the condidered results from that study showed that the effect of trapezoidal inclination angle on heat transfer and flow strength is more than the side wall inclination angle.

Sharif [5] performed a numerical study on mixed convection heat transfer in an inclined lid-driven enclosure filled with viscous fluid. He chose non-porous medium and observed that the average Nusselt number increases with cavity inclination angle. Yasin *et. al.* [6] investigated the free convection in porous media filled right-angle triangular enclosure. The governing equations were obtained using Darcy model and solved by a finite difference techniques. Abdalla et al. [7] analyzed the mixed convection heat transfer in a lid driven cavity with a sinusoidal wavy hot surface. The results of his study showed that the average Nusselt number is increased with an increase of amplitude of the wavy surface and Reynolds number. Kumar [8] investigated the free convection induced by a vertical wavy surface with heat flow in a porous enclosure. He verified that the surface temperature was very sensitive to the drifts in the undulations and amplitude. Dalal and Das [9] presented a numerical study for the natural convection in a cavity with a vertical wavy wall. Their results showed that the local rate of heat transfer and the flow field were significantly affected due to the undulation in the right wall. The effect of surface undulations on the free convection heat transfer from a horizontal wavy surface in a porous wavy enclosure was studied by Murthy et al. [10]. They assumed valid Darcy flow model. Their results showed that the waviness of the

surface reduced the ratio of heat transfer compared with that of a flat surface. Xu *et. al.* [11] performed a numerical study on unsteady natural convection in differentially heated cavity with a fin on a side wall. Different lengths for  $Ra=3.8 \times 10^9$  were performed. The obtained results showed that the fin length significantly impacts on transient thermal flow around the fin and heat transfer through the finned side wall in the early stage of the transient flow development. Hakan and Oztop [12] presented a numerical study to obtain the combined convection field in an inclined porous lid driven enclosure heated from one wall. The study was performed for  $10 \leq Ra \leq 1000$ . It was reported that the flow field, temperature distribution and heat transfer are affected by the angle of inclination. As shown in the literature, there is no study on laminar natural convection in a porous cavity with two wavy walls. So the present work aims to enhance the academic research in this field. As shown in Fig.1, the laminar natural convection heat transfer inside a porous square cavity with two vertical wavy walls has been investigated. The cavity is differentially heated. Different values of wavy walls amplitudes, undulations and Rayleigh numbers are examined. The effect of location of the two wavy walls and thermal boundary conditions are examined. The grid generation system used in this work is based on the procedure proposed by Thompson et al. [13].

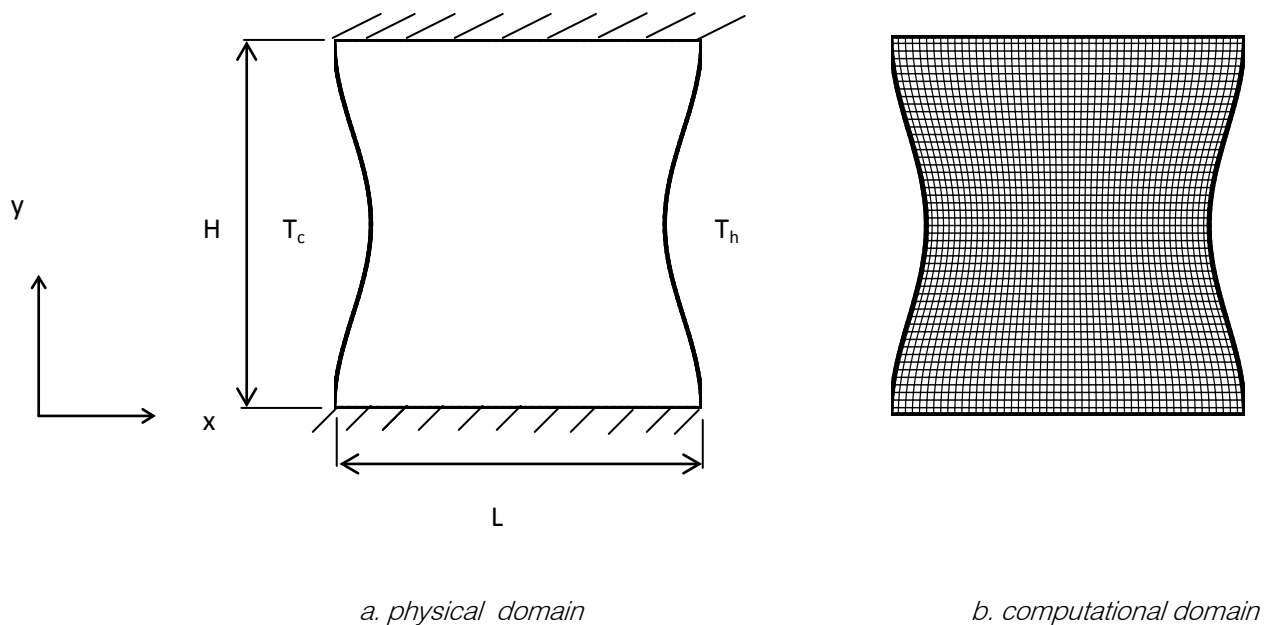


Fig.1 schematic diagram of the problem under consideration

### 1. Grid generation

The numerical calculation of a flow field needs a suitable treatment of boundary conditions which are

difficult to incorporate for complex boundary conditions. A grid generation is used to transfer a physical space in to a computational space. The grid generation method is

used to map the non-rectangular grid in the physical space into a rectangular uniform grid in the computational space. The grid generation method proposed by Thompson [13] is used to transform the

region shown in Fig.1.a in to computation region shown in Fig.1.b. The most common partial differential equation used for grid generation in 2-D is an elliptic Poisson equation.

$$\zeta_{xx} + \zeta_{yy} = P(\zeta, \eta) \text{ ----- (1)}$$

$$\eta_{xx} + \eta_{yy} = Q(\zeta, \eta) \text{ ----- (2)}$$

Where P and Q are known functions used to control interior grid clustering. All grids used in this work are generated with  $P(\zeta, \eta) = Q(\zeta, \eta) = 0$ . The system is completed by addition of Dirichlet boundary conditions which specify  $\zeta$  and  $\eta$  as functions of  $x$  and  $y$  on the

boundary of the region shown in Fig.1. Calculations were performed on the rectangular region so that dependent and independent variables are interchanged to produce a system of two partial differential equations in the form of:

$$\alpha X_{\zeta\zeta} - 2\beta X_{\zeta\eta} + \gamma X_{\eta\eta} = -J^{-2}[X_{\zeta}P(\zeta, \eta) + X_{\eta}Q(\zeta, \eta)] \text{ ----- (3)}$$

$$\alpha Y_{\zeta\zeta} - 2\beta Y_{\zeta\eta} + \gamma Y_{\eta\eta} = -J^{-2}[Y_{\zeta}P(\zeta, \eta) + Y_{\eta}Q(\zeta, \eta)] \text{ ----- (4)}$$

Where

$$\alpha = X_{\eta}^2 + Y_{\eta}^2, \quad \gamma = X_{\zeta}^2 + Y_{\zeta}^2, \quad \beta = X_{\zeta}X_{\eta} + Y_{\zeta}Y_{\eta}, \quad J = X_{\zeta}Y_{\eta} - X_{\eta}Y_{\zeta}$$

The discretization of equations (3-4) is obtained by using a second order central difference procedure and are solved by iteration method with SOR.

## II. MATHEMATICAL MODEL

The two dimensional laminar natural convection heat transfer in a wavy square cavity filled with a porous media has been investigated. To consider the related mathematical model, some assumptions were reported:

- The properties of the porous media is assumed to be constant
- The viscous and inertia effects are ignored and the Boussinesque approximation is valid.

The governing differential equations of the mass continuity, momentum and energy are described as follows. Also some assumption are made on the continuity equation [14,15].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \text{ ----- (5)}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g\beta K}{\nu} \frac{\partial T}{\partial x} \text{ ----- (6)}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \text{ ----- (7)}$$

This is called Darcy model and the equations can be written in dimensionless form after using the following parameters.

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad \Psi = \frac{\psi}{\alpha}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad Ra = \frac{g\beta K(T_h - T_c)H}{a\nu}, \quad \tau = \frac{\alpha t}{H^2}$$

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X} \text{ ----- (8)}$$

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} = \nabla^2 \theta \quad \text{----- (9)}$$

The transformation of the new dependent variables  $(\zeta, \eta)$  defined in the preceding section leads to replacement of  $\psi(x, y)$  in to  $\psi(\zeta, \eta)$  and  $\theta(x, y)$  to  $\theta(\zeta, \eta)$  [13].

$$\lambda \Psi_{\zeta} + \sigma \Psi_{\eta} + \alpha \Psi_{\zeta\zeta} - 2\beta \Psi_{\zeta\eta} + \gamma \Psi_{\eta\eta} = -JRa(\theta_{\zeta} Y_{\eta} - \theta_{\eta} Y_{\zeta}) \quad \text{----- (10)}$$

$$\theta_{\tau} + (-\Psi_{\zeta} \theta_{\eta} + \Psi_{\eta} \theta_{\zeta}) / J = (\lambda \theta_{\zeta} + \sigma \theta_{\eta} + \alpha \theta_{\zeta\zeta} - 2\beta \theta_{\zeta\eta} + \gamma \theta_{\eta\eta}) / J^2 \quad \text{---- (11)}$$

Where

$$\lambda = (X_{\eta} D_y - Y_{\eta} D_x) / J \quad \text{----- (12)}$$

$$\sigma = (Y_{\zeta} D_x - X_{\zeta} D_y) / J \quad \text{----- (13)}$$

$$D_x = \alpha Y_{\zeta\zeta} - 2\beta Y_{\zeta\eta} + \gamma Y_{\eta\eta} \quad \text{----- (14)}$$

$$D_y = \alpha X_{\zeta\zeta} - 2\beta X_{\zeta\eta} + \gamma X_{\eta\eta} \quad \text{----- (15)}$$

#### 1) Boundary conditions

In order to solve the mathematical model, the following boundary conditions are used.

$U = V = 0, \theta = -0.5, \psi = 0$  on the cold wall

$U = V = 0, \theta = 0.5, \psi = 0$  on the hot wall

$U = V = 0, \frac{\partial \theta}{\partial X} = \left( \alpha \frac{\partial \theta}{\partial \zeta} - \beta \frac{\partial \theta}{\partial \eta} \right) / J \sqrt{\alpha} = 0, \psi = 0$  on the two insulated walls

The local and average Nusselt number along the hot wavy wall is calculated as follows.

$$Nu = - \int_0^1 \frac{d\theta}{dx}$$

$$Nu_{av} = \int_0^1 Nudy$$

### III. NUMERICAL SOLUTION

Finite difference formulation is used to discretize the considered partial differential equations. The resulting algebraic equations for temperature distribution, eq. 11 were solved by using alternate difference implicit (ADI) method. The iteration method with successive overrelaxation scheme (SOR) was used for solving the discretization equation of the stream function, eq. 10. The Relaxation factor used for stream function had the value of 1. A home computer program using Fortran 90 language was constructed to handle

the considered problem. In order to ensure that the flow and heat transfer characteristics are not affected by the mesh, different grids were used,  $(31 \times 31)$ ,  $(41 \times 41)$  and  $(51 \times 51)$  respectively. As shown in table 1, there is no noticeable change between the used grids and the grid  $(51 \times 51)$  is adopted in this work.

Table1. Effect of mesh on  $Nu_{av}$  for  $Ra=500$ ,  $A=1$  and for one undulations

Mesh	$Nu_{av}$
31×31	4.49
41×41	4.41
51×51	4.39

#### IV. RESULTS AND DISCUSSION

In this section, the computed results of stream function, isotherm lines, local and average Nusselt numbers will be reported. These factors were plotted for different values of wavy walls undulations, amplitude and Rayleigh number. Also the effect of boundary conditions and the location of the two wavy walls are examined.

Fig.2 demonstrates the stream function and isotherm lines distribution for different Rayleigh numbers and one undulation. For (a), when  $Ra=50$ , it can be seen that there is four rotating vortices symmetrical about the centerline of the cavity. These vortices occupy the upper and lower part of the cavity especially at the troughs. The isotherm line for  $Ra=50$  seem to be parallel and take the cavity shape. This manifest that the convection currents are very small. When  $Ra$  increases to 150, the rotating vortices are seem to be close to each other and the isotherm lines start to deviate and that confirm the presence of convection currents due to increase of the buoyancy forces. When  $Ra=500$ , the size of rotating vortices becomes larger and they be closer to each other at both vertical and horizontal centerlines of the cavity. Also the isotherm lines are significantly deviate and symmetrical about the cavity centerline. The thermal boundary is thick near upper and lower parts of the two wavy walls while it is less thick at the cavity centerline especially at the wavy wall crests. This is confirmed at Fig.5.a where the local Nusselt number value is minimum at  $Y=0.5$ . For one undulation, when  $Ra$  extends to 1000, the size of rotating vortices is larger and more elongation. This indicates that the convection currents are dominant. Also the isotherm lines intensity are larger at the upper and lower part of the wavy walls and less at the cavity centerline. This will enhance heat transfer as shown in Fig.5.b for one undulation. In this Figure, the values of maximum Nusselt number for one undulation are increased nearly by 35%. The effect of  $Ra$  on stream function and isotherm lines distribution for two undulations is depicted in Fig.3. it can be seen that the area of crest and trough is decreased but there is multiple crests and trough. For  $Ra=50$ , there is six

rotating vortices symmetrical about the cavity centerline. The size of vortices are larger at the upper and lower parts of the cavity compared with that near the middle of the cavity. The size of these vortices are increased with the increase of  $Ra$ . Also the shape is elongated to semi elliptical type especially at  $Ra=500$  and  $Ra=1000$ . As a results the isotherm lines deviation is increased with the increase of  $Ra$  due to increase the buoyancy forces. This behavior is disclosed at Fig.5 (a and b) where the heat transfer is increased at the trough regions and decreased at crest. The lower values is not lies at the cavity centerline ( $Y=0.5$ ). When the undulation number increase to three, eight rotating vortices is appearing as shown in Fig.4. However the shape and size of these vortices are significantly changed with the increase of  $Ra$ . Also the deviation of stream lines are increased with the increase of  $Ra$ . This demonstrate at Fig.5( a and b). here the minimum values occur at  $Y=0$ . The effect of wavy walls amplitude ( $A$ ) on stream function and isotherm lines is exhibited at Fig. 7. When  $A=0.075$ , the shape of counter vortices is elongated and the size of these vortices are seem to be larger. When  $A=0.15$ , there is dramatic changes in stream function distribution. Two elongated upper vortices and two elongated lower vortices are appeared. Two of these vortices are very close to the hot wavy wall crest. There is a significant change in isotherm lines distribution (d and e). For f, the density of isotherm lines is very high near the wavy hot wall crest at  $Y=0.5$ . This will enhance heat transfer as shown in Fig.9 where higher values of  $Nu$  occur at  $Y=0.5$ . Fig.8 demonstrates the effect of wavy wall amplitude for three undulations and  $Ra=500$ . For  $A=0.075$ , there is a little change in the size of rotating vortices especially near the centerline ( $Y=0$ ). When  $A=0.085$ , the size of these vortices is seem to be less especially at the upper and lower region of the cavity and the isotherm lines seem to be thicker. When  $A=0.15$ , the size of the resulting vortices is smaller and the boundary layer is thicker at the trough regions. Fig.6 discloses the variation of average Nusselt number versus  $Ra$  at different undulations values. It can be seen



that the average Nusselt number is decreased with the increase of undulations number. When the boundary condition for the dimensionless temperature is changed for  $\theta = 1$  (on the hot wall) and  $\theta = 0$  (on the cold wall) as shown in Fig.10, there is significant changes in the stream function and isotherm lines distribution has been occurred. However the values of local heat transfer are increased. This is dominant for all the studied cases. The two wavy walls are placed at horizontal sides as found in Fig.12, where there is a significant change in number and size of resulting vortices. The isotherm lines densities are less and that leads to decrease the rate of heat transfer as shown in Fig.13. To increase our understanding to the flow field, velocity vectors are plotted for three undulations and for two positions of the wavy walls as shown in Fig.14. As the Figure, shows a part of the velocities hit the crest and the remaining forming recirculation zones. The velocities in core region in (a) is larger than (b). To perform the validation of the present numerical method used, a comparison with the available published results is made as shown in Fig. 15. As the Figure shows, a good agreement is obtained.

## V. CONCLUDING REMARKS

In this study, the 2D laminar natural convection heat transfer and fluid flow inside a square porous cavity with two vertical wavy walls has been performed. Thus from the current computed results, the following conclusions can be summarized.

1. For one and three undulations, the minimum local heat transfer occurs at the middle of the hot wavy wall ( $Y=0$ ). However this is not seen with the two undulations.
2. The average rate of heat transfer is decreased with the increase of undulation number for  $Ra > 200$ .
3. The Rate of heat transfer is increased with the increase of the amplitude values for  $0.05 \leq A \leq 0.075$ . After that the local rate of heat transfer is decreased. However the local rate of heat transfer is enhanced significantly at  $Y=0.5$
4. The number and size of rotating vortices is increased with the increase of undulations number.
5. The location of the two wavy walls is better at the vertical sides compared with that of the horizontal sides.
6. The local Nusselt number exhibited periodic distribution

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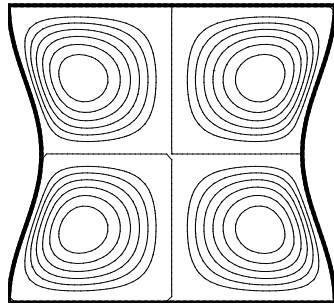
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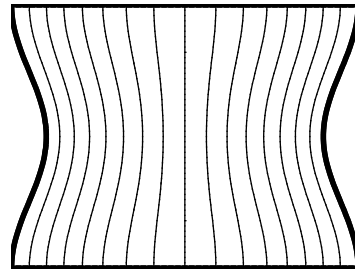
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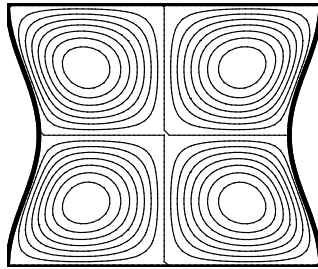




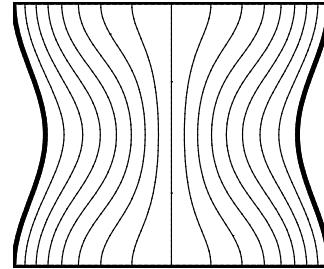
*a.  $Ra=50$*



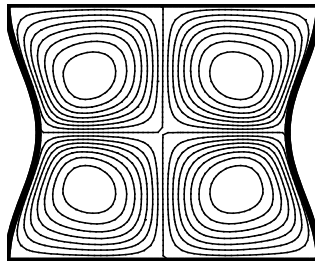
*e.  $Ra=50$*



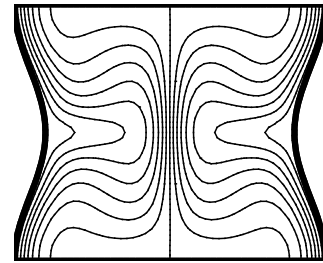
*b.  $Ra=150$*



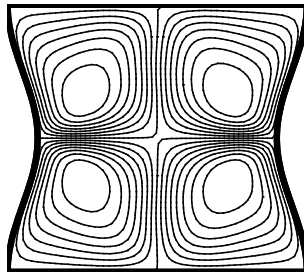
*f.  $Ra=150$*



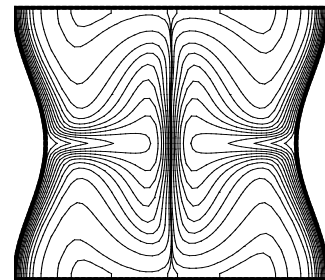
*c.  $Ra=500$*



*g.  $Ra=500$*

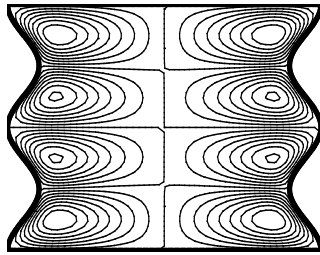


*d.  $Ra=1000$*

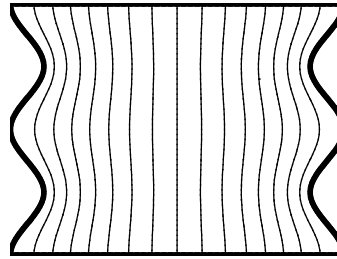


*h.  $Ra=1000$*

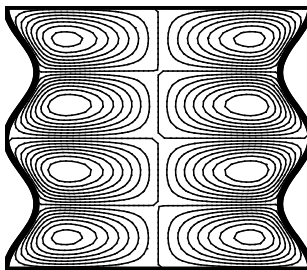
*Fig. 2 effect of  $Ra$  on stream function and isotherm lines distribution for one undulation and  $A=0.05$*



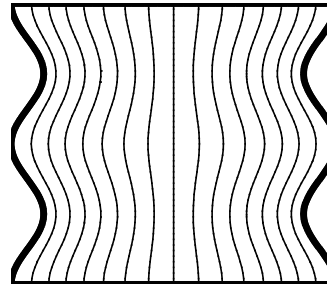
*a. Ra=50*



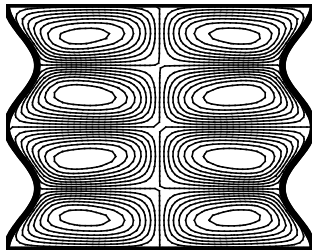
*e. Ra=50*



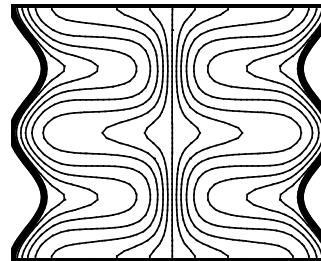
*b. Ra=150*



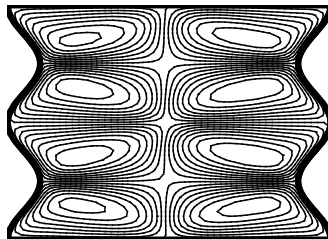
*f. Ra=50*



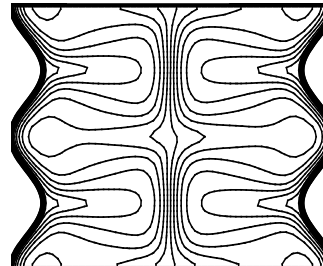
*c. Ra=500*



*g. Ra=500*

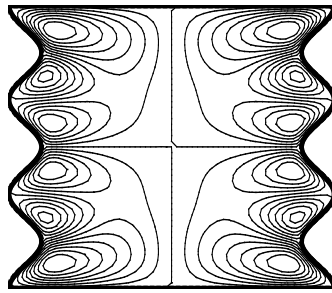
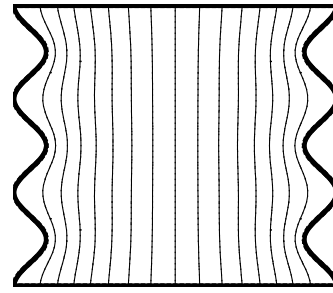
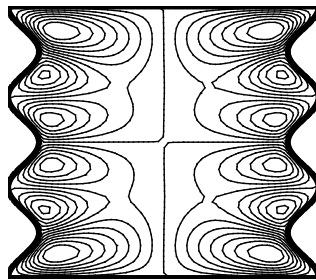
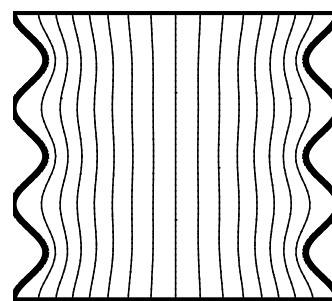
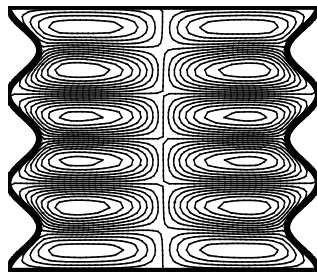
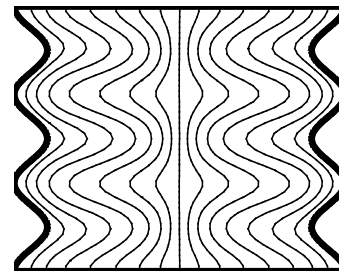
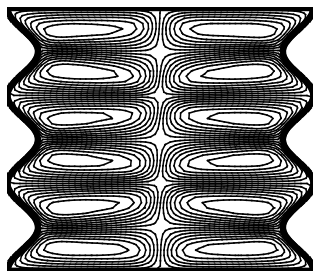
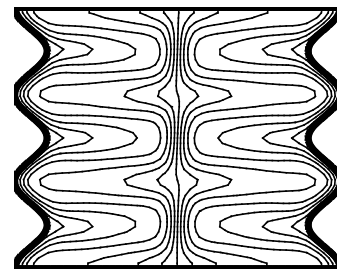


*d. Ra=1000*

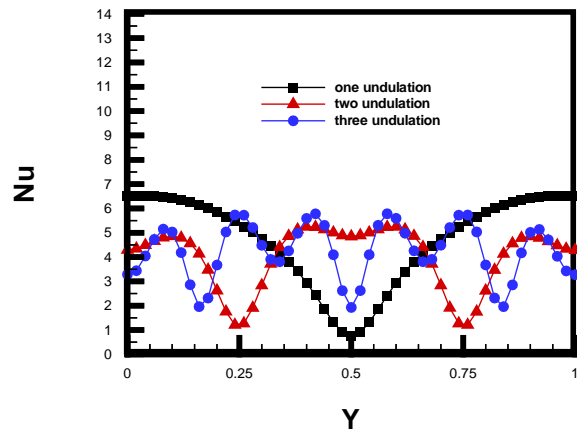


*h. Ra=1000*

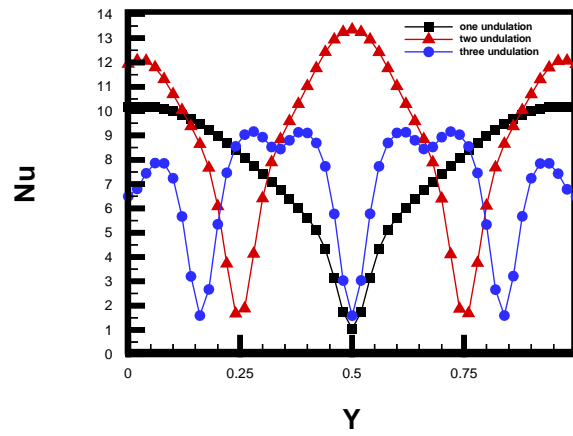
*Fig. 3 effect of Ra on stream function and isotherm lines distribution for two undulation and A=0.05*

*a.  $Ra=50$* *e.  $Ra=50$* *b.  $Ra=150$* *f.  $Ra=150$* *c.  $Ra=500$* *g.  $Ra=500$* *d.  $Ra=1000$* *h.  $Ra=1000$* 

*Fig. 4 effect of  $Ra$  on stream function and isotherm lines distribution for three undulation and  $A=0.05$*



a.  $Ra = 500$



b.  $Ra = 1000$

Fig5 variation of  $Nu$  for different undulations and different Rayleigh Numbers and  $A=0.05$

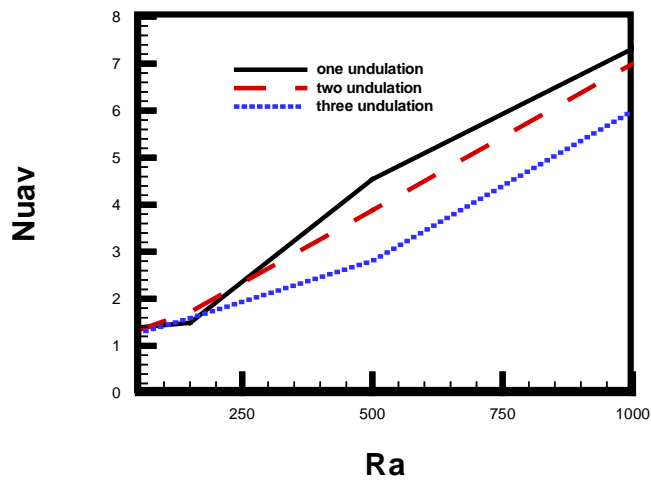
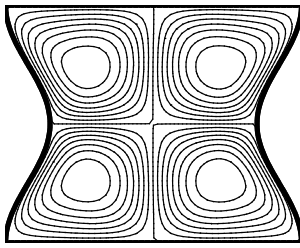
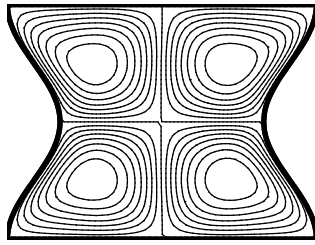


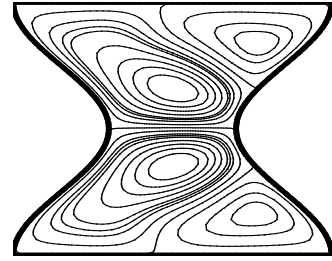
Fig. 6 variation of average Nusselt number versus Rayleigh for different undulation Numbers and  $A=0.05$



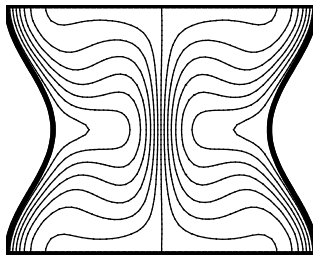
a.  $A=0.075$



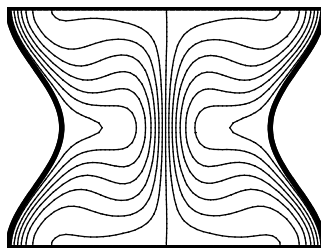
b.  $A=0.085$



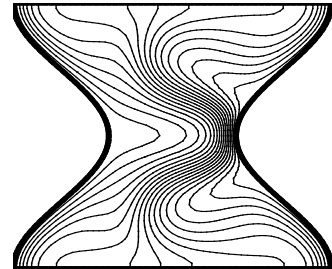
c.  $A=0.15$



d.  $A=0.075$

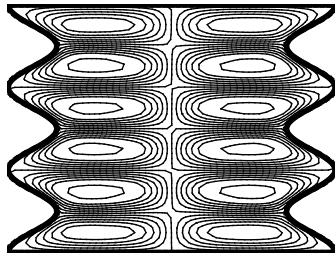


e.  $A=0.085$

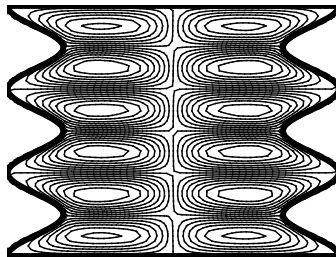


f.  $A=0.15$

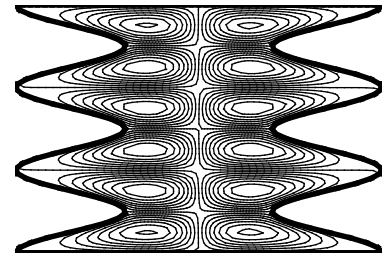
Fig. 7 effect of wavy walls amplitude on stream function and isotherm lines distribution for one undulation and  $Ra = 500$



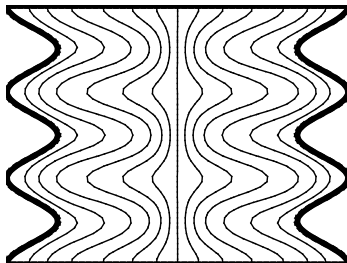
a.  $A=0.075$



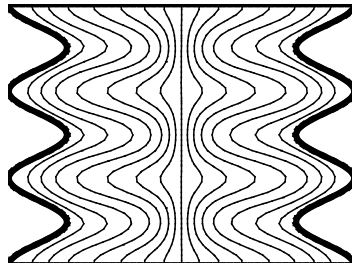
b.  $A=0.085$



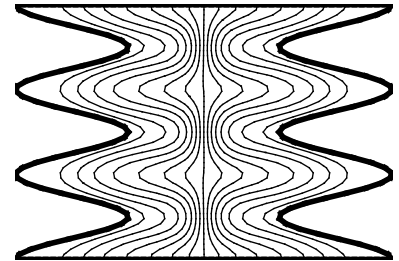
c.  $A=0.15$



d.  $A=0.075$



e.  $A=0.085$



f.  $A=0.15$

Fig. 8 effect of wavy walls amplitude on stream function and isotherm lines distribution for three undulations and  $Ra = 500$

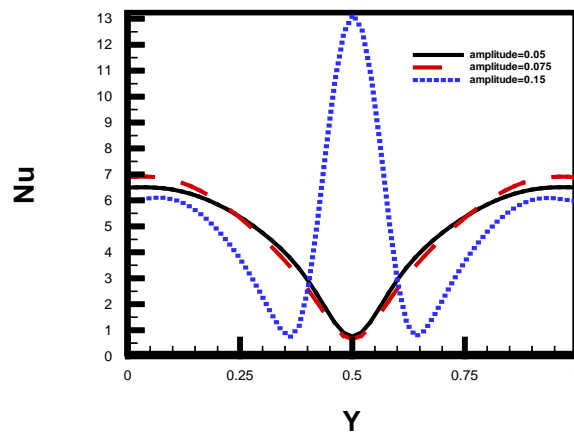


Fig.9 effect of amplitude on variation of Nu for one undulation and  $Ra=500$

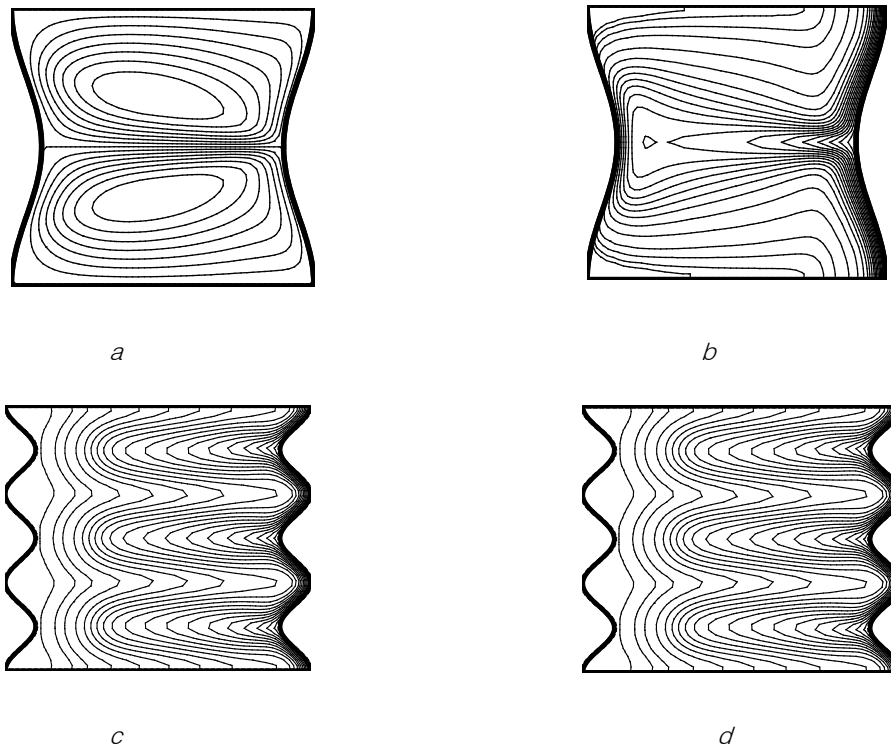


Fig.10 effect of thermal boundary conditions on stream function and isotherm lines distribution for  $Ra=500$  and  $A=0.05$

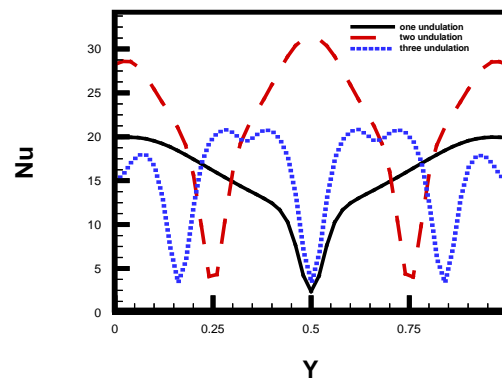
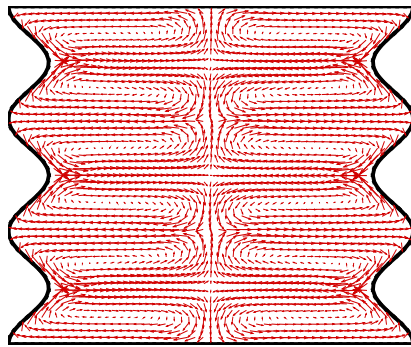
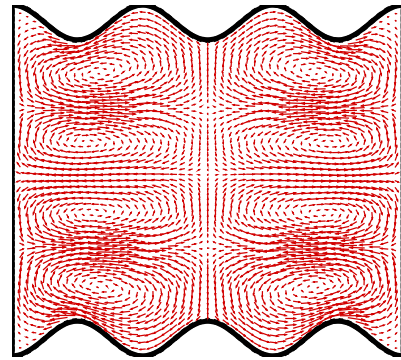


Fig. 11 variation of Nu for different undulations,  $A=0.05$  and  $Ra=500$



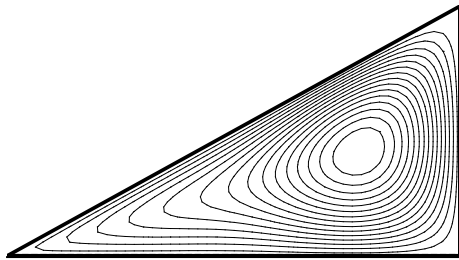


*a. two vertical wavy walls*

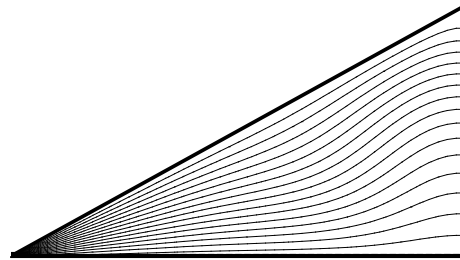


*b. two horizontal wavy walls*

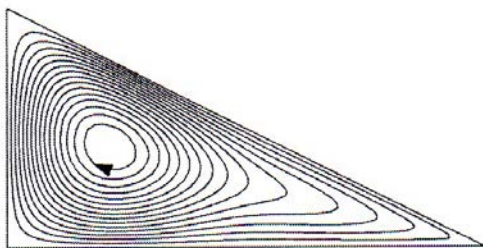
*Fig. 12 velocity vectors for  $Ra = 500$ ,  $A = 0.05$  and with three undulations*



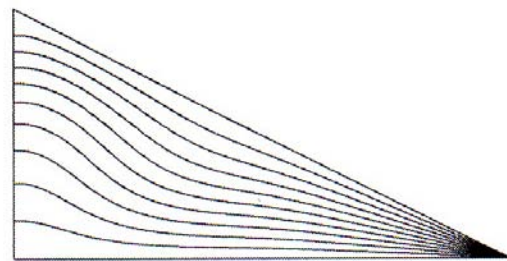
*a. present results of stream function*



*b. present results of isotherm contours*



*c. published results of stream function*



*d. published results of isotherm contours*

*Fig. 13 validation of the present code with published results [6] For  $Ra = 50$ ,  $AR = 0.5$*