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I. Introduction

The singular phenomenon arises in the modeling of physical structures, random processes, control theory, networks synthesis and other areas of the applied sciences and engineering [1-2]. These singular non-linear systems have substantial significance in classical and modern science, as it defines the dynamics of a system [3]. The singular non-linear system of polytrophic and isothermal sphere is represented by the following non-linear second order homogenous equation:

$$\frac{d^2y(t)}{dt^2} = -\frac{2}{t}\left(\frac{dy(t)}{dt}\right) + y^{\lambda} \qquad 0 \le t \le 1, \qquad (1)$$

The tropic index is taken to be $\lambda = 5$, and subject to the following initial conditions

$$y(0) = 1\,,\ \dot{y}(0) = 0, \qquad (2)$$

It is essentially a Poisson's equation applied is the various modeling application like radioactively cooling, self-gravitating clouds and in the clusters of galaxies [4]. The analytic solution of the model given in equation(1) has been considered by the authors in the recent years by handling the singularity having the index λ. In the recent years, the merging of Homotopy idea with perturbation is applied for non-linear systems. [5]. Optimal Homotopy Asymptotic Method (OHAM) is established very recently by Marinca et al [6]. The OHAM have built in convergence criteria like HAM but is more flexible. From the contribution of Marinca et al [7-9] and Iqbal et al. [10] have proved generalization, effectiveness and reliability of the method to important engineering applications. The linear/nonlinear singular initial value problems have been solved by Iqbal et al. by using OHAM method. But no one yet tried to solve the singular non-linear system of Polytrophic and Isothermal sphere by using artificial neural network optimized by evolutionary computation. The applicability of artificial neural network (ANN) is remarkable for problem involving singularity and convex nature.In the last decay various scientists exploited ANN along with stochastic computational techniques, some of them are provided as a reference. Radial Bases functions neural networks were exploited to design a moving mass attitude control system [11] to control a vehicle with three axis stabilization in intra-atmospheric space. C. Monterola and C. Saloma [12-13] works on non-linear system represented by Schrodinger equation, this system is also formulated by FFNN. The flexibility of neural network and GA can be seen in the work carried out by Stackelberg [14] where Nash equilibrium was achieved by a hybrid intelligent algorithm in which fuzzy simulation, neural networks and genetic algorithms were integrated. Bumps and ruts on the roads cause the abrupt change in the rectilinear motion of the vehicles and adhesion coefficient. For a stable movement of the vehicle an appropriate breaking torque is needed. The optimization of this required torque in this problem can also be accomplished using the neural networks and genetic algorithms [15].

In this paper the mathematical modeling for singular non-linear system is performed by ANN with log-sigmoid as activation function. The optimization of the weights of the model is performed by genetic

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algorithm. The result of the GA is provided as a start point for rapid local search. The optimal Homtopy asymptotic method and exact solution of the problem are analyzed with the proposed scheme. The scheme is run for 100 independent runs to get a comprehensive statistical analysis based on the minimum fitness achieved, maximum fitness and spread in the results on the based of mean and standard deviation. The timing analysis is provided to see the computational complexity of the method along with the level of accuracy. According to best of the author knowledge this is the first article in which stochastic methods are incorporated with NN to optimize the non-linear singular systems of Polytrophic and Isothermal sphere.

The remainder of the paper is organized in the following way. The section 2 describes the importance of evolutionary techniques in optimization .In section 3 a mathematical modeling that contains neural network architecture of the singular system is explained. The numerical results along with discussion on the results are revealed in section 4. Finally section 5 presents some concluding remarks on the results along with directions to the future research.

II. Evolutionary Computation

The term evolutionary computation (EC), used vigorously for all evolutionary algorithms (EAs) this describes field of investigations. The major advantage of these techniques can be seen in practical difficult optimization problems. The major benefits are multifold, simplicity of the concept, robustness in changing circumstances, flexibility and other facets [16]. By this the EC has received special interest in the researchers for its applications in science and engineering. Because it is conceptually simple so no gradient information needs to be presented to the algorithms. The domain of EC is for all those problems that can have a function optimization task. The phenomenon of EC depends upon a data structure to represent solutions, the index of performance to evaluate solutions, and operators to generate new solutions from old [17]. The operator should takes care a behavioral link between parents and offspring's. A disjoint state space for possible solutions is formed which encompass infeasible regions, and time varying index or a function of competing solutions in the population [18]. The procedure of applicability is same for problems like, continuous-valued parameter optimization problems, discrete combinatorial problems, mixed-integer problems, and so forth [19].

a) Genetic Algorithms

The modern researches in genetic algorithms (GAs) has outline that the initial proposals were incapable of solving hard problems in a robust and efficient way. In large-scale Optimization problems, the execution time of first-generation GAs increases dramatically whereas solution quality decreases. Moreover the things such as encoding schemes, selection procedures, and self-adaptive and knowledge-based operators play a key role in the optimization of highly convex and stochastic ion nature problems. The birth of the GAs is also for optimization of various fields of interest [20-21]. Over and above the problems in which optimization itself is the final goal, it is also a way for achieving modeling, forecasting, control, simulation, and so forth. Traditional optimization techniques begin with a single candidate and search iteratively for the optimal solution by applying static heuristics. On the other hand, the GA approach uses a population of candidates to search several areas of a solution space, simultaneously and adaptively.The most popular methods that go beyond simple local search are GAs [22], simulated annealing (SA) [23], and tabu search (TS) [24]. Genetic algorithms operate on a population of individuals. Each individual is a potential solution to a given problem and is typically encoded as a fixed-length binary string, which is an analogy with an actual chromosome. After an initial population is randomly or heuristically generated, the algorithm evolves population through sequential and iterative application of three operators: selection, crossover, and mutation. A new generation is formed at the end of each iteration. The strongest aspect of the GA is, it does not get stuck in local minimum.

III. Neural Network Mathematical Modeling

The linear combination of log-sigmoid functions can be used a mathematical model of feed-forward ANN. The log-sigmoid is used as universal function approximator [25-26] as it has the tendency to model the non-linear systems effectively and efficiently in diverse fields of engineering [27-29]. Any network suitably trained to approximate a mapping satisfying some ODE will have an output function that will also approximate the DE [30]. In this feed-forward ANN, the input and output layers used linear function as activation while log-sigmoid is used for hidden layers. The following continuous mapping is employed for the function, its first and second derivatives respectively,

$$\hat{y}(t) = \sum_{i=1}^{m} \alpha_i \varphi(w_i t + b_i), \tag{3}$$

$$\frac{d\hat{y}(t)}{dt} = \sum_{i=1}^{m} \alpha_i \frac{d}{dt}\varphi(w_i t + b_i) \tag{4}$$

$$\frac{d^2\hat{y}(t)}{dt^2} = \sum_{i=1}^{m} \alpha_i \frac{d^2}{dt^2}\varphi(w_i t + b_i), \tag{5}$$

where the activation function is considered to be log-sigmoid and is given in expression (5)

$$\varphi(t) = \frac{1}{1+e^{-t}} \tag{6}$$

Therefore the error function formed by the neural networks given (3) to (5) is formed as

$$\varepsilon_j = \varepsilon_j^1 + \varepsilon_j^2 \qquad j = 1,2,3\cdots\cdots, \tag{7}$$

where j is the number of generations.

$$\varepsilon_j^1 = \frac{1}{s}\sum_{i=1}^{s}\left(\frac{d^2\hat{y}(t_i)}{dt^2} + \frac{2}{t_i}\frac{d\hat{y}(t_i)}{dt} - \hat{y}(t_i)^5\right)^2\Bigg|_j \tag{8}$$

while the error subject to initial condition is given in (9).

$$\varepsilon_j^2 = \frac{1}{2}\left\{\left(\hat{y}(0)-1\right)^2 + \left(\frac{d}{dt}\hat{y}(0)\right)^2\right\} \tag{9}$$

The linear combinations of networks from (3) to (5) can approximately model the system given in (1). It is named as differential equation neural (DEN) network, whose architecture is given in the Fig. 1; the activation function used in this is log-sigmoid. The learning procedure adopted in GA and basic flow chart is provided in one of our last article [31].

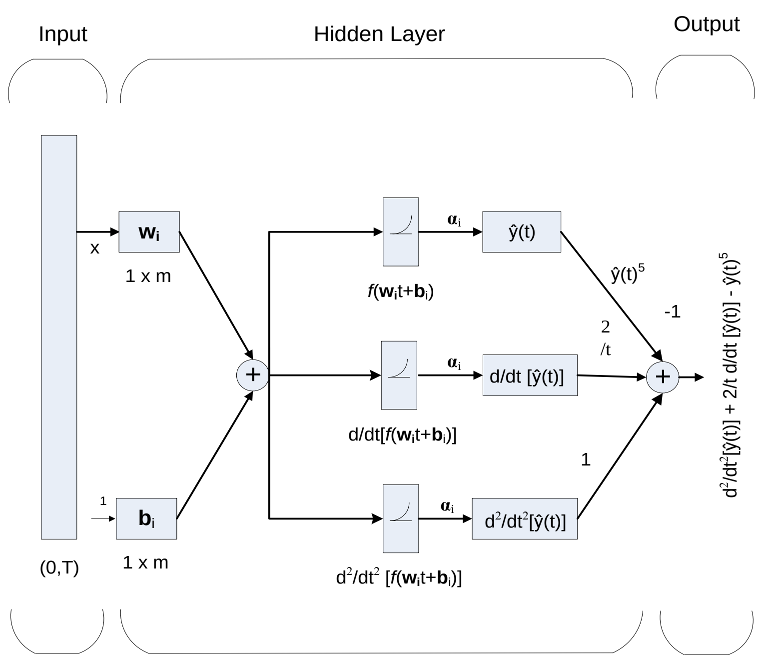

Fig. 1 : The DEN network of the Singular system

IV. Results and Discussion

In this section, we shell consider the solution of (1) by the proposed scheme. In order to prove the applicability and effectiveness of the proposed scheme for singular system the solution of OHAM and exact solution is compared with the results attained by the given approach. However, the statistical analysis is also carried out for the solvers to see the reliability and depth of the algorithm. Moreover, the time complexity of the scheme is also provided in the discussion.

The exact solution of expression (1) is [32]:

$$y(t) = \left(1+\frac{t^2}{3}\right)^{-\frac{1}{2}}. \tag{10}$$

The OHAM method series solution is generated by taking zero-order, first-order and second-order solution respectively [10], and the final reported expression is as follow:

$$y_{OHAM}(t) = 1 - 0.16252076779054878\,x^2 + 0.028096429522343383\,x^4 \tag{11}$$

Now, the singular non-linear system of Polytrophic and Isothermal sphere is solved by the proposed scheme as well, the number of neurons in each hidden layer of the DEN network are taken to be m = 10 that results in 30 unknown adaptive weights (α_i, w_i and b_i). The optimization of these weights is carried out using built-in function for GA in MATLAB. The parameter setting used for the execution of the algorithm is given in Table 1.

Table 1 : Parameters setting of the algorithms

GA		ASA	
Parameters	*Setting*	*Parameters*	*Setting*
Population Size	240	Start Point	Weights from GA
Chromosome size	30	Chromosome size	30
No. of runs	1500	Number of iterations	1500
Selection	Stochastic uniform	Maximum function evaluation	150000
Scaling function	Rank	Function tolerance	1e-12
Reproduction	Elite Count of 3 crossover fraction 0.6	Non-Linear constraints tolerance	1e-20
Mutation	Adaptive feasible	Derivative type	Central difference
Crossover	Scattered	X-Tolerance	1e-06
Migration interval	15	Bound	(-30 30)
Hybridization	ASA	Minimum fitness value	1e-12

The input of the training set is taken from time $t \in (0, 1)$ with a step size of 0.1. It means that the total time steps $m = 11$ so the fitness function is formulated as:

$$\varepsilon_j = \frac{1}{11}\sum_{i=1}^{11}\left(\frac{d^2\hat{y}(t_i)}{dt^2} + \frac{2}{t_i}\frac{d\hat{y}(t_i)}{dt} - \hat{y}(t_i)^5\right)^2 + \frac{1}{2}\left\{\left(\hat{y}(0)-1\right)^2 + \left(\frac{d}{dt}\hat{y}(0)\right)^2\right\}\Bigg|_j, \quad j = 1,2,3\cdots \tag{12}$$

where j be the iteration index, $\frac{d^2\hat{y}(t)}{dt^2}$, $\frac{d\hat{y}(t)}{dt}$ and $\hat{y}(t)$ are the networks given in (3) to (5) respectively. The scheme runs iteratively to find the minimum of fitness function *ej*, with stoppage criteria as 3000 number of runs or fitness value $e_j \le 10^{-9}$ whichever comes earlier. One of the unknown weights learned by given scheme with fitness value *7.2039e-11* is provided in Table 2. These weights can be used in equation (3) to obtain the solution of the equation for any input time *t* between *0* and *1*.

Table 2 : Adaptive Parameters Obtained by DE-NN Networks

index	w_i	α_i	β_i
1	0.629640981406070	0.734362550675546	-0.200888390149721
2	-0.385947956618289	-0.442473051469874	-0.994494465387963
3	0.988325374583115	1.944524168475940	1.252751222355890
4	-0.428119414616133	0.497504101676858	0.686299892190351
5	0.066289997417983	-1.203306475712070	0.096988771307929
6	0.960650440363807	-0.791782203756771	0.081407828807695
7	-1.806500080437780	-0.904970658665490	-0.489752157826344
8	-1.434684547506530	1.372140554331030	-0.153510274865171
9	-1.861314323302590	-0.066247775539287	0.335975293394860
10	0.803834370819629	-0.892632934401436	-0.875741872774712

Table 3 : Comparison of the Results with exact and numerical method

t	y_{exact}	y_{ohtm}	y_{GA-AsA}	$\lvert y_{exact} - y_{ohtm} \rvert$	$\lvert y_{exact} - y_{GA-ASA} \rvert$
0.0	1.00000000	1.00000000	0.99999987	0.00000E+00	1.27568E-07
0.1	0.99833749	0.99837760	0.99833660	4.01135E-05	8.84555E-07
0.2	0.99339927	0.99354412	0.99339754	1.44856E-04	1.73096E-06
0.3	0.98532928	0.98560071	0.98532741	2.71434E-04	1.87168E-06
0.4	0.97435470	0.97471595	0.97435312	3.61242E-04	1.58756E-06
0.5	0.96076892	0.96112583	0.96076766	3.56912E-04	1.26070E-06
0.6	0.94491118	0.94513382	0.94491012	2.22638E-04	1.06611E-06
0.7	0.92714554	0.92711078	0.92714455	3.47643E-05	9.88417E-07
0.8	0.90784130	0.90749501	0.90784036	3.46293E-04	9.37937E-07
0.9	0.88735651	0.88679225	0.88735566	5.64264E-04	8.50186E-07
1.0	0.86602540	0.86557566	0.86602468	4.49742E-04	7.25942E-07

The comparison of the results is made with OHAM for the same ranges of the inputs as taken for stochastic numerical method. The results are summarized in Table 3 in comparison of the exact solution. It is clear from the results that the accuracy of the given method is in a good agreement with exact solution and also comparatively excellent with OHAM method. The absolute error of the OHAM is in the range *1.0e-5 to 1.0e-4* while the proposed scheme has the error from *1.0e-7 to1.0e-6.*

The derivative of the system representing in expression (1) is also approximated by the given scheme to check the depth in the method. The results are summarized in Table 4 upto twelve decimal places. It is quite evident from the table that the results obtained by neural network optimized by evolutionary computation is cable to find the derivative of the systems as well by using the same weights as given in Table 2.

Moreover, the reliability of the stochastic algorithm is being tested by a comprehensive statistical analysis. The analysis is performed in the complete range of the time between *0* to *1* and the results are narrated in table 5. The references of the analysis are the mean, standard deviation, best and worst values of the absolute error of proposed method with exact solution. It is quite evident from the table 5, that the Best and Mean absolute error of both the system is in the range *1e-04* to *1e-06* and *1e-06* respectively. The value of the worst of the absolute error is *1e-04* that is even remarkable.

Table 4 : Comparison of the Results with exact for the derivative of the system

t	y'_{exact}	y'_{GA-AsA}	$\lvert y'_{exact} - y'_{GA-ASA} \rvert$
0.0	0.000000000000	0.000001840886	-0.000001840886
0.1	-0.033277916282	-0.033178401419	-0.000099514863
0.2	-0.066226617853	-0.065360105484	-0.000866512369
0.3	-0.098532927816	-0.095661581246	-0.002871346570
0.4	-0.129913960492	-0.123332491326	-0.006581469167
0.5	-0.160128153805	-0.147807897045	-0.012320256761
0.6	-0.188982236505	-0.168732906775	-0.020249329729
0.7	-0.216333959525	-0.185959935851	-0.030374023674
0.8	-0.242091013068	-0.199524924233	-0.042566088834
0.9	-0.266206952825	-0.209610660651	-0.056596292174
1.0	-0.288675134595	-0.216505060841	-0.072170073754

Table 5 : Statistical Analysis of the solution by Proposed Scheme

t	Best	Worst	Mean	STD
0.0	-6.342636E-06	4.594012E-04	9.394565E-06	5.692623E-05
0.1	-9.995311E-05	3.793471E-04	7.351355E-06	5.810525E-05
0.2	-1.253426E-04	3.782469E-04	4.657521E-06	5.879943E-05
0.3	-1.147922E-04	3.337455E-04	3.722699E-06	5.432348E-05
0.4	-1.180902E-04	3.074559E-04	4.684137E-06	5.170384E-05
0.5	-1.197289E-04	3.298016E-04	6.264152E-06	5.263826E-05

0.6	-1.072861E-04	3.398575E-04	7.059596E-06	5.351095E-05
0.7	-8.684856E-05	3.364792E-04	6.459919E-06	5.092706E-05
0.8	-7.119308E-05	3.116579E-04	4.779545E-06	4.380955E-05
0.9	-6.673061E-05	2.635342E-04	2.851818E-06	3.402397E-05
1.0	-6.732281E-05	2.155496E-04	1.478341E-06	2.624317E-05

The value of the fitness functions are computed for 100 independent runs to have a close look on the optimization behavior of various input times. The value of the functions for some of the input times are drawn the descending order. The results are plotted on the semi log scale as the difference between the results for various inputs times are merely negligible. The optimization behavior is drawn in the Fig.2 in descending order for 100 independent runs. Finally it has been concluded from the figure that the convergence capability of the given scheme is 100% for all input times between *0* and *1*. Moreover, the absolute error for 12% of the independent runs is in the range 10^{-03} to 10^{-05} while 88% lies in the range 10^{-05} to 10^{-09}.

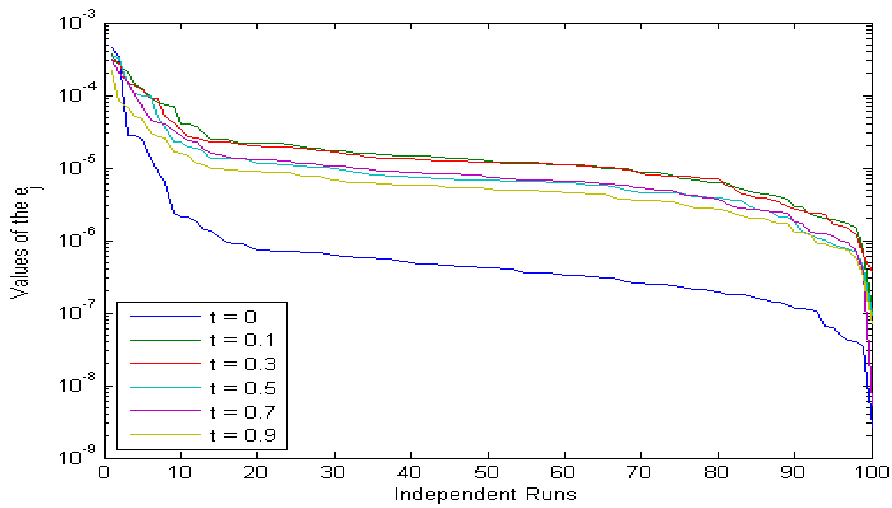

Fig 2 : The behavior of the optimization error for 100 independent runs

V. Conclusions and Future Remarks

On the basis of simulation and results provided in the last section it can be concluded that:

The stochastic solvers based on DEN networks optimized with hybridized genetic algorithm can effectively provides the solution of the Non-Linear Singular System of Polytrophic and Isothermal sphere model. The mean of the absolute error lies in range of 10^{-06}.

The reliability and effectiveness of proposed artificial intelligence techniques are validated from statistical analysis base on 100 independent runs. It is found that the confidence interval for the convergence of the given approach 100% to get an approximate solution in a acceptable error range.

It has been observed that the proposed scheme show the supremacy on the optimal Homotopy asymptotic method in comparison with the exact solution. Moreover, the proposed scheme can readily provide the solution on the continuous grid of time. Thus this provides an alternate approach to researchers to apply the solver to complex real life problems in engineering.

In future, one can look for application of other artificial intelligence techniques base on neural networks optimized with ant/bee colony optimization, genetic programming, particle swarm optimization and differential evolution etc. for solving such vast applications.

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