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Relativistic Elasticity & the Universal Equation of Elasticity for Next Generation Aircrafts & Spacecrafts

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Abstract - The theory of "Relativistic Elasticity" is proposed for the design of the new generation large aircrafts with turbojet engines and speeds in the range of 50,000 km/h. This theory shows that there is a considerable difference between the absolute stress tensor and the stress tensor of the moving frame even in the range of speeds of 50,000 km/h. For bigger speeds like c/3, c/2 or 3c/4 (c=speed of light), the difference between the two stress tensors is very much increased. Therefore, for the next generation spacecrafts with very high speeds, then the relative stress tensor will be very much different than the absolute stress tensor. Furthermore, for velocities near the speed of light, the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The proposed theory of "Relativistic Elasticity" is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results to the "Universal Equation of Elasticity". For the structural design of the new generation aircrafts and spacecrafts the stress tensor of the airframe will be used in combination to the singular integral equations method. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

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RELATIVISTIC ELASTICITY THE UNIVERSAL EQUATION OFELASTICITY FOR NEXT GENERATION AIRCRAFTS SPACECRAFTS

Strictly as per the compliance and regulations of:

Relativistic Elasticity & the Universal Equation of Elasticity for Next Generation Aircrafts & Spacecrafts

E.G. Ladopoulos

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Furthermore, for velocities near the speed of light, the values of the relative stress tensor are very much bigger than the corresponding values of the absolute stress tensor. The proposed theory of "Relativistic Elasticity" is a combination between the theories of "Classical Elasticity" and "Special Relativity" and results to the "Universal Equation of Elasticity". For the structural design of the new generation aircrafts and spacecrafts the stress tensor of the airframe will be used in combination to the singular integral equations method. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

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I. Future Applications of Aircrafts and Spacecrafts Design

The possibilities of turbomachines applied in aircrafts have been very much increased because of the big evolution of the jet engines and the high performance axial – flow compressor. The concern for very light weight in the aircraft propulsion application, and the desire to achieve the highest possible isentropic efficiency by minimizing parasitic losses, led inevitably speed operation. The increasing evolution of aeroelasticity in aircraft turbomachines to axial-flow compressors with cantilever airfoils of high aspect ratio. Also, the turbojet engines were found to experience severe vibration of the rotor blades at part Continues to be under active investigation, driven by the needs of aircraft powerplant and turbine designers.

The target of international Aeronautical Industries is therefore to achieve a competitive technological advantage in certain strategic areas of new and rapidly developing advanced technologies, by which in the longer terms, can be achieved increased market share. This considerably big market share includes the design of a new generation large aircraft with speeds even in the range of 50,000 km/h. The application of new generation turbojet engines makes possible the design of such type of large aircrafts and therefore there is a need of elastic stress analysis for the construction of the total parts of such type of new generation aircrafts.

Furthermore, the target of the International Space Agencies (ESA, NASA, etc.) is to achieve in the future, next generation spacecrafts moving with very high speeds, even approaching the speed of light. In such cases the relative stress tensor will be much different than the absolute stress tensor and so special material will be used for the construction of such spacecrafts. The type of the proper material for the construction of the next generation spacecrafts is under investigation and will be very much different than the usual composite materials.

In the present investigation it will be shown that there is a difference between the absolute stress tensor and the stress tensor of the airframe even in the range of speeds of 50,000 km/h. On the other hand, for bigger speeds the difference of the two stress tensors is very much increased. Thus, for bigger velocities like c/3, c/2 or 3c/4 (c=speed of light) the relative stress tensor is very much different than the absolute one, while for velocities near the speed of light the values of the relative stress tensor are much bigger than the corresponding values of the absolute stress tensor. The study of the connection between the stress tensors of the absolute frame and the airframe is included in the theory proposed by E.G.Ladopoulos [30] - [32] under the term *"Relativistic Elasticity"* and the final formula which results from the above theory is called the *"Universal Equation of Elasticity"*. Hence, in the present study the theory of *"Relativistic Elasticity"* will be applied for the elastic stress analysis design of the next generation aircrafts and spacecrafts.

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Beyond the above, E.G.Ladopoulos [1]-[16] and E.G.Ladopoulos et al. [17]-[22] proposed several linear singular integral equation methods applied to elasticity, plasticity and fracture mechanics applications. In the above studies the Singular Integral Operators Method (S.I.O.M.) is investigated for the numerical evaluation of the multidimensional singular integral equations in which is reduced the stress tensor analysis of the linear elastic or plastic theory. Also, the theory of linear singular integral equations was extended to non-linear singular integral equations, too. [23]-[29]. The theory of *"Relativistic Elasticity"* will be applied to the design of the elastic stress analysis for the airframes. *"Relativistic Elasticity"* is derived as a generalization of the classical theory of elastic stress analysis for stationary frames. For future aerospace applications the difference between the relative and the absolute stress tensors will be of increasing interest. Furthermore, the classical theory of elastic stress analysis began to be analyzed in the early nineteenth century and was further developed in the twentieth century. In the past were written several important monographs on the classical theory of elasticity. [33]- [52}.

On the other hand, during the past years special attention has been concentrated on the theoretical aspects of the special theory of relativity. Hence, some classical monographs were written, dealing with the theoretical foundations and investigations of the special and the general theory of relativity. [53]–[60].Furthermore, a very important point which will be shown in the present research is that the *"relative stress tensor is not symmetrical"*, while, as it is well known, the *"absolute stress tensor is symmetrical"*. This difference is very important for the design of the next generation aircrafts and spacecerafts of very high speeds. Thus, the foundations of the theory of *"Relativistic Elasticity"* for airstructures lead to a general theory, in which no restriction is made with regard to the relative motion. This general theory is further reduced to one class of relative motion, uniform in direction and velocity.

II. Relative Stress Tensor Formulation for Airframes

The state of stress at a point in the stationary frame S^0, is defined by the following symmetrical stress tensor: (Fig.1).

$$\sigma^0 = \begin{bmatrix} \sigma^0_{11} & \sigma^0_{12} & \sigma^0_{13} \\ \sigma^0_{21} & \sigma^0_{22} & \sigma^0_{23} \\ \sigma^0_{31} & \sigma^0_{32} & \sigma^0_{33} \end{bmatrix} \tag{2.1}$$

Where: $$\sigma^0_{21} = \sigma^0_{12},\ \sigma^0_{31} = \sigma^0_{13},\ \sigma^0_{32} = \sigma^0_{23} \tag{2.2}$$

Consider an infinitesimal face element df with a directed normal, defined by a unit vector n, at definite point p in the three-space of a Lorenz system. The matter on either side of this face element experiences a force which is proportional to df.

Thus, the force is valid as:

$$\mathrm{d}\,\boldsymbol{\sigma}(\mathbf{n}) = \boldsymbol{\sigma}(\mathbf{n})\,\mathrm{d}\,f \tag{2.3}$$

The components σi(n) of σ(n) are linear functions of the components $\mathrm{n_k}$ of n:

$$\sigma_i(\mathbf{n}) = \sigma_{ik} n_k,\ i,k = 1,2,3 \tag{2.4}$$

Where $\boldsymbol{\sigma_{ik}}$ is the elastic stress tensor, which can be also called the relative stress tensor, in contrast to the space part σ^0_{ik} of the total energy-momentum tensor T_{ik}, referred as the absolute stress tensor. [53], [54} (Fig. 2).

The connection between the absolute and relative stress tensors is:

$$\sigma^0_{ik} = \sigma_{ik} + g_i u_k,\ i,k = 1,2,3 \tag{2.5}$$

where gi are the components of the momentum density g and u_k the components of the velocity u of the matter.

Furthermore, the connection between g and the energy flux s, is valid as:

$$\mathbf{g} = \mathbf{s}/c^2 \tag{2.6}$$

in which c denotes the speed of light (= 300.000 km/sec).

The total work done per unit time by elastic forces on the matter inside the closed surface f is equal to:

$$W = \int_f (\boldsymbol{\sigma}(\mathbf{n})\cdot\mathbf{u})\mathrm{d}\,f = \int_f \sigma_{ik} n_k u_i\,\mathrm{d}\,f = -\int_\upsilon \frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\mathrm{d}\,\upsilon,\ i,k = 1,2,3 \tag{2.7}$$

Where the integration in the last integral is extended over the interior υ of the surface f.

Hence, the work done on an infinitesimal piece of matter of volume $\delta\upsilon$ is valid as:

$$\delta W = -\frac{\vartheta(u_i\sigma_{ik})}{\vartheta x_k}\delta\upsilon \tag{2.8}$$

Moreover, (2.8) must be equal to the increase per unit time of the energy inside $\delta\upsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}(h\delta\upsilon) = \delta W \tag{2.9}$$

where h is the total energy density, including the elastic energy and denotes the substantial time derivative.

Eq. (2.9) is valid as:

$$\frac{\mathrm{d}}{\mathrm{d}t}(h\delta\upsilon)=\left(\frac{\vartheta h}{\vartheta t}+\frac{\vartheta h}{\vartheta x_k}u_k\right)\delta\upsilon+h\delta\upsilon\frac{\vartheta u_k}{\vartheta x_k}=\left[\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k)\right]\delta\upsilon \tag{2.10}$$

which leads to the relation:

$$\frac{\vartheta h}{\vartheta t}+\frac{\vartheta}{\vartheta x_k}(hu_k+u_i\sigma_{ik})=0 \tag{2.11}$$

So, the total energy flow is valid as:

$$\mathbf{s}=\mathbf{hu}+(\mathbf{u}\cdot\boldsymbol{\sigma}) \tag{2.12}$$

Where $(\mathbf{u}\cdot\boldsymbol{\sigma})$ is a space vector with components $(\mathbf{u}\cdot\boldsymbol{\sigma})_k=u_i\sigma_{ik}$.

Hence, the total momentum density can be written as:

$$\mathbf{g}=\frac{\mathbf{s}}{c^2}=\mu\mathbf{u}+\frac{(\mathbf{u}\cdot\boldsymbol{\sigma})}{c^2} \tag{2.13}$$

Where $\mu=h/c^2$ is the total mass density, including the mass of the elastic energy.

From (2.5) and (2.13) one obtains:

$$\sigma_{ik}-\sigma_{ki}=-g_iu_k+g_ku_i=[-(\mathbf{u}\cdot\boldsymbol{\sigma})_iu_k+(\mathbf{u}\cdot\boldsymbol{\sigma})_ku_i]/c^2\neq 0 \tag{2.14}$$

which shows that the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor (2.1) which is symmetrical.

In the stationary frame S^0 the velocity $u^0=0$ and hence, from (2.5), (2.12) and (2.13) one obtains the following expressions:

$$\sigma^0_{ik}=\sigma_{ik}=\sigma_{ki}=\sigma^0_{ki}\ (i,k=1,2,3) \tag{2.15}$$

Beyond the above, the mechanical energy-momentum tensor satisfies the following relation:

$$T_{ik}U_k=-h^0U_i \tag{2.16}$$

where U_i is the four-velocity of the matter, in the Lorentz system and $U^0_i=(0,0,0,ic)$.

Thus, the following scalar can be formed:

$$U_iT_{ik}U_k/c^2=U^0_iT^0_{ik}U^0_k/c^2=-T^0_{44}=h^0(x_1) \tag{2.17}$$

With $h^0(x_1)$ the invariant rest energy density considered as a scalar function of the coordinates (x_i) $(i=1,2,3)$ in S. (Fig. 2)

By applying further the tensor:

$$\Delta_{ik}=\delta_{ik}+U_iU_k/c^2 \tag{2.18}$$

which satisfies the relations:

$$U_i\Delta_{ik}=\Delta_{ik}U_k=0 \tag{2.19}$$

then, we can form the following symmetrical tensor:

$$S_{ik}=\Delta_{i1}T_{1m}\Delta_{mk}=S_{ki} \tag{2.20}$$

which is orthogonal to U_i:

$$U_iS_{ik}=S_{ik}U_k=0 \tag{2.21}$$

By combining eqs. (2.16), (2.17) and (2.20) we obtain:

$$S_{ik}=T_{ik}-h^0U_iU_k/c^2 \tag{2.22}$$

Furthermore, in the stationary system S_0 one has:

$$S^0_{ik}=\sigma^0_{ik}=\sigma_{ik},\ S^0_{i4}=S^0_{4i}=0 \tag{2.23}$$

Eq. (2.22) may also be written as:

$$T_{ik}=\xi_{ik}+S_{ik} \tag{2.24}$$

where:

$$\xi_{ik}=h^0U_iU_k/c^2=\mu^0U_iU_k \tag{2.25}$$

is the kinetic energy-momentum tensor for an elastic body and:

$$\mu^0=h^0/c^2 \tag{2.26}$$

is the proper mass density.

Also, let us introduce in every system S the quantity:

$$\sigma_{ik}=S_{ik}-S_{i4}U_k/U_4 \tag{2.27}$$

which, on account of (2.24) and (2.25) is valid as:

$$\sigma_{ik}=T_{ik}-T_{i4}U_k/U_4 \tag{2.28}$$

From (2.1) and (2.2) the three-tensor:

$$S^0_{ik}=\sigma^0_{ik}=\sigma_{ik}$$

in the stationary system is a real symmetrical matrix. The corresponding normalized eigenvectors $\mathbf{h}^{0(j)}$ satisfy the orthonormality relations:

$$\mathbf{h}^{(j)0} \cdot \mathbf{h}^{(\rho)0} = \delta^{je} \tag{2.29a}$$

and:

$$h_i^{(j)0} h_k^{(j)0} = \delta_{ik} \quad (j, \rho = 1,2,3) \tag{2.29b}$$

The eigenvalues $p_{(j)}^0$, the principal stresses, are the three roots of the following algebraic equation, where λ is the unknown:

$$\left|S_{ik}^0 - \lambda\delta_{ik}\right| = \left|\sigma_{ik}^0 - \lambda\delta_{ik}\right| = 0 \tag{2.30}$$

The matrix S_{ik}^0 may also be written in terms of the eigenvalues and eigenvectors as:

$$S_{ik}^0 = \sigma_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.31}$$

From eqs. (2.23) and (2.31) one obtains the following form of the stress four-tensor in S^o:

$$S_{ik}^0 = p_{(j)}^0 h_i^{(j)0} h_k^{(j)0} \tag{2.32}$$

Hence, in any system S we have

$$S_{ik} = p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.33}$$

From (2.24), (2.25), (2.27) and (2.33) we obtain the following expressions

$$T_{ik} = \mu^0 U_i U_k + p_{(j)}^0 h_i^{(j)} h_k^{(j)} \tag{2.34}$$

$$\sigma_{ik} = S_{ik} - S_{i4} U_k / U_4 = p_{(j)}^0 h_k^{(j)} \left(h_k^{(j)} + i h_4^{(j)} u_k / c\right) \tag{2.35}$$

By putting:

$$h_i^{(j)} = (\mathbf{h}^{(j)}, h_4^{(j)}) \tag{2.36}$$

and introducing the notation $\mathbf{a} \bullet \mathbf{b}$ for the direct product of the vectors a and b, we may write (2.35) for the relative stress tensor σ as:

$$\boldsymbol{\sigma} = p_{(j)}^0 \left[\mathbf{h}^{(j)} \bullet \mathbf{h}^{(j)} + \frac{i}{c} h_4^{(j)} (\mathbf{h}^{(j)} \bullet \mathbf{u})\right], \; j = 1,2,3 \tag{2.37}$$

Beyond the above, the triad vectors $h_i^{(j)}$ satisfy the tensor relations:

$$h_i^{(j)} h_i^{(\rho)} = \delta^{j\rho} \tag{2.38}$$

$$h_i^{(j)} h_k^{(j)} = \Delta_{ik} \tag{2.39}$$

with Δ_{ik} given by (2.18).

If the stationary system S^0 for every event point is chosen in such a way that the spatial axes in S^0 and in S have the same orientation, one obtains:

$$\mathbf{h}^{(j)} = \mathbf{h}^{(j)0} + \left\{\mathbf{u}(\mathbf{u} \cdot \mathbf{h}^{(j)0})(\gamma - 1)\right\} / u^2$$

$$h_4^{(j)} = i\mathbf{u} \cdot \mathbf{h}^{(j)0} \gamma / c$$

with:

$$\gamma = 1 / (1 - u^2/c^2)^{1/2} \tag{2.41}$$

From (2.34) and (2.40) with $i = k = 4$ we obtain:

$$h = -T_{44} = -\mu^0 U_4^2 - p_{(j)}^0 (\mathbf{u} \cdot \mathbf{h}^{(j)0})^2 \cdot \gamma^2 / c^2 \tag{2.42}$$

In the stationary system, (2.37) reduces to:

$$\boldsymbol{\sigma}^0 = p_{(j)}^0 \left(\mathbf{h}^{(j)0} \bullet \mathbf{h}^{(j)0}\right) \tag{2.43}$$

Thus, from (2.42) we obtain the following transformation law for the energy density:

$$h = \frac{h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} / c^2}{1 - u^2/c^2}$$

$$\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} = u_i \sigma_{ik}^0 u_k \tag{2.44}$$

and the mass density:

$$\mu = \frac{\mu^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u} / c^4}{1 - u^2/c^2} \tag{2.45}$$

From (2.40) and (2.34) with k = 4, one obtains the momentum density $\mathbf{g}$ with the components $g_i = T_{i4} / ic$:

$$\mathbf{g} = \mathbf{u}\left[h^0 + \mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u}(1 - \gamma^{-1}) / u^2\right] \gamma^2 / c^2 + (\boldsymbol{\sigma}^0 \cdot \mathbf{u}) \gamma / c^2 \tag{2.46}$$

Also, from (2.40) and (2.35) we obtain the relative stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \mathbf{u} \bullet (\boldsymbol{\sigma}^0 \cdot \mathbf{u})(\gamma - 1)/u^2 - (\boldsymbol{\sigma}^0 \cdot \mathbf{u}) \bullet \mathbf{u}(\gamma - 1)/\gamma u^2$$
$$- (\mathbf{u} \bullet \mathbf{u})(\mathbf{u} \cdot \boldsymbol{\sigma}^0 \cdot \mathbf{u})(\gamma - 1)^2 / \gamma u^4 \tag{2.47}$$

In the special case u = (u,0,0), where the notation of the matter at the point considered is parallel to the x1-axis (see Figs.1 and 2), the transformation equations (2.44), (2.46) and (2.47) reduce to:

$$h = \left(h^0 + \frac{u^2}{c^2} \sigma_{11}^0\right) \gamma^2$$

$$g_{x_1} = \gamma^2 \left(\mu^0 + \frac{\sigma_{11}^0}{c^2}\right) u \tag{2.48}$$

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$$g_{x_2} = \frac{\gamma\sigma_{21}^{0}}{c^2} u$$

$$g_{x_3} = \frac{\gamma\sigma_{31}^{0}}{c^2} u$$

and the relative stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^{0} & \gamma\sigma_{12}^{0} & \gamma\sigma_{13}^{0} \\ \frac{1}{\gamma}\sigma_{21}^{0} & \sigma_{22}^{0} & \sigma_{23}^{0} \\ \frac{1}{\gamma}\sigma_{31}^{0} & \sigma_{32}^{0} & \sigma_{33}^{0} \end{bmatrix} \quad (2.49)$$

where γ is given by (2.41). Finally, as it could be easily seen the relative stress tensor is not symmetrical, in contrast to the absolute stress tensor which is symmetrical.

III. ELASTIC STRESS ANALYSIS FOR STATIONARY FRAMES AND AIRFRAMES

Let us consider the stationary frame of Fig. 1 with Γ_1 the portion of the boundary of the body on which displacements are presented, Γ_2 the surface of the body on which the force tractions are employed and Γ the total surface of the body equal to Γ_1+Γ_2.

For the principal of virtual displacements, for linear elastic problems then the following formula is valid:

$$\int_{\Omega} (\sigma_{jk,j}^{0} + b_k) u_k \,\mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \overline{p}_k) u_k \,\mathrm{d}\Gamma \quad (3.1)$$

Where u_k are the virtual displacements, which satisfy the homogeneous boundary conditions $\overline{u}_k \equiv 0$ on Γ_1, b_k the body forces (Fig. 1) and p_k the surface tractions at the point k of the body. (Fig. 3)

Beyond the above, (3.1) takes the following form if u_k do not satisfy the previous conditions on Γ_1:

$$\int_{\Omega} (\sigma_{jk,j}^{0} + b_k) u_k \,\mathrm{d}\Omega = \int_{\Gamma_2} (p_k - \overline{p}_k) u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} (\overline{u}_k - u_k) p_k \,\mathrm{d}\Gamma \quad (3.2)$$

where $p_k = n_j \sigma_{jk}^{0}$ are the surface tractions corresponding to the u_k system. By integrating (3.2) follows:

$$\int_{\Omega} b_k u_k \,\mathrm{d}\Omega - \int_{\Omega} \sigma_{jk}^{0} \varepsilon_{jk} \,\mathrm{d}\Omega = -\int_{\Gamma_2} \overline{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} (\overline{u}_k - u_k) p_k \,\mathrm{d}\Gamma \quad (3.3)$$

in which ε_{jk} are the strains.

By a second integration (3.3) reduces to:

$$\int_{\Omega} b_k u_k \,\mathrm{d}\Omega + \int_{\Omega} \sigma_{jk,j}^{0} u_k \,\mathrm{d}\Omega = -\int_{\Gamma_2} \overline{p}_k u_k \,\mathrm{d}\Gamma - \int_{\Gamma_1} p_k u_k \,\mathrm{d}\Gamma + \int_{\Gamma_1} \overline{u}_k p_k \,\mathrm{d}\Gamma + \int_{\Gamma_2} u_k p_k \,\mathrm{d}\Gamma \quad (3.4)$$

Furthermore, a fundamental solution should be found, satisfying the equilibrium equations, of the following type:

$$\sigma_{jk,j}^{0} + \Delta_l^i = 0 \quad (3.5)$$

Where Δ_l^i is the Dirac delta function which represents a unit load at i in the l direction.

The fundamental solution for a three-dimensional isotropic body is: [31]

$$u_{lk}^{*} = \frac{1}{16\pi G(1-v) r} \left[(3-4v)\Delta_{lk} + \frac{\vartheta r}{\vartheta x_l} \frac{\vartheta r}{\vartheta x_k} \right]$$

$$p_{lk}^{*} = -\frac{1}{8\pi(1-v) r^2} \left[\frac{\vartheta r}{\vartheta n} \left[(1-2v)\Delta_{lk} + 3 \frac{\vartheta r}{\vartheta x_l} \frac{\vartheta r}{\vartheta x_k} \right] - (1-2v) \left[\frac{\vartheta r}{\vartheta x_l} n_k - \frac{\vartheta r}{\vartheta x_k} n_l \right] \right] \quad (3.6)$$

where G is the shear modulus, v Poisson's ratio, n the normal to the surface of the body, Δ_{lk} Kronecker's delta, r the distance from the point of application of the load to the point under consideration and n_j the direction cosines (Fig.3).

The displacements at a point are given by the formula:

$$u^i = \int_\Gamma up \,\mathrm{d}\Gamma - \int_\Gamma pu \,\mathrm{d}\Gamma + \int_\Omega bu \,\mathrm{d}\Omega \tag{3.7}$$

Hence, (3.7) takes the following form for the "l" component:

$$u_l^i = \int_\Gamma u_{lk} p_k \,\mathrm{d}\Gamma - \int_\Gamma p_{lk} u_k \,\mathrm{d}\Gamma + \int_\Omega b_k u_{lk} \,\mathrm{d}\Omega \tag{3.8}$$

By differentiating u at the internal points, one obtains the stress-tensor for an isotropic medium:

$$\sigma_{ij}^0 = \frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_l}{\vartheta x_l} + G\left(\frac{\vartheta u_i}{\vartheta x_j} + \frac{\vartheta u_j}{\vartheta x_i}\right) \tag{3.9}$$

Also, after carrying out the differentiation we have:

$$\sigma_{ij}^0 = \int_\Gamma \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right] p_k \,\mathrm{d}\Gamma +$$

$$+ \int_\Omega \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta u_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta u_{ik}}{\vartheta x_j} + \frac{\vartheta u_{jk}}{\vartheta x_i}\right)\right] b_k \,\mathrm{d}\Omega - \tag{3.10}$$

$$- \int_\Gamma \left[\frac{2Gv}{1-2v}\Delta_{ij}\frac{\vartheta p_{lk}}{\vartheta x_l} + G\left(\frac{\vartheta p_{ik}}{\vartheta x_j} + \frac{\vartheta p_{jk}}{\vartheta x_i}\right)\right] u_k \,\mathrm{d}\Gamma$$

Eq. (3.10) can be further written as following:

$$\sigma_{ij}^0 = \int_\Gamma D_{kij} p_k \,\mathrm{d}\Gamma - \int_\Gamma S_{kij} u_k \,\mathrm{d}\Gamma + \int_\Omega D_{kij} b_k \,\mathrm{d}\Omega \tag{3.11}$$

Where the third order tensor components D_{kij} and S_{kij} are:

$$D_{kij} = \frac{1}{8\pi(1-v)r^2}\left[(1-2v)\left[\Delta_{ki}r_{,j} + \Delta_{kj}r_{,i} - \Delta_{ij}r_{,k}\right] + 3r_{,i}r_{,j}r_{,k}\right] \tag{3.12}$$

$$S_{kij} = \frac{G}{4\pi(1-v)r^3}\Big[3\frac{\vartheta r}{\vartheta n}\left[(1-2v)\Delta_{ij}r_{,k} + v(\Delta_{ik}r_{,j} + \Delta_{jk}r_{,i}) - 5r_{,i}r_{,j}r_{,k}\right]$$
$$+ 3v(n_i r_{,j} r_{,k} + n_j r_{,i} r_{,k}) + (1-2v)(3n_k r_{,i} r_{,j} + n_j\Delta_{ik} + n_i\Delta_{jk}) - (1-4v)n_k\Delta_{ij}\Big] \tag{3.13}$$

whith: $r_{,i} = \dfrac{\vartheta r}{\vartheta x_i}$

Finally, because of eqs (2.49) and (3.11) by considering the moving system S of Fig. 2, then the stress-tensor reduces to the following form:

$$\begin{aligned}
\sigma_{11} &= \sigma_{11}^0 \\
\sigma_{12} &= \gamma\sigma_{12}^0 \\
\sigma_{13} &= \gamma\sigma_{13}^0 \\
\sigma_{21} &= \frac{1}{\gamma}\sigma_{21}^0 \\
\sigma_{22} &= \sigma_{22}^0 \\
\sigma_{23} &= \sigma_{23}^0 \\
\sigma_{31} &= \frac{1}{\gamma}\sigma_{31}^0 \\
\sigma_{32} &= \sigma_{32}^0 \\
\sigma_{33} &= \sigma_{33}^0
\end{aligned} \tag{3.14}$$

Where σ_{ij}^0 are given by. (3.11) to (3.13).

The following Table 1 shows the values of γ as given by (2.41) for some arbitrary values of the velocity u of the moving aerospace structure:

Table 1

Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$	Velocity u	$\gamma = 1/\sqrt{1-u^2/c^2}$
50,000 *km/h*	1.000000001	0.800c	1.666666667
100,000 *km/h*	1.000000004	0.900c	2.294157339
200,000 *km/h*	1.000000017	0.950c	3.202563076
500,000 *km/h*	1.000000107	0.990c	7.088812050
10E+06 *km/h*	1.000000429	0.999c	22.36627204
10E+07 *km/h*	1.000042870	0.9999c	70.71244596
10E+08 *km/h*	1.004314456	0.99999c	223.6073568
2x10E+8 *km/h*	1.017600788	0.999999c	707.1067812
c/3	1.060660172	0.9999999c	2236.067978
c/2	1.154700538	0.99999999c	7071.067812
2c/3	1.341640786	0.999999999c	22360.67978
3c/4	1.511857892	C	∞

From the above Table follows that for small velocities 50,000 km/h to 200,000 km/h, the absolute and the relative stress tensor are nearly the same. On the other hand, for bigger velocities like c/3, c/2 or 3c/4 (c = speed of light), the variable γ takes values more than the unit and thus, relative stress tensor is very different from the absolute one. Finally, for values of the velocity of the moving structure near the speed of light, the variable γ takes bigger values, while when the velocity is equal to the speed of light, then γ tends to the infinity.

The Singular Integral Operators Method (S.I.O.M.) as was proposed by E.G.Ladopoulos [4], [8], [9], [11], [12], [13], [15] and E.G.Ladopoulos et all [22] will be used for the numerical evaluation of the stress tensor (3.11), for every specific case.

IV. Conclusions

In the present investigation in the area of aeronautics technologies the theory of "Relativistic Elasticity" has been introduced and applied for the design of a new generation large aircraft with turbojet engines and speeds in the range of 50,000 km/h. Such a design and construction of the new generation aircraft will be applied to an increased market share of International Aeronautical Industries. Furthermore, the theory of "Relativistic Elasticity" has been applied for the design of the next generation spacecrafts moving with very high speeds, even approaching the speed of light, as the target of the International Space Agencies (ESA, NASA, etc.) is to achieve such spacecrafts in the future. The future investigation concerns to the determination of the proper composite materials for the construction of the next generation spacecfracts, as usual composite solids are not proper for such a construction.

The theory of "Relativistic Elasticity" and the "Universal Equation of Elasticity" show that there is a considerable difference between the absolute stress tensor of the airframe even in the range of speeds of 50,000 km/h. For bigger speeds the difference between the two stress tensors is very much increased. "Relativistic Mechanics" is a combination of the theories of "Classical Elasticity" and "Special Relativity".

For the structural design of the next generation aircrafts and spacecrafts will be used the stress tensor of the airframe in combination to the singular integral equations. Such a stress tensor is reduced to the solution of a multidimensional singular integral equation and for its numerical evaluation will be used the Singular Integral Operators Method (S.I.O.M.).

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Figure Captions

Figure 1 : The state of stress σ_{ik}^{0} in the stationary system.

Figure 2 : The state of stress σ_{ik}^{0} in the stationary system S^{o} and σ_{ik} in the airframe system S, with velocity u parallel to the x_1 - axis.

Figure 3 : The stationary system S^{o}.

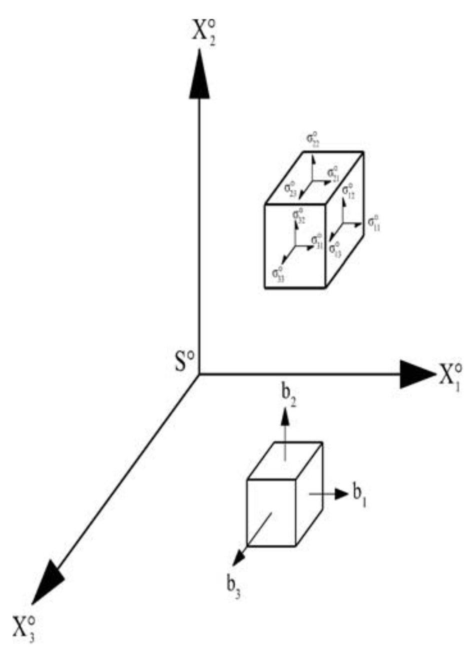

Figure 1

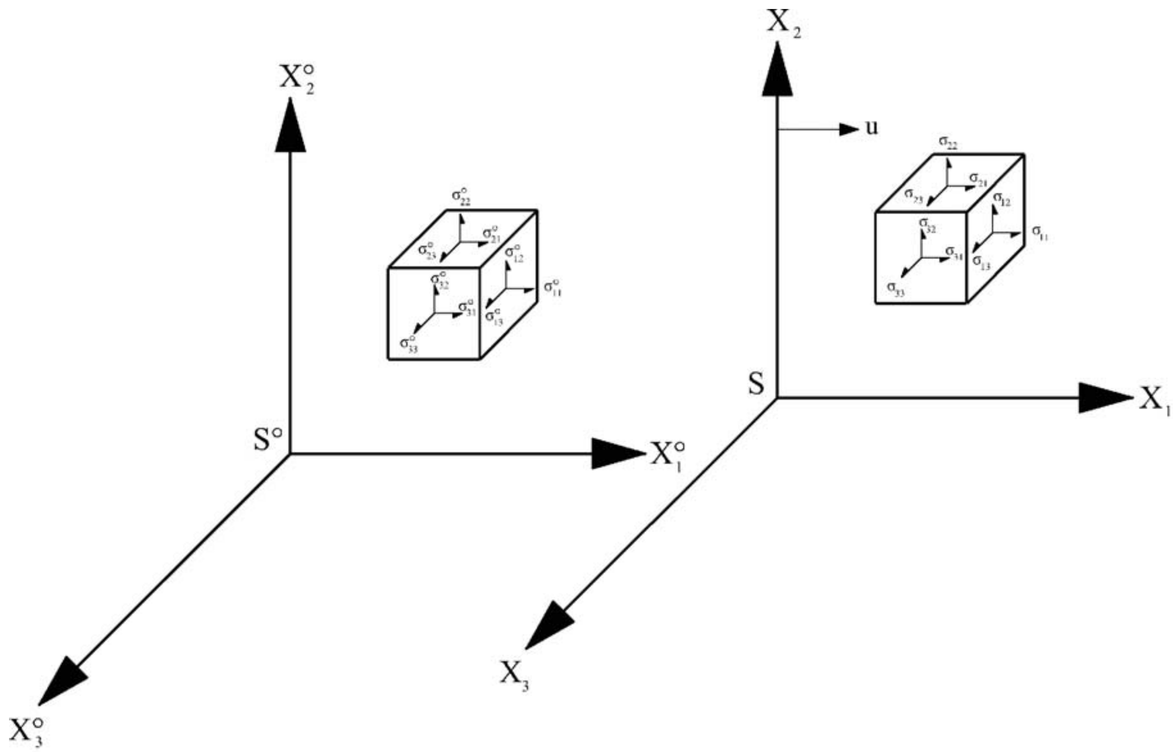

Figure 2

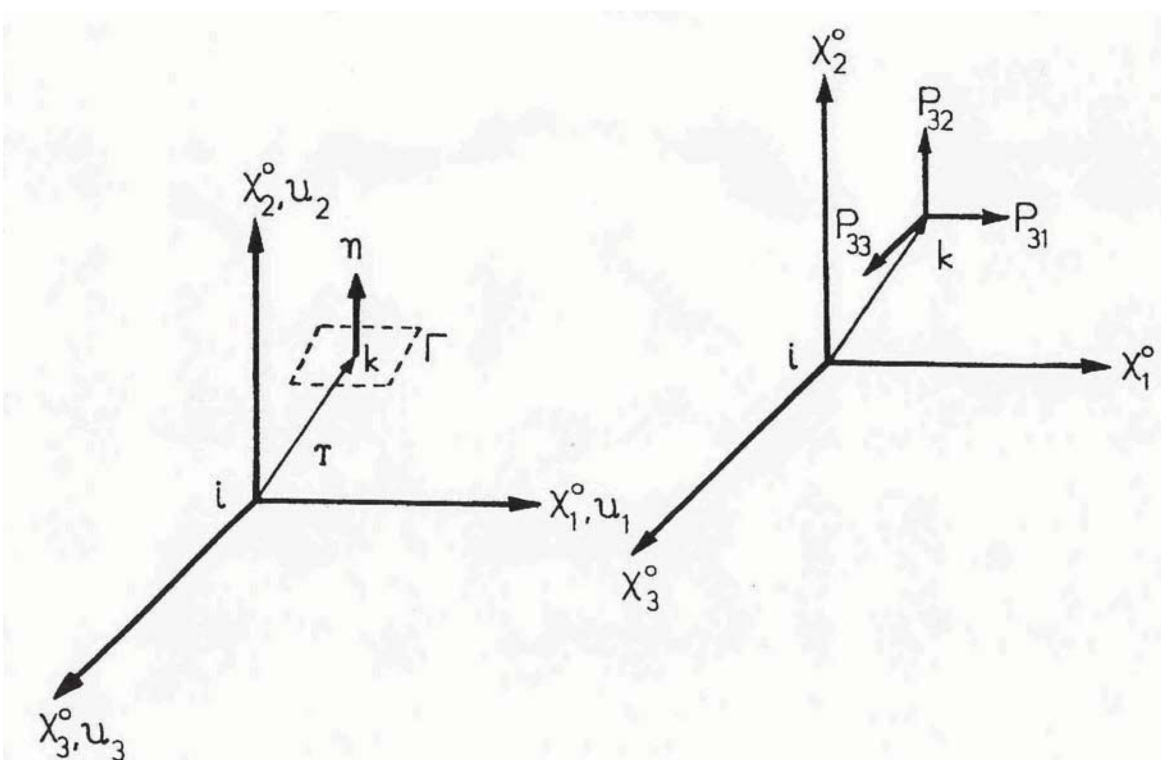

Figure 3