Global Journal of researches in engineering
Electrical and electronics engineering
Volume 12 Issue 6 Version 1.0 May 2012
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

A Novel Approach for the design of 2D Sharp Circularly Symmetric FIR Filters

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GJRE-F Classification : FOR Code: 100508,100599

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Abstract - In applications like weather monitoring, remote sensing, seismic signal processing etc, the boundaries and sharp features of the images have to be enhanced to obtain useful results and interpretations. The two dimensional(2D) highpass filters are used for such image processing applications. A direct design of the 2D high pass filter using frequency transformation has not been reported in the literature so far. In this paper, we present the design of a sharp 2D high pass filter using a direct transformation of the 1D low pass filter into a 2D high pass filter. Chebyshev polynomial approximation is used in mapping the one dimensional (1D)filter into its 2D equivalent. A promising aspect of the proposed transformation is that it is multiplier-free. Therefore the total complexity is decided by the multipliers of the 1D prototype filter. The complexity of designing a sharp 1D filter can be reduced by using the technique of Frequency Response Masking (FRM). Better circularity of the contour in the cutoff radius of the 2D filter is achieved compared to the traditional McClellan transformation. Reduced computation time and complexity are the highlights of the proposed approach.

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I. Introduction

Over the last few decades, the area of the design and implementation of 2D filters has attracted much attention of the researchers owing to its wide deployment in various areas like biomedical signal processing, seismic signal processing, radar and sonar procesing, genomic signal processing, satellite image processing etc(Williams et.al.,2007, Boudjelaba et.al.,2011). The various methods for the design of 2D linear phase FIR filter include frequency sampling, windowing, and frequency transformation(Lim,1990). Although the first two methods produce a better approximation of the response to the ideal one, the filter design involves large computational efforts especially for higher order filters. In contrast, the McClellan transformation has been bestowed with the features of reduced computation time and lesser implementation complexity. The higher order designs of the 2D zero phase filter can be done with much less computation efforts(Merserau,1980). The above features have contributed to lot of significant research performed in this area and very recently, this technique has been extended to the design of 3D filters(Mollova and Mecklenbrauker,2009).

Two dimensional high pass filters are extremely useful in detecting the sharp edges and boundaries of a given image. For instance, in weather monitoring, the discrete 2D high-pass filters are used to isolate the boundaries and edges of the weather data to make appropriate inferences and predictions(Feser and Storch,2005). Besides, high order high pass filter is shown to be a useful tool to evaluate and interpret meteorological data in finite areas in the work by Raymond,1989. The 2D high pass filter along with its complimentary lowpass filter has also been deployed in applications like the two channel quadrature mirror filter bank and the crossover network for image coding(Mitra and Yu,1986). There are several techniques for the design of 2D low pass filters(Lim,1990) and hence in most cases, the high pass filter is obtained using the complimentarity property(Mitra and Yu, 1986). Hence the direct design of high pass filters has not been addressed properly. In this paper, we propose a novel method for the direct design of 2D high pass filter. To this end, we use an approach similar to that is used in McClellan transformation using Chebyshev polynomial approximation for mapping the 1D filter into its 2D equivalent. But the striking difference is that the proposed transformation is used to derive a High Pass 2D filter directly from the 1D low pass prototype filter. Such an approach has not been reported in the literature so far. An advantage of using this approach is that this method preserves most of the characteristics of the 1D filter, especially the transition width and ripple characteristics. Once the high pass 2D filter is obtained, the complimentary low pass filter can be derived. This technique also avoids the drawback of the traditional McClellan transformation giving squarish contours at wide-band cutoff radii. The above transformations being multiplier-less, the number of multipliers needed for the realization of the 2D filter is decided by the order of the 1D FIR filter. Hence the number of multipliers of the 2D filter is fully decided by that of the 1D prototype filter.

In order to obtain a two dimensional filter whose performance specifications are close to an ideal 2D filter, the one dimensional prototype filter should be sharp. Even though 1D linear phase FIR filters are

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acclaimed for their inherent stability, computational complexity becomes exceedingly large as the transition width is reduced. To circumvent this drawback, we utilize the Frequency Response Masking (FRM) approach(Lim,1986) for the design of the sharp 1D prototype low pass filter and it results in sparse coefficients. FRM technique offers significant savings in the number of multipliers compared to the traditional mini-max design for the design of 1D filter. Due to the enormous computational saving possible for the design of an FRM filter, it has been extended to a variety of applications like array beam forming, FPGA, audio processing etc(Lim.et.al.,2007). Thus the 1D filter designed using FRM is converted into the 2D high filter using the proposed transformation. In many of the previous work using McClellan transformation, either optimization procedures or numerical methods were performed to obtain good circularity of the 2D radius. But in the very recent work by Liu and Tai, 2011, none of the above techniques were needed to obtain circular contours even at wideband. Instead a cascading term was added to the traditional McClellan transformation and two transformations namely T1 and T2 were proposed in the above paper. Since an analytical expression has been derived, the contour mapping problem is avoided in the T1 and T2 transformations. Our approach for the design of circular symmetric filters is based on the derivation of a totally new transformation instead of the mapping function in the traditional McClellan transformation. The contour mapping problem is avoided in our proposed method also. Hence the computation time is reduced drastically. In short, the sharp 2D filters designed using the proposed technique is better for high speed and low power applications while offering good performance close to the ideal one. This paper is organized as follows. Section II deals with an overview of frequency response masking approach. In Section III, the proposed transformation is briefed. The design of the 1D FRM filter is explained in Section IV A. Section IV B addresses the proposed design of the 2D circularly symmetric sharp filters. Simulation results are presented in Section V and conclusions are made in Section VI.

II. Overview of Frequency Response Masking

Frequency Response Masking is a much acclaimed technique for the design of an arbitrary bandwidth sharp FIR filter with reduced computational complexity. Efficient hardware implementation of the filter designed using FRM is possible due to the large number of zero valued coefficients. It consists of a prototype filter $H_a(z)$, complimentary filter $H_c(z)$, masking filter $H_{ma}(z)$ and a complimentary masking filter $H_{mc}(z)$. The complimentary filter $H_c(z^M)$ can be realized by substracting the output of the low pass filter $H_a(z^M)$ from the delay block $z^{-0.5(N_a-1)M}$, provided, the model filters are FIR in the nature. The block diagram of the FRM filter is shown in Fig. 1(Y.C.Lim, 1986).

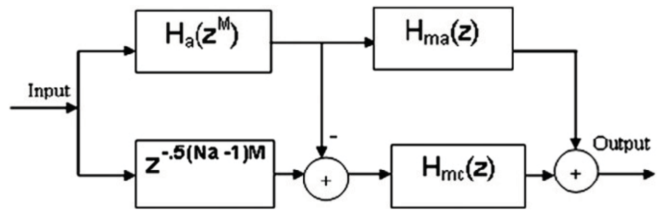

Figure.1 : Block Diagram of an FRM FIR filter.

The overall transfer function of the FRM filter can be written as follows

$$H(z) = H_a(z^M)H_{ma}(z) + H_c(z^M)H_{mc}(z) \tag{1}$$

$$\text{where} \quad H_c(z^M) = \left(z^{-0.5(N_a-1)M} - H_a(z^M)\right)$$

The prototype filter, also called the band edge shaping filter, is interpolated by a factor M and therefore its transition width is reduced by a factor of M. The masking filters are used to retain the necessary spectrum repetitions for the formation of any arbitrary bandwidth filter under consideration.

III. Proposed Frequency Transformation

a) *Derivation of the Proposed transformation*

Generalised McClellan transformation converts 1-D linear phase filter $H(\Omega)$ into a 2-D linear phase filter $H(\omega_1,\omega_2)$ by means of the substitution of variables (McClellan, 1973).

$$H(\Omega) = \sum_{n=0}^{N} a(n)\cos\Omega n \tag{2}$$

$$\text{where} \quad a(n) = \begin{cases} h(0), for\ n = 0 \\ 2h(n), otherwise \end{cases}$$

$h(n)$ corresponds to the 1D filter coefficients. Approximating $H(\Omega)$ using n-th order Chebyshev polynomial,

$$H(\Omega) = \sum_{n=0}^{N} a(n)\,T_n\,[\cos\Omega] \tag{3}$$

Now applying the 2D transformation by substitution of variables,

$$H(\omega_1,\omega_2) = \sum_{n=0}^{N} a(n)\,T_n\,[F(\omega_1,\omega_2)] \tag{4}$$

In the traditional McClellan transformation for designing the 2D filter with circular symmetry, the following expression was used(Liu and Tai, 2011).

$$\cos\Omega = F(\omega_1,\omega_2) = 2\cos^2(\omega_1/2)\cos^2(\omega_2/2) - 1 \tag{5}$$

Several investigations have been done in this direction to obtain low pass filters with different contours like circular, fan shaped, rectangular, diamond shaped etc.(Liu and Tai, 2011). But all of them were intended to obtain a 2D low pass filter. High pass filter was derived from it using the complimentarity property(Mitra and Yu,1986).

The advantage of using McClellan transformation technique compared to other filter design methods is that N number of multiplications are only needed to realize the 2D filter against N^2 number of multiplications in the other methods that use direct convolution if the filter is of size NxN. Making use of this advantage, in this work we propose a totally new transformation namely, H1 transformation for directly obtaining the 2D High pass filter from a 1D low-pass prototype filter.

This novel transformation is briefed below.

$$F(\omega_1,\omega_2)=(0.5\sin^2(\omega_1/2)+0.5\sin^2(\omega_2/2)-1)g(\omega_1,\omega_2) \quad (6)$$

where $g(\omega_1,\omega_2)=\cos^2(\omega_1/2)\cos^2(\omega_2/2)$

Linear Least square estimator has been used in deriving the transformation. The proof is given in Appendix 1. The features of the transformation are given as follows.

a. It is circularly symmetric at all radii.
b. It is quadrantly symmetric.
c. At all radii, the magnitude of $F(\omega_1, \omega_2)<1$.
d. The frequency mapping enables 1D prototype low-pass filter to become a 2D high pass filter.

The contour mapping of the proposed transformation is shown in Figure.2. A promising feature of the proposed design of the 2D filter is that the contour mapping problem is avoided in our work, since, an analytical expression is available for the proposed transformation. Hence, the 2D filter is obtained by the direct application of the proposed transformation. This calls for reduction in the computation time of the 2D filter.

b) Frequency Mapping of the proposed H1 Transformation

The relationship between the one dimensional frequency Ω and the two dimensional radius ω is derived below.

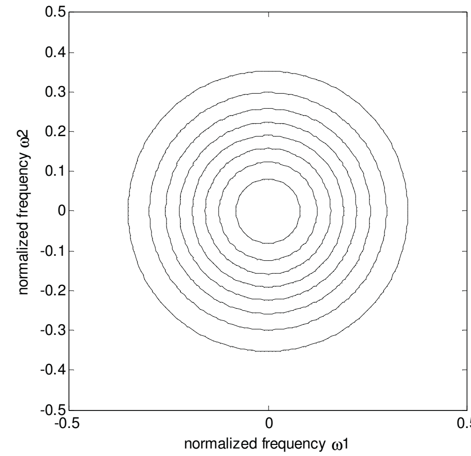

Fig.2 : Contour of the proposed transformation

Utilizing the quadrantal symmetry of the proposed transformation, let us consider only one direction in the 2D plane namely the ω_1 axis and make $\omega_1=\omega$ and $\omega=0$. This is done to obtain a one to one mapping between the frequencies in the 1D and 2D.

After applying the above condition, Equation.6. is modified as follows

$$\cos\Omega= F(\omega_1,\omega_2)= (0.5\sin^2(\omega/2)-1))\cos^2(\omega/2) \quad (7)$$

From Equation.7., it can be found that the 1D frequency can be obtained as given below.

$$\Omega=|\cos^{-1}((0.5\sin^2(\omega/2)-1))\cos^2(\omega/2))| \quad (8)$$

More clearly, the mapping is given by

$$\Omega=|\cos^{-1}(0.5\sin^2(\omega/2)\cos^2(\omega/2)- \cos^2(\omega/2))| \quad (9)$$

The figure showing the relation between the 1D and the 2D frequencies is given in Figure.3.

c) Realization of the proposed H1 transformation

In order to realize the transformation it has to be expressed in terms of z_1 and z_2.
Hence using the trigonometric identities

$$\cos\Omega= 2\cos^2(\omega/2) - 1 \text{ and}$$
$$\cos\Omega= 1 - 2\sin^2(\omega/2),$$

the transformation in Equation (6)can be rewritten as

$$F(\omega_1,\omega_2)=(1/4(1-\cos\omega_1)+1/4(1-\cos\omega_2)-1)(1/4 (1+\cos\omega_1)(1+\cos\omega_2)) \quad (10)$$

Therefore the final transfer function of the transformation is given by

$$F(z_1,z_2)=(1/4(1-Q_1(z_1))+1/4(1-Q_2(z_2))-1) (1/4(1+Q_1(z_1))(1+Q_2(z_2))) \quad (11)$$

where $Q_1(z_1)=(z_1^{1}+z_1^{-1})/2$ and $Q_2(z_2)=(z_2^{1}+ z_2^{-1})/2$

After rearranging the terms, above expression can also be written as follows

$$F(z_1,z_2)=(-1/2(1+Q_1(z_1)+Q_2(z_2))(1/4(1+Q_1(z_1)(1+Q_2z_2))$$

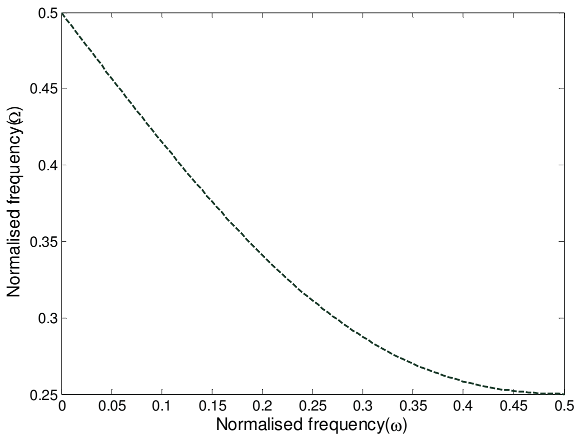

Fig.3 : Frequency Mapping of the proposed transformation

The block diagram of the proposed H1 transformation is shown in Figure.4. From the realization, it can be found that the transformation is totally multiplierless. Hence the total number of multipliers of the 2D filter is decided by the multipliers of the 1D filter. It is found that the computational complexity of the proposed H1 transformation, is least when compared to the T1, T2 and the McClellan transformation. The generalized McClellan transformation for obtaining circular contours as given in Kidambi, 1995 is used to make the comparison of complexity. The expression for McClellan transformation in Kidambi,1995 is given as follows.

$$\cos \Omega=0.5(\cos(\omega_1)+\cos(\omega_2))+t_{11}(\cos(\omega_1)\cos(\omega_2)-1) \quad (12)$$

The computational complexity of the transformation is found in terms of the number of adders and multipliers and the comparison is given in Table 1. MC refers to generalized McClellan transformation.

Table1 : Complexity comparison of various transformations

Complexity	H1	T1	T2	MC (Kidambi, 1995)
No of adders	8	10	9	6
No of multipliers	0	0	0	1

IV. Proposed Design of the Sharp 2D Circularly Symmetric Filters

A direct design of 2D high pass filter using frequency transformation has not been reported in the literature so far. All the work in this direction relied on the design of the 2D low-pass filter first, followed by finding out the complimentary high pass filter. The proposed design technique can be extended to the design of a 2D filter with the 1D prototype being any linear phase 1D filter. FRM has been used in our work to generate sharp filters with low complexity. Once the design specifications of the 2D High pass filter are obtained, using the relationship in Equation.9., the band edges of the 1D prototype low-pass filter are obtained. The design of the 1D prototype low pass filter using the FRM technique is briefed below.

a) Design of 1D FRM FIR Low Pass filter

The band edges of the various sub-filters of the FRM filter are obtained as given in the original work on FRM by Lim,1986. The optimal interpolation factor M is obtained in such a way that the total number of multipliers needed for the realization of the overall FRM filter is minimum. The sub-filters namely the model filter $H_a(z)$, masking filter $H_{ma}(z)$ and complimentary masking filter $H_{mc}(z)$ are designed as per the Remez Exchange algorithm. The conditions as outlined in the work by Lu .et.al., 2003 have been used to obtain a linear phase for the overall FRM FIR filter. As the objective of the work is to design a 2D high pass sharp filter with reduced complexity, FRM technique is used to obtain a sharp 1D filter with reduced complexity. The transformation preserves the properties of the 1D filter like the transition width, ripple characteristics etc.

b) Proposed Design of the 2D circularly symmetric Filters

A direct transformation is applied to the 1D prototype filter to obtain the sharp circularly symmetric High pass filter. A promising aspect of this design approach is that better circularity is obtained for all the contours when compared to the traditional McClellan Transformation. Also circularly symmetric 2D FRM based FIR filters have been reported for the narrow band case only.

Besides, a 2D circularly symmetric low pass filter can be obtained by the complimentarity property as given in the work by Mitra and Yu, 1986. Here, only one adder is needed in addition to number of adders needed for the realization of the high pass filter. This low pass filter has the advantage that it has lesser complexity compared to the T1 and T2 transformations proposed very recently for the design of circularly symmetric low pass filter(Liu and Tai,2011). The realization of the 2D filter is shown in Figure.5. This realization is quite similar to that used for the McClellan Transformation and the only difference being the use of a new $F(\omega_1, \omega_2)$. In this figure, a(n) corresponds to the coefficients of the 1D FRM FIR filter. A typical application of the complimentary pair of 2D filters is in the image coding system using the crossover network as pointed out by Mitra and Yu,1986.

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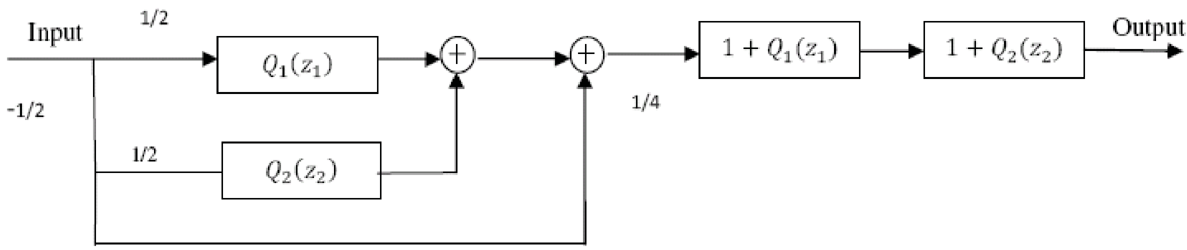

Fig.4 : Transfer Function of the proposed transformation $F(z_1,z_2)$

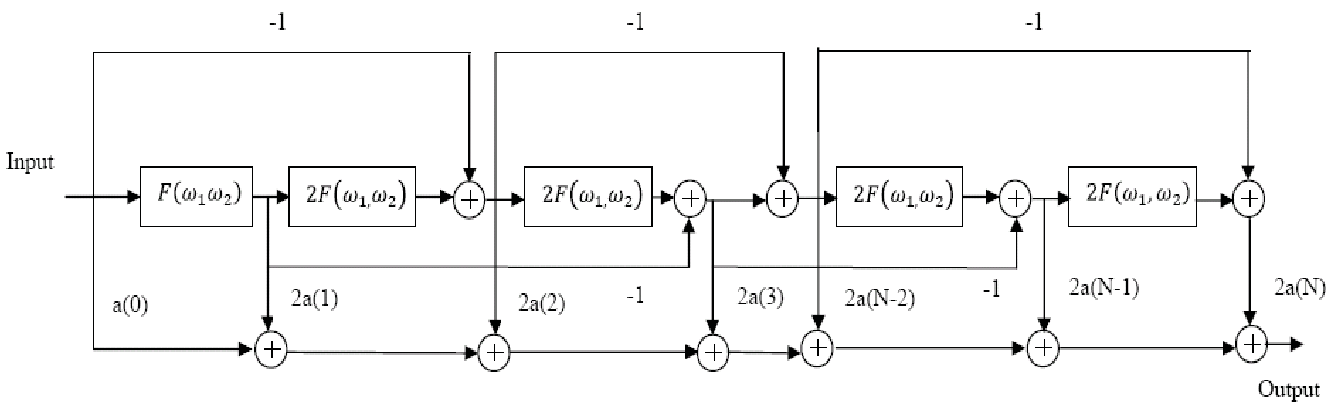

Fig.5 : Realization of the 2-D sharp High Pass filter

ω_r	.1π	.2 π	.3 π	.4 π	.5 π	.6 π	.7 π	.8 π	.9 π
$E(\omega_r)$	0.000051	0.000254	0.00085	0.0020	0.0037	0.0057	0.0073	0.007	0.006

Table 2 : Contour approximation error for varying 2D radius ω_r

In this paper, a new measure is proposed for evaluating the contour approximation error. Define C as the 1D frequency and ω_r as the 2D radius. The error measure $E(\omega_r)$is briefed as the L_2 norm of the error of approximation. It is given below.

$$E(\omega_r)=||\cos(\Omega)-F(\omega_r,\theta)||_2 \quad (13)$$

The algorithm for finding the error is briefed below.

1. Fix the value of the 2D filter cutoff radius ω_r .
2. Vary theta for given cutoff radius and obtain $F(\omega_r, \theta)$.
3. For each radius, using inverse mapping, find the value of C
4. Compute $E(\omega_r)$ as per eqn.13.
5. Repeat the steps 2-4 for varying values of ω_r. The approximation error is tabulated in Table 2.

The computational complexity of a 2D high pass filter designed using the proposed H1 transformation is shown in Table.3. Here 2N+1 is the length of the 1D prototype filter. To derive the high pass filter, the complimentarity property was used. Here, only one adder is needed in addition to number of adders needed for the realization of the low pass filter. This high pass filter has the advantage that it has lesser complexity compared to the T1 and T2 transformation proposed very recently for the design of circularly symmetric low pass filter(Liu and Tai, 2011).

Complexity	H1	T1	T2	MC (Kidambi, 1995)
No. of adders	9N	11N+1	10N+1	7N+1
No. of multipliers	N+1	N+1	N+1	2N+1

Table 3 : Complexity comparison of implementation of 2D high pass filter.

V. Simulation Results

Simulation is done using MATAB 7.10.0 on a Dual Core Opteron processor. The proposed approach has been deployed to obtain typical FIR 2D high pass filter. From the high pass filter, the complimentary low pass filter can also be obtained. Two design examples have been illustrated to demonstrate the usefulness of the proposed approach. One wideband and one narrowband high pass filter example have been taken and the equivalent low-pass filter was found out in each case. The narrowband 2D high pass example is given as Case1. The wideband 2D high pass filter is given as Case 2.

Case 1

$$H(\omega_1,\omega_2,) = \begin{cases} \delta_s, & 0 \leq \sqrt{{\omega_1}^2+{\omega_2}^2} \leq 0.6\pi \\ 1\pm\delta_p, & 0.61\pi \leq \sqrt{{\omega_1}^2+{\omega_2}^2} \leq \pi \end{cases}$$

$\delta_s = \delta_p = 0.01$.

The band-edges of the 1D prototype filter to be designed are found as $\Omega p=0.5884\pi$, $\Omega s= 0.6101\pi$.

The magnitude response of the 1D prototype filter using FRM technique is shown in Fig.6. The magnitude response and contour of the 2-D circularly symmetric high pass filter for Case 1 are shown in Fig.7 and 8 respectively. The magnitude response and contour of the complimentary 2-D low pass filter for Case 1 are shown in Fig.9 and 10 respectively.

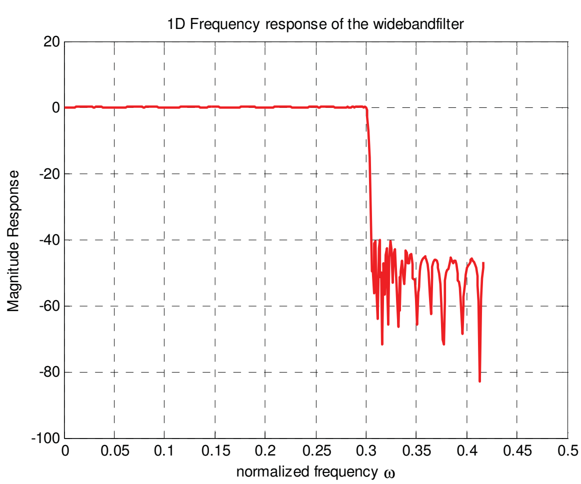

Fig.6 : Magnitude response of the 1D low pass prototype filter (Case 1)

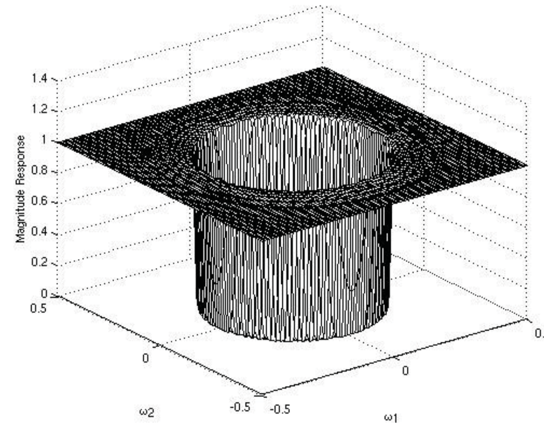

Fig.7 : Magnitude response of the 2D sharp high pass filter using the proposed H1 transformation (Case 1)

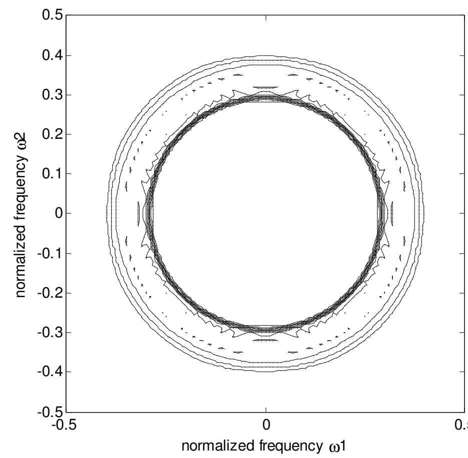

Fig.8 : Contour of the 2D sharp circularly symmetric high pass filter using the proposed H1 transformation (Case1)

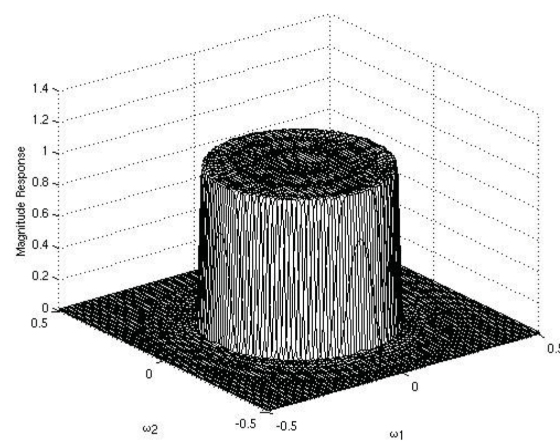

Fig. 9 : Magnitude response of the 2D complimentary low pass filter using the proposed H1 transformation (Case1)

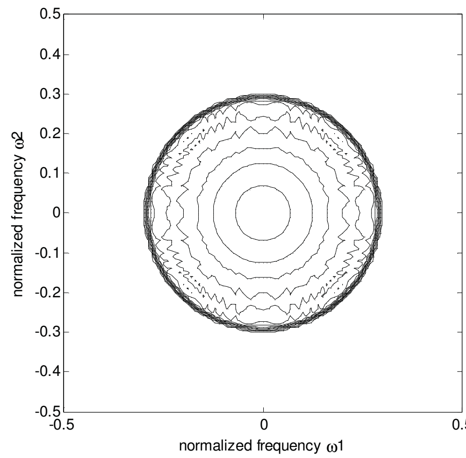

Fig. 10 : Contour of the sharp circularly symmetric 2D complimentary low pass filter using the proposed H1 transformation (Case1)

Case 2.

$$H(\omega_1\,,\omega_2,) = \begin{cases} \delta_s, & 0 \le \sqrt{{\omega_1}^2 + {\omega_2}^2} \le 0.2\pi \\ 1 \pm \delta_p, & 0.21\pi \le \sqrt{{\omega_1}^2 + {\omega_2}^2} \le \pi \end{cases}$$

$\delta_s = \delta_p = 0.01.$

The band-edges of the 1D prototype filter to be designed are found as $\Omega_p = 0.7933\pi$, $\Omega_s = 0.8086\pi$. The magnitude response and contour of the 2-D high pass filter for Case 2 is shown in Fig.11 and 12 respectively. Fig.13 shows the magnitude response of the complimentary low pass filter for Case 2.

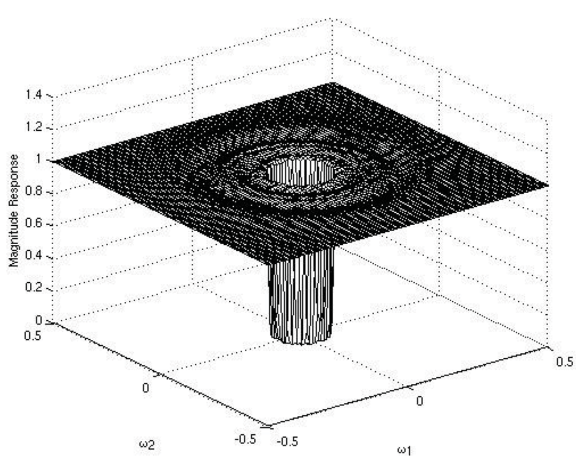

Fig.11 : Magnitude Response of the wide-band 2D sharp high pass filter using the proposed H1 transformation (Case 2)

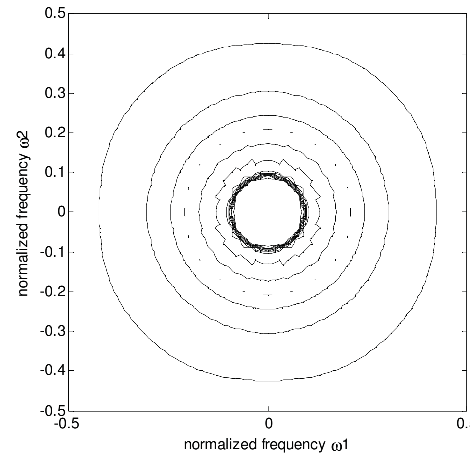

Fig.12 : Contour of the wide-band 2D circularly symmetric sharp high pass filter using the proposed H1 transformation (Case 2)

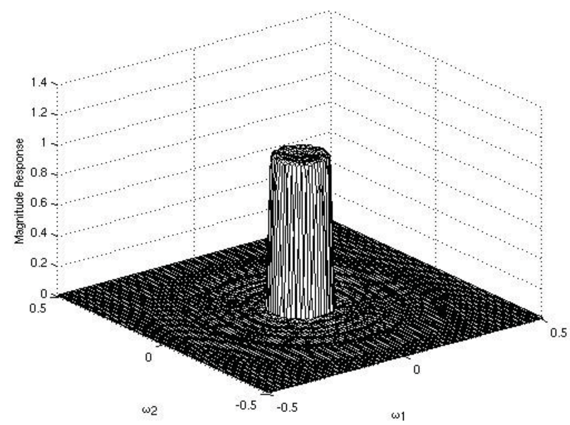

Fig.13 : Magnitude Response of the complimentary 2D sharp low pass filter using the proposed H1 transformation (Case 2)

VI. Conclusion

A novel and direct design approach for obtaining the 2D high pass, circularly symmetric, zero phase filter using the frequency transformation method has been presented. This transformation is then applied to a sharp 1D filter designed using Frequency Response Masking(FRM) technique. FRM is used to reduce the computational complexity of the sharp filter. From the 2D high pass filter, the complimentary low-pass filter can also be derived. The promising features of the proposed design technique is the reduced computation time and reduced complexity compared to the traditional methods. Good circularity is obtained at both narrowband and wideband radius of the 2D filter compared to McClellan transformation. Regarding the performance of the resulting 2D high-pass filter, the complexity is less compared to the recently proposed T1 and T2 transformations.

Appendix I

The proposed transformation for the direct design of 2D high pass filter has been found out using the Linear Least Square Estimator(Kay,1993).

Linear Least Square Estimator attempts to minimize the difference between the estimated and true parameter values. Here the unknown parameters are the coefficients of the basis functions being $\sin^2(\omega_1/2)$, $\sin^2(\omega_2/2)$ and 1 respectively. Consider the mapping given by

$$S=HU$$

S is the vector of dimension Nx1, H is an NxP matrix where N is the number of observations. P is the dimension of the vector U. X is the actual value. The least Square Estimate is found by minimizing the least squared error J(U) given by the following expression.

$$J(U)=(X-HU)^T(X-HU).$$

Setting the gradient of the the above =0, we get, the estimate of *U* as

$$\hat{U}=(H^T H)^{-1} H^T X$$

In the above method, the values of S are the known values of the expected contour defined at various values of the 2D frequencies(ω_1, ω_2). There are N observations. H is the matrix obtained by evaluating the values of the basis functions at various values of (ω_1, ω_2). In our work, we have N=20 and P=3. The estimated values of the coefficients of the constituents in the mapping are $\hat{u}_1=0.4848$, $\hat{u}_2=0.4848$, $\hat{u}_3=1$. In order to get a multiplier-les realization, and were taken to be 0.5. Hence the transformation is as follows.

$$0.5\sin^2(\omega_1/2) + 0.5\sin^2(\omega_2/2) - 1$$

But the contours of this transformation had noncircularity in the outermost radii. To correct this, a cascading term is added to the above transformation. Hence the resulting transformation was found to be as follows.

$$F(\omega_1,\omega_2)=(0.5\sin^2(\omega_1/2)+0.5\sin^2(\omega_2/2)-1)g(\omega_1,\omega_2)$$

Where $g(\omega_1,\omega_2)= \cos^2(\omega_1/2)\cos^2(\omega_2/2)$

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