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## Turbogenerator-System Controller Design Via H∞- Control Techniques

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Keywords :  $H\infty$  -control, disturbance attenuation, multirate digital control, power system. GJRE-F Classification: FOR Code: 090607



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# Turbogenerator-System Controller Design Via H<sup>∞</sup>- Control Techniques

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Abstract - In the present work an  $H^{\infty}$  -control technique is presented in concise form and applied to the design of discrete optimal multirate-output controllers. The technique used is based on multirate-output controllers having a multirate sampling mechanism with different sampling period in each measured output quantity of the system. It relies mainly, under appropriate conditions, on the reduction of the original H<sup>®</sup> -disturbance attenuation problem to an associated discrete H<sup>∞</sup> -control problem for which a fictitious static state feedback controller is to be designed, even though some state variables may not be available (measurable) for feedback purposes. The discrete linear open-loop system model under consideration (which includes a disturbance term) is systematically derived from the associated continuous 3thorder MIMO linearized open-loop model of a practical power system, having an 160MVA synchronous generator supplying power to an infinite grid through a step-up transformer and an appropriate transmission line. Through the obtained pertinent simulation results the validity and practical usefulness of this design control technique is assessed.

Keywords :  $H^{\infty}$  -control, disturbance attenuation, multirate digital control, power system.

#### I. INTRODUCTION

he  $H^{\infty}$  - optimization control problem as known has drawn great attention [1-17]. In particular, the  $H^{\infty}$ -control problem for discrete-time and sampled-data singlerate and multirate systems has been treated successfully before [3-15]. Generally speaking, when the state vector is not available for feedback purposes, the  $H^{\infty}$ -control problem is usually solved in both the continuous and discrete-time cases using dynamic measurement feedback approach.

Recently, a new technique [5,6,9] has been presented for the solution of the  $H^\infty$ -disturbance attenuation problem. This technique is based on optimal multirate-output controllers (OMOCs) and, in order to solve the sampled-data  $H^\infty$ -disturbance attenuation problem, relies mainly on the reduction, under appropriate conditions, of the original  $H^\infty$ -disturbance attenuation problem to an associated discrete  $H^\infty$ -control problem for which a fictitious static statefeedback controller is to be designed, even though some state variables are not available for feedback use.

In the present work the ultimately investigated discrete linear open-loop power system model was

Author <sup>a</sup> : Professor, Technological Education Institution (TEI) of Kavala Faculty of Applied Technology, St. Loukas, 654 04 Kavala, GREECE, E-mail : akbogl@teikav.edu.gr obtained through a systematic procedure using a linearized continuous 3<sup>th</sup> -order MIMO open-loop model, containing an input disturbance, representing a practical power system having an 160MVA synchronous generator supplying power to an infinite grid through a step-up transformer and transmission line [18]. The digital controller leading to the associated designed discrete closed-loop power system model is displaying enhanced dynamic stability characteristics, and it is accomplished by applying properly the presented OMC technique.

## II. OVERVIEW OF RELEVANT MATHEMATICAL CONSIDERATIONS

The general description of the controllable and observable continuous, linear, time-invariant, multivariable MIMO dynamical open-loop system expressed in state-space form is

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$
(1)

where:  $x\left(t\right)\in R^{n}$ ,  $u(t)\in R^{m}$ ,  $y(t)\in R^{p}$  are state, input and output vectors respectively; and A, B and C are real constant system matrices with proper dimensions.

The associated general discrete description of the system of eq. (1) is as follows

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$
  
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$$
 (2)

Where  $\mathbf{x}(k) \in \mathbf{R}^n$ ,  $\mathbf{u}(k) \in \mathbf{R}^m$ ,  $\mathbf{y}(k) \in \mathbf{R}^p$ are state, input and output vectors respectively; and  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are real constant system matrices with proper dimensions.

### III. BRIEF REVIEW OF H∞-CONTROL WITH OMOC

Consider the controllable and observable continuous linear state-space system model in the following general form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{q}(t) \quad \mathbf{x}(0) = \mathbf{0}$$
 (3a)

$$\mathbf{y}_{m}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{J}_{1}\mathbf{u}(t),$$
  

$$\mathbf{y}_{c}(t) = \mathbf{E}\mathbf{x}(t) + \mathbf{J}_{2}\mathbf{u}(t)$$
(3b)

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where:  $\mathbf{x}(t) \in \mathbf{R}^{n}$ ,  $\mathbf{u}(t) \in \mathbf{R}^{m}$ ,  $\mathbf{q}(t) \in \mathbf{L}_{2}^{d}$ ,  $\mathbf{y}_{\mathbf{m}}(t) = \in \mathbf{R}^{p_{1}}$ ,  $\mathbf{y}_{\mathbf{c}}(t) \in \mathbf{R}^{p_{2}}$  are the state, input, external disturbance, measured output and controlled output vectors, respectively. In equ. (3) all matrices have real elements and appropriate dimensions. Next follows a useful definition.

**Definition** For an observable matrix pair  $(\mathbf{A}, \mathbf{C})$ , with  $\mathbf{C}^{\mathrm{T}} = \begin{bmatrix} \mathbf{c}_{1}^{\mathrm{T}} & \mathbf{c}_{2}^{\mathrm{T}} & \cdots & \mathbf{c}_{p_{1}}^{\mathrm{T}} \end{bmatrix}$  and  $\mathbf{c}_{i}$  with  $i=1,\ldots,p_{1}$ , the *t*h row of the matrix  $\mathbf{C}$ , a collection of  $p_{1}$  integers  $\{\mathbf{n}_{1}, \mathbf{n}_{2}, \cdots, \mathbf{n}_{p_{1}}\}$  is called an *observability index vector* of the pair  $(\mathbf{A}, \mathbf{C})_{,}$  if the following relationships hold simultaneously

$$\sum_{i=1}^{p_1} \mathbf{n}_i = \mathbf{n}$$

$$rank \begin{bmatrix} \mathbf{c}_1^T & \dots & (\mathbf{A}^T)^{p_1-1} \mathbf{c}_1^T & \dots & \mathbf{c}_{p_1}^T & \dots \\ \dots & \dots & \mathbf{c}_{p_1}^T (\mathbf{A}^T)^{n_{p_1}-1} \mathbf{c}_{p_1}^T \end{bmatrix} = n$$
(4)

The multirate sampling mechanism shown in Fig. 1. is applied to eq. (3).



Fig. 1 : Simplified block diagram for the control of linear systems using OMC technique

Assuming that all samplers start simultaneously at t = 0, a sampler and a zero-order hold with period  $T_{\rm 0}$  is connected to each plant input  ${}^{u_{\rm i}(t)}$ , i=1,2,...,m, such that

$$\mathbf{u}(t) = \mathbf{u}(kT_0) \quad , \quad t \in [kT_0, (k+1)T_0) \tag{5}$$

while the ith disturbance  $q_{i}\left(t\right)$ ,  $i{=}1,...,d$ , and the ith controlled output  $y_{c,i}\left(t\right)$ ,  $i{=}1,...,p_{2}$ , are detected at time  $kT_{0}$ , such that for  $t\in\left[kT_{0},(k+1)T_{0}\right)$ 

$$\mathbf{q}(t) = \mathbf{q}(\mathbf{k}\mathbf{T}_0)$$
,  $\mathbf{y}_c(\mathbf{k}\mathbf{T}_0) = \mathbf{E}\mathbf{x}(\mathbf{k}\mathbf{T}_0) + \mathbf{J}_2(\mathbf{k}\mathbf{T}_0)$  (6)

where  $(\mathbf{J}_2)_i$  is the ith row of the matrix  $\mathbf{J}_2$ .

The ith measured output  $y_{m,i}\left(t\right)$  , i=1,...,  $p_{1}$  , is detected at every  $T_{i}$  period, such that for  $\mu=0,...,N_{i}$  -1

$$y_{m,i}(kT_0 + \mu T_i) = c_i x(kT_0 + \mu T_i) + (J_1)_i u(kT_0)$$

Here  $N_i \in \mathbf{Z}^+$  are the output multiplicities of the sampling and  $T_i \in \mathbf{R}^+$  are the output sampling periods having rational ratio, i.e.  $T_i = T_0 / N_i$  with  $i = 1, \ldots, p_1$ .

The sampled values of the plant measured outputs obtained over  $[kT_0, (k+1)T_0)$  are stored in the  $N^*$ -dimensional column vector given by

$$\hat{\gamma}(kT_0) = \begin{bmatrix} y_{m,1}(kT_0) & \cdots & y_{m,1}(kT_0 + (N_1 - 1)T_1) \dots \\ \dots & y_{m,p_1}(kT_o) \dots & y_{m,p_1}[kT_o + (N_{p_1} - 1)T_{p_1}] \end{bmatrix}^T$$
(7)

(where  $N^{\ast} = \sum_{i=l}^{p_{1}} N_{i}$  ), which is used in the OMC in the form

$$\mathbf{u}[(\mathbf{k}+1)\mathbf{T}_{0}] = \mathbf{L}_{\mathbf{u}}\mathbf{u}(\mathbf{k}\mathbf{T}_{0}) - \mathbf{L}_{\gamma}\hat{\gamma}(\mathbf{k}\mathbf{T}_{0})$$
(8)

where  $\mathbf{L}_{u} \in \mathbf{R}^{mxm}, \ \mathbf{L}_{\gamma} \in \mathbf{R}^{mxN^{*}}$ 

#### Assumptions:

a) The matrix triplets (A, B, C) and (A, D, E) are stabilizable and detectable.

b) 
$$\operatorname{rank}\begin{bmatrix} A & D \\ C & \partial_{p_{1}xd} \end{bmatrix} = n + d,$$
  
 $\operatorname{rank}\begin{bmatrix} A & B & D \\ C & \partial_{p_{1}xm} & \partial_{p_{1}xd} \end{bmatrix} = n + m + d$ 

- c)  $\mathbf{J}_{2}^{\mathrm{T}} \begin{bmatrix} \mathbf{E} & \mathbf{J}_{2} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{\mathrm{m} \times \mathrm{n}} & \mathbf{I}_{\mathrm{m} \times \mathrm{m}} \end{bmatrix}$ .
- d) There is a sampling period  $T_{\rm 0},\,$  such that the open-loop discrete-time system model in general form becomes

$$\mathbf{x}[(\mathbf{k}+1)\mathbf{T}_{0}] = \mathbf{\Phi}\mathbf{x}(\mathbf{k}\mathbf{T}_{0}) + \mathbf{\hat{B}}\mathbf{u}(\mathbf{k}\mathbf{T}_{0}) + \mathbf{\hat{D}}\mathbf{q}(\mathbf{k}\mathbf{T}_{0})$$
$$\mathbf{y}_{c}(\mathbf{k}\mathbf{T}_{0}) = \mathbf{E}\mathbf{x}(\mathbf{k}\mathbf{T}_{0}) + \mathbf{J}_{2}\mathbf{u}(\mathbf{k}\mathbf{T}_{0})$$
(9)

where

$$\boldsymbol{\Phi} = \exp(\mathbf{A}\mathbf{T}_0), \ \left(\hat{\mathbf{B}}, \hat{\mathbf{D}}\right) = \int_0^{\mathbf{T}_0} \exp(\mathbf{A}\lambda)(\mathbf{B}, \mathbf{D}) d\lambda \quad (10)$$

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is stabilizable and observable and does not have invariant zeros on the unit circle.

Consequently it follows that the procedure for  $H^\infty$ -disturbance attenuation, using OMC essentially consists in finding for the control law a fictitious state matrix F which equivalently solves the problem, and then either determining the OMC pair  $\left(L_\gamma,L_u\right)$  or choosing a desired  $L_u$  and determining the  $L_\gamma$ . Based in [3] matrix F takes the following form

$$\mathbf{F} = \left(\mathbf{I} + \hat{\mathbf{B}}^{\mathrm{T}} \mathbf{P} \hat{\mathbf{B}}\right)^{-1} \hat{\mathbf{B}}^{\mathrm{T}} \mathbf{P} \boldsymbol{\Phi}$$

where  ${\bf P}$  is an appropriate solution of the following Riccati equation

$$\mathbf{P} = \mathbf{E}^{\mathrm{T}}\mathbf{E} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{P}\mathbf{\Phi} - \mathbf{\Phi}^{\mathrm{T}}\mathbf{P}\mathbf{\hat{B}}\left(\mathbf{I} + \mathbf{\hat{B}}^{\mathrm{T}}\mathbf{P}\mathbf{\hat{B}}\right)^{-1}\mathbf{\hat{B}}\mathbf{P}\mathbf{\Phi} + \mathbf{P}\mathbf{\hat{D}}_{\gamma}\left(\mathbf{I} + \mathbf{\hat{D}}_{\gamma}^{\mathrm{T}}\mathbf{P}\mathbf{\hat{D}}_{\gamma}\right)\mathbf{\hat{D}}_{\gamma}^{\mathrm{T}}\mathbf{P}, \mathbf{\hat{D}}_{\gamma} = \gamma^{-1}\mathbf{\hat{D}},$$
(11)

It is to be noted that  $\gamma \in \mathbf{R}^+$ , such that  $\|\mathbf{T}_{qy_c}(z)\| \ge \gamma$  where  $\|\mathbf{T}_{qy_c}(z)\|_{\infty}$  is the H<sup> $\infty$ </sup>-norm of the proper stable discrete transfer function  $\mathbf{T}_{qy_c}(z)$ , from sampled-data external disturbances  $\mathbf{q}(kT_o) \in \ell_2^d$  to sampled-data controlled output  $\mathbf{y}_c(kT_o)$ 

. Once matrix F is obtained the OMC matrices  $L_\gamma$  and  $L_u$  (in the case where  $L_u$  is free) can be computed as follows

$$\mathbf{L}_{\gamma} = \begin{bmatrix} \mathbf{F} & \mathbf{0}_{m \times d} \end{bmatrix} \widetilde{\mathbf{H}} + \Lambda \left( \mathbf{I}_{\mathcal{N}^{*} \times \mathcal{N}^{*}} - \begin{bmatrix} \mathbf{H} & \Theta_{\mathbf{q}} \end{bmatrix} \widetilde{\mathbf{H}} \right)$$
$$\mathbf{L}_{\mathbf{u}} = \left\{ \begin{bmatrix} \mathbf{F} & \mathbf{0}_{m \times d} \end{bmatrix} \widetilde{\mathbf{H}} + \Lambda \left( \mathbf{I}_{\mathcal{N}^{*} \times \mathcal{N}^{*}} - \begin{bmatrix} \mathbf{H} & \Theta_{\mathbf{q}} \end{bmatrix} \widetilde{\mathbf{H}} \right) \right\} \Theta_{\mathbf{u}}$$
(12)

where  $\widetilde{H} \begin{bmatrix} H & \Theta_q \end{bmatrix} = I$  and  $\Lambda \in R^{mxN^*}$  is an arbitrary specified matrix. In the case where  $L_u = L_{u,sp}$ , we have

$$\mathbf{L}_{\gamma} = \begin{bmatrix} \mathbf{F} & \mathbf{L}_{\mathbf{u},sp} & \mathbf{0}_{m \times d} \end{bmatrix} \mathbf{\hat{H}} + \dots \\ + \Sigma \left( \mathbf{I}_{N^{^{*}} \times N^{^{*}}} - \begin{bmatrix} \mathbf{H} & \boldsymbol{\Theta}_{\mathbf{u}} & \boldsymbol{\Theta}_{\mathbf{q}} \end{bmatrix} \mathbf{\hat{H}} \right)$$
(13)

where  $\hat{H} \begin{bmatrix} H & \Theta_u & \Theta_q \end{bmatrix} = I$  and  $\Sigma \in R^{mxN^*}$  is arbitrary.

 $\begin{array}{cccc} & \text{The} & \text{explicit} & \text{expressions} & \text{for} \\ \textbf{H}, \ \widetilde{\textbf{H}}, \ \widehat{\textbf{H}}, \ \textbf{A}, \ \textbf{\Theta}_{\text{u}} \text{ and } \textbf{\Theta}_{\text{q}} \text{ are given in [5,8]}. \\ & \text{The} \ \text{resulting} \ \text{discrete} \ \text{closed-loop} \ \text{system} \\ & \text{matrix} \left( \textbf{A}_{\mathbf{d}/\mathbf{d}} \right) \text{ takes the following general form} \end{array}$ 

$$\mathbf{A}_{\mathbf{cl/d}} = \mathbf{A}_{\mathbf{ol/d}} - \mathbf{B}_{\mathbf{ol/d}}\mathbf{F}$$
(14)

where cl = closed-loop, ol = open-loop and d = discrete.

## IV. OMOC DESIGN AND SIMULATIONS OF Resulting Discrete Closed-Loop Power System Model

The power system under study is taken from [18] and is shown here in Fig. 2 (it consists of an 160MVA synchronous generator with conventional exciter supplying power through a transformer and a transmission line to an infinite grid).





The continuous open-loop model describing this power system (taken from [18]) in the form of eq. (1) (with  $p_1=4$  and  $p_2=8$ ) is as follows

$$\begin{bmatrix} \mathbf{x}_{1} \\ \mathbf{x}_{2} \\ \mathbf{x}_{3} \\ \mathbf{x}_{3} \end{bmatrix} = \begin{bmatrix} 0 & 377 & 0 \\ -0.1030 & -0.1818 & -0.1209 \\ -0.3091 & 0 & -5.5517 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.0909 & 0 \\ 0 & 0.1695 \end{bmatrix} \begin{bmatrix} \Delta T_{m} \\ \Delta E_{FD} \end{bmatrix}$$
$$\begin{bmatrix} y_{1} \\ y_{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -0.0433 & 0 & 0.4777 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix}$$

Based on the transformed continuous openloop power system model, the associated discrete one, in relation to equs. (3a), (3b), is given in Appendix B.

Where:

$$x = \begin{bmatrix} \delta & \omega & \mathbf{E}_{q} \end{bmatrix}, \quad u = \begin{bmatrix} \Delta T_{m} & \Delta \mathbf{E}_{FD} \end{bmatrix}^{T}, \quad q = u,$$

$$y_{m} = \begin{bmatrix} \delta & V_{t} \end{bmatrix}^{T}, \quad y_{c} = x, \quad E = I_{3x3}, \\ J_{1} = 0_{2x2}, \quad J_{2} = 0_{3x2} \end{bmatrix}$$

The associated computed discrete linear openloop power system model, based on the above linearized continuous open-loop system model, is given in Appendix B with a properly selected sampling period, i.e.  $T_0 = 0.25 \text{sec}$ 

Based on Fig. 1 and using the H<sup> $\infty$ </sup>-control with OMCOs and the computed discrete linear open-loop model of the power system under study, the discrete closed-loop power system model is designed, considering the case with  $\gamma$ =5.68 and the computed values of **K**, **L**<sub>I</sub> and **F** feedback gain matrices which are:

$$\mathbf{K} = \begin{bmatrix} 0.1790 & -0.4971 & -24.1404 & 52.3042 & -30.1999 \\ -0.0233 & 0.0655 & -0.6778 & 0.6982 & 0.6985 \end{bmatrix}$$
$$\mathbf{L}_{u} = 10^{-14} \times \begin{bmatrix} -0.0444 & 0.1082 \\ 0.0022 & -0.0329 \end{bmatrix}$$
$$\mathbf{F} = \begin{bmatrix} -0.2299 & 30.1281 & -0.9726 \\ 0.0110 & -0.2976 & 0.3434 \end{bmatrix}$$

The magnitude of the eigenvalues of the discrete original open-loop and of the designed closed-loop power system model are shown in Table 1.

*Table 1 :* Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

| Original open-loop | 1 1               | 0.9342 |
|--------------------|-------------------|--------|
| power system       | $ \lambda $       | 0.9342 |
| model              | -                 | 0.9539 |
| Designed closed-   |                   | 0.5524 |
| loop power         | $ \hat{\lambda} $ | 0.5524 |
| system model       |                   | 0.8817 |

The simulated responses of the output variables  $(\delta, \omega, v_t)$  of the discrete original open-loop and designed closed-loop power system models, for zero initial conditions and unit step input disturbance, are shown in Fig. 3 and Fig. 4.

By comparing the computed eigenvalues of the simulated responses of the discrete original openloop power system model and the associated designed discrete closed-loop models, it is clear that the resulting enhancement in the dynamic system stability of the closed-loop system model is remarkable.

It is to be noted that the solution results of the discrete system models (i.e. eigenvalues, eigenvectors, responses of system variables etc.) for zero initial conditions were obtained using a special software program (which is based on the theory of § 2 and runs on MATLAB program environment).



Fig. 3 : Responses of  $\delta$  and  $v_t$  outputs subject to unit step input change and external disturbance where: (a) and (b) refer to discrete open-loop and closed-loop to step input change  $\Delta T_m$ =0.05 and  $\Delta E_{FD}$ =0.0p.u., (c) and (d) refer to discrete open-loop and closed-loop to step input change  $\Delta T_m$ =0.10 and  $\Delta E_{FD}$ =0.0p.u.)

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Fig. 4 : Response of  $\omega$  output subject to unit step input change and external disturbance where: (a) and (b) refer to discrete open-loop and closed-loop to step input change: (A):  $\Delta T_m$ =0.05 and  $\Delta E_{\text{FD}}$ =0.0p.u., and (B):  $\Delta T_m$ =0.10 and  $\Delta E_{\text{FD}}$ =0.0p.u.

In Fig. 5, the maximum singular value of  $T_{qy^c}(z)$  is depicted, as a function of the frequency  $\omega$ .

Clearly, the design requirement  $\|T_{qy_c}(z)\|_{\infty} \leq 5.68$ , is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.

Not that, in the case of unsaturated machine the H<sup> $\infty$ </sup> -norm of the open-loop system transfer function between disturbances and controlled outputs has the value  $\left\|C(j\omega I - A)^{-1}D\right\|_{\infty} = 71.9845$  while the minimum achievable disturbance attenuation level is  $\gamma_{inf} = 0.325$ .



*Fig. 5 :* The maximum singular value of  $Tqy_c(z)$  over  $\omega$ , for the unsaturated machine and for  $\gamma$ =5.68.

## v. Conclusions

An  $H^{\infty}$  -control technique based on optimal multirate-output controllers (OMOCs) has been presented in concise form for the purpose of attenuating in an effective manner system disturbances which otherwise degrade the performance of the synchronous machine. The application of the OMOC technique on the discrete open-loop power system model obtained vielded the associated (or designed ) discrete closedloop power system model. The respective simulation results of the open- and closed-loop models demonstrated clearly that the dynamic stability characteristics of the closed-loop model is far more superior than the associated one of the open-loop model. Therefore the presented  $H^{\infty}$  -control technique proved to be a reliable tool for the design of implementable OMCOs.

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### APPENDIX A

Numerical values of the system parameters and the operating point (p.u. values on generator ratings)

Turbogenerator:

160 MVA, 2-pole, pf= 0.894,  $x_q$ =1.7,  $x_q$ =1.6,  $x'_d$ =0.245p.u.

$$\tau_{d0} = 5.9, H = 5.5$$
s;  $\omega_R = 377$ rad/s  $D = 2,0$ p.u.

External system:

$$R_e = 0.02, X_e = 0.40$$
 p.u. (on a 160 MVA base).

Operating point:

$$P_0=1.0, Q_0=0.5, E_{FDo}=2.5128, E_{qo}=0.9986, V_{t0}=1.0, T_{mo}=1.0p.u.; \delta_0=1,1966rad; K_1=1,1330, K_2=1.3295, K_3=0.3072, K_4=1.8235, K_5=-0,0433, K_6=0.4777.$$

#### APPENDIX B

Constant matrices of discrete open-loop power system model based on Figs. 1 and 2 with sampling period  $T_0=0.25$ sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} 0.0543 & 59.8421 & -1.0910 \\ -0.0155 & 0.0255 & -0.0176 \\ -0.0463 & -2.7891 & 0.9011 \end{bmatrix}$$
$$\mathbf{B}_{ol/d} = \begin{bmatrix} 0.8620 & -0.0170 \\ 0.0144 & -0.0005 \\ -0.0234 & 0.0399 \end{bmatrix}$$
$$\mathbf{C}_{ol/d} = \begin{bmatrix} 1 & 0 & 0 \\ -0.0433 & 0 & 0.4777 \end{bmatrix}$$
$$\mathbf{D}_{ol/d} = \begin{bmatrix} 0.08620 & -0.0170 \\ 0.0144 & -0.0005 \\ -0.0234 & 0.0399 \end{bmatrix}$$