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Abstract - Over the last 20 years spacecraft formation flying has been the subject of numerous research activities due to the advantages offered when compared with large, complex, single purpose satellites. With the obvious advantages of increased functionality and enhanced reliability, come however, also substantial challenges in the maintenance and reconfiguration of the spacecraft formation. The present paper addresses these problems by proposing two approaches that can be mathematically validated thus making it attractive for safety critical applications such as proximity operations. The first approach hinges on the implementation of pursuit algorithms first studied by French scientist Pierre Bouguer in the 18th century. The proposed approach separates the control law into two distinct stages: planar movement control and orthogonal displacement suppression. The second approach relies on the use of motion camouflage which is a hunting technique widely used in the natural world that allows a predator to approach a prey while appearing to remain stationary. A number of different scenarios are presented and the two approaches implemented within them. Numerical results shows that both methods are robust to dynamical uncertainties and do ensure the correct reconfiguration manoeuvres.

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ROBUST ALGORITHMS FOR FORMATION FLYING RECONFIGURATION

Robust Algorithms for Formation Flying Reconfiguration

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Abstract - Over the last 20 years spacecraft formation flying has been the subject of numerous research activities due to the advantages offered when compared with large, complex, single purpose satellites. With the obvious advantages of increased functionality and enhanced reliability, come however, also substantial challenges in the maintenance and reconfiguration of the spacecraft formation. The present paper addresses these problems by proposing two approaches that can be mathematically validated thus making it attractive for safety critical applications such as proximity operations. The first approach hinges on the implementation of pursuit algorithms first studied by French scientist Pierre Bouguer in the 18th century. The proposed approach separates the control law into two distinct stages: planar movement control and orthogonal displacement suppression. The second approach relies on the use of motion camouflage which is a hunting technique widely used in the natural world that allows a predator to approach a prey while appearing to remain stationary. A number of different scenarios are presented and the two approaches implemented within them. Numerical results shows that both methods are robust to dynamical uncertainties and do ensure the correct reconfiguration manoeuvres.

I. Introduction

In recent years, the idea of distributing the functionality of large satellites among smaller has become increasingly popular as a traditional, large single spacecraft may not be sufficient to meet mission requirements [1]. Several scenarios entailing cooperative satellites have been considered for numerous space missions. To this end spacecraft formation flying has become a promising means of reducing operational costs and increase mission flexibility and functionality [2-6]. Due to the often precise navigation and positioning requirements of these missions, the spacecraft station keeping and orbit and increase mission flexibility and functionality [2-6]. Due to the often precise navigation and positioning requirements of these missions, the spacecraft station keeping and orbit control become crucial for mission success. Different approaches exist and have been proposed in literature to tackle these challenging problems [7-12]. The main drawback of these approaches is that they generally require costly computational resources making them thus unsuitable for on-board scheduling. The development of autonomy technologies is the key to three vastly important strategic technical challenges facing future spacecraft missions. The reduction of mission operation costs, the continuing return of quality science products through increasingly limited communications bandwidth and the launching of a new era of solar system exploration, characterised by sustained presence and in depth scientific studies. Spacecraft autonomy will bring significant advantages by improving resource management, increasing fault tolerance and simplifying payload operations. Also, when considering the communication delays in deep space missions, the requirement for autonomy becomes clear. Ground stations and controllers will not be able to communicate and control distant spacecraft in real-time to guarantee pointing precision and safety. As the number of satellites within the formation and the distance of the operational orbit from the Earth increase, conventional methods show their limits and become less practical. New control methods are therefore required; approaches that enhance the automation of the system, enabling the formation to perform deployment, maintenance and re-configuration manoeuvres autonomously.

a) Pursuit Algorithms

An interesting line of research, inspired by pursuit algorithms, was first studied by French scientist Pierre Bouguer in the 18th century. Simply put, if a point *A* in space moves along a known curve, then another point *P* describes a pursuit curve if its motion is always directed towards *A* and the two points move with equal speeds. More than a century later, scholars found that if three agents, initially placed at the vertices of an equilateral triangle, were to run one after the other, then their pursuit curves would be a logarithmic spiral and they would eventually meet at a common point, known now as the Brocard point of a triangle as shown in Figure1[13].

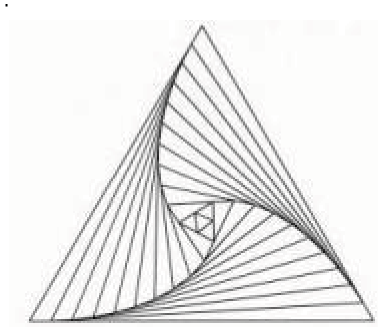

Figure 1: Pursuit curve pattern for an equilateral triangle.

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b) Motion Camouflage

Motion camouflage is a stealth technique that allows a predator to approach a moving target (e.g. the prey) whilst appearing to remain stationary. To achieve this, the predator follows a path such that it always lies on the line connecting the predator and a fixed point (knowas the camouflage background) as shown in Figure 2. Biologists have used stereo cameras to reconstruct the movements in three dimensions of dragonflies, and verify that these insects successfully use motion camouflage to disguise themselves as stationary during aerial maneuvers. A more elaborate behavior is performed by the male dragonflies that periodically appear to switch fixed point locations, sometimes to nearby points, sometimes to points at infinity [14].

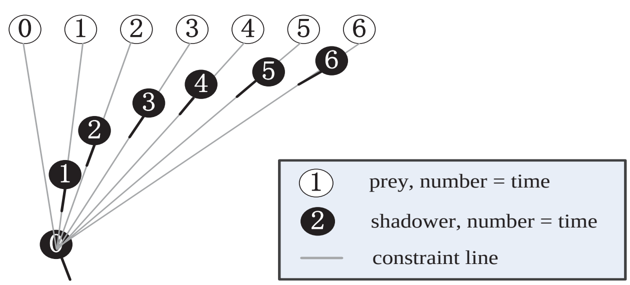

Figure 2 : A predator motion camouflage trajectory

The only visual cue to the predator's approach is its graduallooming. In a psychophysical experiment based on a 3D computergame, humans became prey, defending themselves against attacks from motion camouflaged missiles. Alternative missile approach Strategies included a homing approach and direct interception approach. The experimental results demonstrated that motioncamouflaged missiles were in general able to get closer to the object before being shot than the alternative strategies.

II. Control Algorithms

We assume that the formation is orbiting the Earth at an altitude which is much larger than the relative distance between the satellites. We can therefore define the equations of motion of a chaser satellite about a target satellite through the Clohessy- Wiltshire equations:

$$\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2 x &= a_x \\ \ddot{y} + 2n\dot{x} &= a_y \\ \ddot{z} + n^2 z &= a_z \end{aligned} \quad (1)$$

Motion in the *z* direction and along the orbital plane is decoupled; hence if necessary the control law can be designed in two stages: planar and orthogonal control.

a) Pursuit Algorithms Control

If the satellite lies on the reference centre, then under cyclic pursuit it will remain stationary. Generally the initial position of the agentis however not superposed to the reference centre, thus it is necessary to combine this with beacon's guidance to achieve reorientation. Suppose the reference centre to be a virtual beacon, together with angular rotation control of $\dot{\theta}_i' = \omega_i$, another control denoted as $\dot{\theta}_i'' = u_i$ would be required to maintain the relative distance maintenance with respect to the beacon. This linear control is expressed as:

$$u(t) = \begin{cases} k_b g_b(\rho(t))\alpha_b(\gamma(t)) & if\ \rho(t) > 0 \\ 0 & if\ \rho(t) = 0 \end{cases} \quad (2a)$$

with

$$g_b(\rho) = \ln\left(\frac{(c_b - 1)\cdot\rho + \rho_e}{c_b \cdot \rho_e}\right) \quad (2b)$$

$$\alpha_b(\gamma) = \begin{cases} \gamma & if\ 0 \le \gamma \le 3\pi/2 \\ \gamma - 2\pi & if\ 3\pi/2 < \gamma < 2\pi \end{cases} \quad (2c)$$

$$\alpha_b(\gamma) = \begin{cases} \gamma & if\ 0 < \gamma < \pi/2 \\ \gamma - 2\pi & if\ \pi/2 \le \gamma \le 2\pi \end{cases} \quad (2d)$$

Where, is the distance between the vehicle and the beacon$\gamma \in [0, 2\pi)$represents the angular distance between the heading of the vehicle and the position vector of the beacon Note that Eq. 2c valid in the case of counterclock wise equilibrium and Eq. 2d valid in the case of clockwise equilibrium. A combined control law for multi-agent motion would then be:

$$\dot{\theta}_i = \dot{\theta}_i' + \dot{\theta}_i'' = \omega_i + u_i(t) = \begin{cases} k_\alpha \alpha_i + k_b g_b(\rho_i)\alpha_b(\gamma) & \text{for} \quad \rho_i > 0 \\ k_\alpha \alpha_i \quad \text{for} \quad \rho_i = 0 \end{cases} \quad (3)$$

In the orthogonal direction, a linear feedback control is designed.

To suppress possible oscillations, the velocity value is taken into account. Here the parameters are adjusted to be:

$$k_z = 0.0002,\ k_v = 0.0002\,.$$

$$u_z = -k_z z - k_v \dot{z} \quad (4)$$

This provides the control in the out of plane direction.

b) Motion Camouflage Control

The ideal motion camouflage equations are built on the assumption that the position of the target is given in advance. Let us assume that the position of the target is $\vec{z}(t)$ and that of the predator is $\vec{r}(t)$, both of which lie either in a plane or three-dimensional Euclidean space.

If the predator uses motion camouflage, then lies $\vec{r}(t)$ on the line connecting the target and some fixed reference point r_0 . This means that:

$$\vec{r}(t) = \vec{r}(0) + u(t)(\vec{z}(t) - \vec{r}_0) \quad (5)$$

Where $u(t)$=[0,1] is the position ratio of $\overrightarrow{r_0 r}$ to $\overrightarrow{r_0 z}$ To perform the formation control we assume impulsive manoeuvres such that the velocity vector changes instantaneously. The chaser transfers from state of $(\rho_1, \dot{\rho}_1^-)$ at t_1 to $(\rho_2, \dot{\rho}_2^+)$ at t_2. Superscripts of " - " and " + " refer to the state of before and after an impulse respectively. Defining $\Delta t = t_2 - t_1, \psi = n\Delta t, s = \sin\psi, c = \cos\psi$, The state transition matrix becomes:

$$\Phi(t_1, t_2) = \Phi(\Delta t) = \begin{bmatrix} \Phi_{\rho\rho} & \Phi_{\rho\dot{\rho}} \\ \Phi_{\dot{\rho}\rho} & \Phi_{\dot{\rho}\dot{\rho}} \end{bmatrix}$$

$$= \begin{bmatrix} 4-3c & 0 & 0 & s/n & 2(1-c)/n & 0 \\ 6(s-\psi) & 1 & 0 & 2(1-c)/n & (4s-3\psi)/n & 0 \\ 0 & 0 & c & 0 & 0 & s/n \\ 3ns & 0 & 0 & c & 2s & 0 \\ 6n(c-1) & 0 & 0 & -2s & 4c-3 & 0 \\ 0 & 0 & -ns & 0 & 0 & c \end{bmatrix} \quad (6)$$

with

$$\begin{cases} \dot{\rho}_1^+ = \Phi_{\rho\dot{\rho}}^{-1}(\rho_2 - \Phi_{\rho\rho}\rho_1) \\ \dot{\rho}_2^- = \Phi_{\dot{\rho}\rho}\rho_1 + \Phi_{\dot{\rho}\dot{\rho}}\dot{\rho}_1^+) \end{cases} \quad (7)$$

and impulse vectors of

$$\begin{cases} \Delta v_1 = \dot{\rho}_1^+ - \dot{\rho}_1^- \\ \Delta v_2 = \dot{\rho}_2^+ - \dot{\rho}_2^- \end{cases} \quad (8)$$

Integrating all the velocity changes provides the fuel mass required for the manoeuvre through the rocket equation.

III. Numerical Results – Pursuit Algorithms

To simplify the stability analysis, a formation of only two satellites is investigated at first. We consider two reconfiguration manoeuvres: separation increase and phase angle adjustment. In this task for the reason of initial symmetric states, cyclic pursuit is sufficient to achieve radius enlargement. Applying cyclic pursuit control to this scenario requires the linear velocity to be constant. Whereas to keep the periodicity invariant to the reference centre under orbital dynamics, the velocity value relative to the reference centre should change as well. Then the cyclic pursuit in the orbit direction and the feedback control in the orthogonal direction are applied. Setting $k_\alpha = 2\omega_e / \pi,\ \omega_i = k_\alpha \alpha_i - \omega_e$ as control input,

Where ω_e is the expected angular rotation rate, $k_z = 0.0002,\ k_v = 0.0002$ Assuming the satellites has the same mass $m = 367kg$ and electric thrusters with $I_{sp} = 1640s,\ Thrust = 7.22e-5kN$ Is used in the first phase of about 20 minutes after which it is decreased to 3.67e-5kN. These correspond to the values of the centripetal forces in initial and target positions respectively. The results are shown in Fig 3-4.

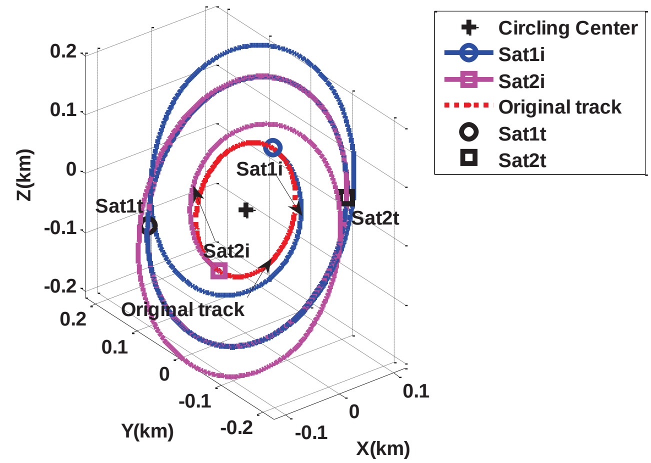

Figure 3 : Propagation of radius enlargement.

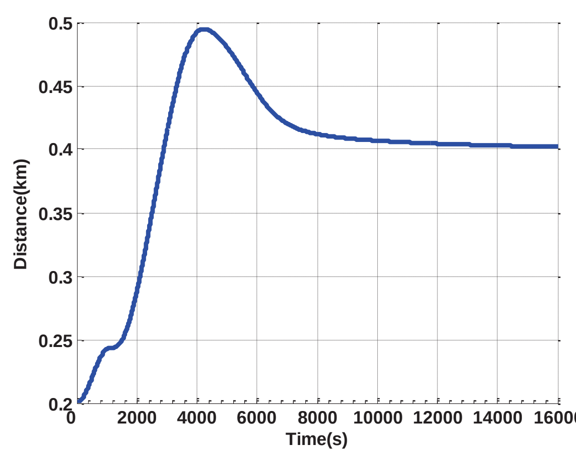

Figure 4 : Spacecraft relative distance

The eigenvalues of this system are $\pm 0.9918ie-3$, - 0.6314, $-0.3157 \pm 0.9402ie-3$, 0. Eliminating the fake eigenvalues of $\pm 0.9918ie-3$ and 0 through coordinate constraints, leaves the remaining with negative real parts. Hence the planar movement is stable. Figure 4 shows the radius of this formation increases while maintaining a constant phase angle. The radius increase following the velocity increase is rapid, but it still takes a relatively long time to finally reach the desired orbital configuration. In the first phase, a higher thrust is required to increase the relative distance. In the second phase, the thrust should be reduced to avoid overshooting the desired relative distance. If the thrust is maintained to the initial level throughout the manoeuvre then, the convergence rate is very slow.

In the second scenario want to modify the relative phase angle between the satellites. Applying control law to planar movement with parameters $\rho_e = 0.1\text{km}$, $c_b = 2$, $k_b = 0.02$ and $k\alpha = 2\omega_e/\pi$. Initial spacecraft mass are the same as before while the propulsion system employs SMART-1 Hall Effect Thrusters with $I_{sp} = 1640s$.

Figure 5 shows that the two satellites gradually evolve to the new required angular phase distance.

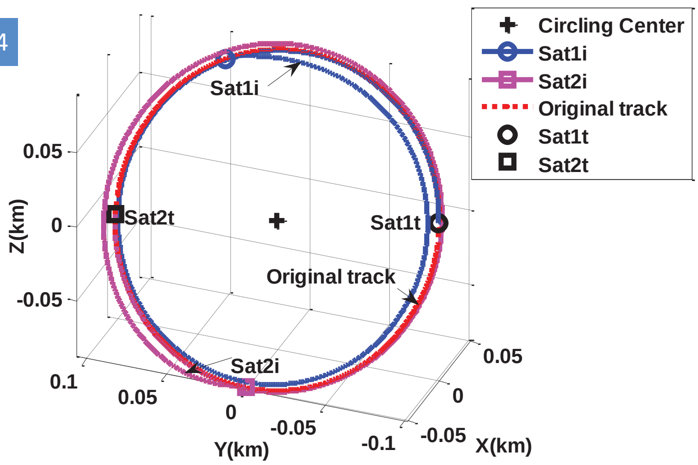

Figure 5 : Propagation of the phase angle adjustment.

If an impulsive propulsion system is used, the relative distance between the satellites would oscillate slightly before reaching the required phase angle separation as shown in Figure 6. It can be seen that the manoeuvre takes more than twice the time than using a low thrust propulsion system, as shown in Figure 7.

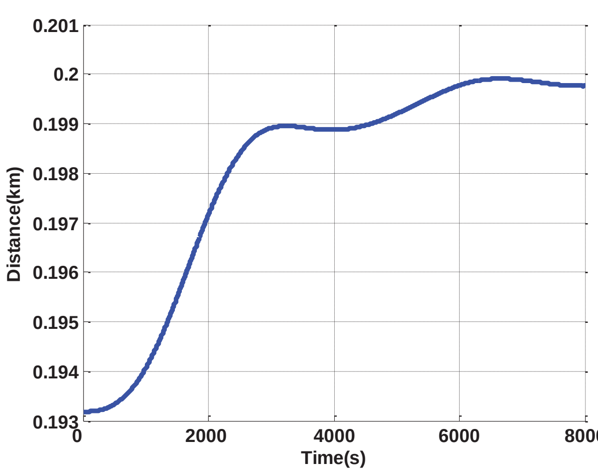

Figure 6 : Spacecraft relative distance.

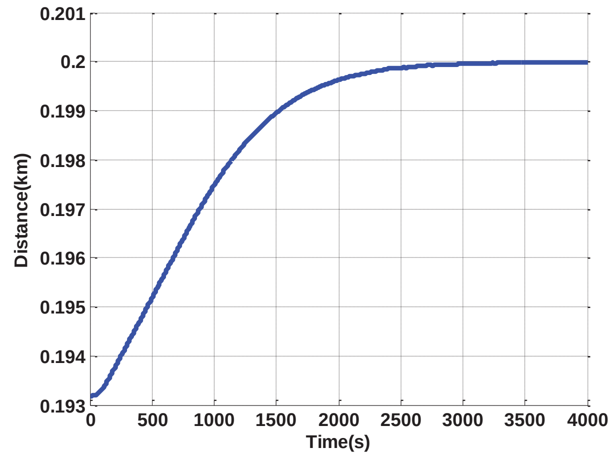

Figure 7 : Spacecraft relative distance.

IV. Numerical Results –Motion Camouflage

To simplify the preliminary analysis, let us assume the target is circling around a spacecraft with parameters of:

$$A_t = 2000m, B_t = 2000\sqrt{3}m/s,\ \phi_t = 0, \psi_t = \pi$$

$n \approx 9.92e-4\ rad/s$ The target's motion can be expressed as

$$\begin{aligned} x_t &= -A_t \cos(nt + \phi_t) \\ y_t &= 2A_t \sin(nt + \phi_t) \\ z_t &= B_t \cos(nt + \phi_t + \psi_t) \end{aligned} \tag{9}$$

We assume the chaser initiates its trajectory takes from the centre of the reference frame. When the parameters, u_c, N_c and φ_c are defined the trajectory shown in Figure 8 with velocity consumption of $\Delta V = 2.717m/s$ is followed.

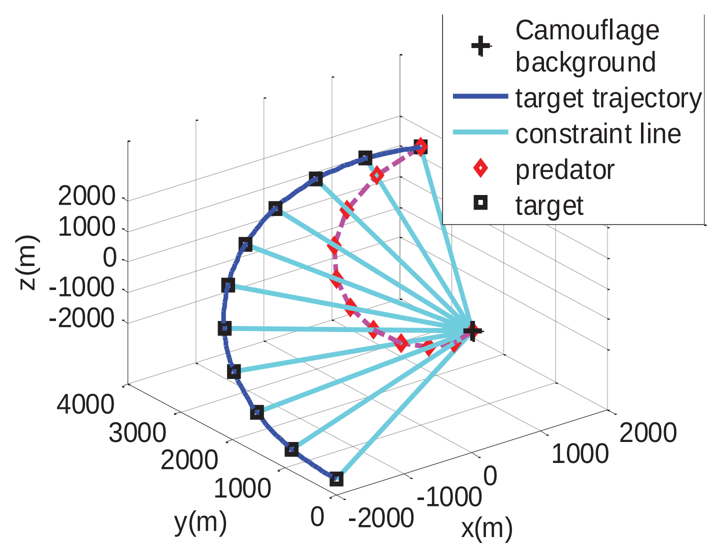

Figure 8 : Motion camouflage trajectory

In between impulsive intervals, the chaser will not be precisely located on the constraint lines all the time. This phenomenon would probably result in the failure of a possible stealthy approach. To address this failing more frequent impulses need to be applied as shown in Figure 9. This however comes at the expense of a more costly manoeuvre with a $\Delta v = 12.33$ m/s

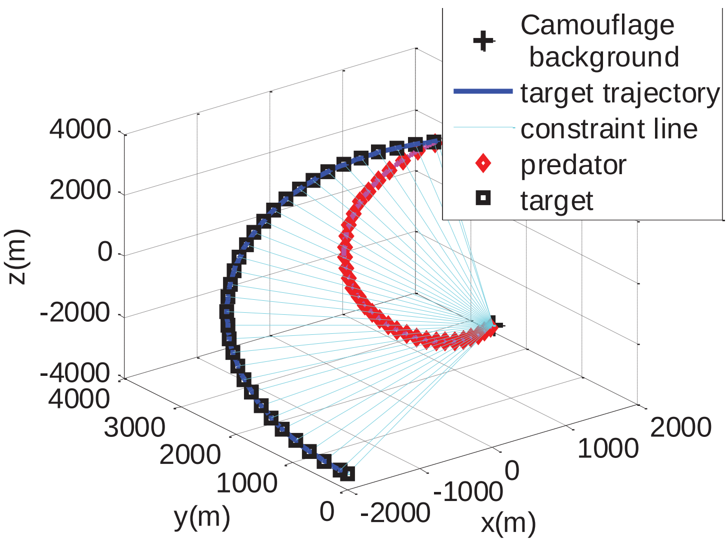

Figure 9 : Motion camouflage trajectory

V. Conclusions

This paper presented two different methodologies for group coordination and cooperative control of *n* satellites to achieve formation reconfiguration and phase angle adjustment. The first approach is based on pursuit algorithms while the second takes inspiration from motion camouflage. To validate the methodologies different scenarios are presented: a formation reconfiguration, an angular phase shift and a rendezvous manoeuvre. In summary, it has been shown that the control schemes proposed in this paper may have some potential for implementation in space missions, particularly since these approaches can be validated analytically. Future work would be the application of these control schemes in various scenarios while optimizing fuel consumption.

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