Global Journal of researches in engineering
General engineering
Volume 12 Issue 2 Version 1.0 May 2012
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Cumulative Production Forecast of An Oil Well Using Simplified "Hyperbolic-Exponential" Decline Models

By Makinde F. A, Orodu O. D. , Ladipo A. O. & Anawe P. A. L.
Covenant University, Ota, Nigeria

Abstract - Decline Curves are important tools employed in the petroleum production industry to establish a good production performance forecast of production wells. Studies have shown that neither hyperbolic nor exponential decline could accurately produce dependable forecast results, which in turn affects the various economic decisions being made on both investment and future production processes. New simplified models for decline curve analysis are developed. The models are applicable to naturally producing wells that have not been secondarily enhanced. These models use exponential decline to extrapolate hyperbolic decline behaviour in making future production performance forecasts. Estimating different needed parameters and engaging some assumptions, the forecasted cumulative production increment using the model is . This compares favourably with the existing models.

Keywords : Production forecast, Reserves, Decline curve, Exponential, Hyperbolic.

GJRE- J Classification : FOR Code: 090999

CUMULATIVE PRODUCTION FORECAST OF AN OIL WELL USING SIMPLIFIED HYPERBOLIC-EXPONENTIAL DECLINE MODELS

Strictly as per the compliance and regulations of:

Cumulative Production Forecast of An Oil Well Using Simplified "Hyperbolic-Exponential" Decline Models

Makinde F. A[α], Orodu O. D.[σ], Ladipo A. O.[σ] & Anawe P. A. L.[σ]

Abstract - Decline Curves are important tools employed in the petroleum production industry to establish a good production performance forecast of production wells. Studies have shown that neither hyperbolic nor exponential decline could accurately produce dependable forecast results, which in turn affects the various economic decisions being made on both investment and future production processes. New simplified models for decline curve analysis are developed. The models are applicable to naturally producing wells that have not been secondarily enhanced. These models use exponential decline to extrapolate hyperbolic decline behaviour in making future production performance forecasts. Estimating different needed parameters and engaging some assumptions, the forecasted cumulative production increment using the model is $Q_{el} \approx 20,705\ bbls$. This compares favourably with the existing models.

Keywords : Production forecast, Reserves, Decline curve, Exponential, Hyperbolic.

I. Introduction

The petroleum industry is summarily driven by profit. This is the topmost factor to consider in the development of petroleum fields. Reserves have to be well estimated before the company's limited available resource is expended on a given project.

One of the most important tasks of a petroleum engineer is estimating, by factual prediction, the amount of oil and gas that could be recovered from a reservoir. Choosing the methodology is critical for accurate forecasts that are, in turn, vital for sound managerial planning[5]. Risks have to be minimized in the process of making decisions based on the recoverable percentage of the original hydrocarbon in place, as well as the residual amount of this oil when the economic limit is reached before the need for any secondary recovery mechanism is involved, for the case of a naturally producing reservoir-well relationship.

Extrapolation of production history has long been considered the most accurate and defendable method of estimating the remaining recoverable reserve from a well and, in turn, a reservoir[5]. Various methods have been developed so far, ranging from the most commonly used Decline Curve Analysis (DCA), to the Type Curve Matching (TCM) method based on the well or reservoir production history. While Decline Curve Analysis is independent of any reservoir characteristics, Type Curve Matching is a very subjective procedure.

DCA is a method used for the prediction of future hydrocarbon production by analyzing past production. A decline curve of a well is simply a plot of the well's production rates against the respective times of recording. It was recognized early in the history of petroleum engineering that calculating reserves and production forecasts were possible by studying past production trends.

Upon all the various works done in the past on forecasting the recoverable amount of hydrocarbons from an oilfield, the problem has been how to forecast with decline curves (production history) without over-estimating or under-estimating cumulative production/reserves, most especially with Arp's Decline Curve Analysis.

This study combines hyperbolic and exponential declines in developing simplified empirical decline curve models (hyperbolic-exponential models), to forecast the oil production performance of a primarily flowing well till economic limit. The production performance includes **predicting future production rates**, **cumulative production increments**, **production time or period**, **ultimate recovery and the residual reserves**[6] but this paper presents only the models for predicting the cumulative production.

Arps[1] published several excellent papers in 1944 and 1945. These papers contained several excellent equations which have essentially remained unchanged and are still referred to as the "Arps" equations.

Arps categorized decline curves into exponential, hyperbolic and harmonic declines. The hyperbolic equation is the universal equation upon which the exponential and harmonic declines are special cases of. The difference is just the changes in the hyperbolic constant used for the two cases. The hyperbolic constant determines the degree of change of each decline. The hyperbolic constant "b" equals 0 for exponential declines, 1 for harmonic declines, and ranges from 0 to 1 for hyperbolic declines.

Arps in his document, coupled with the work of Curtler showed that approximately two-third of the various cases of declines studied have a hyperbolic constant between 0.1 and 0.4, suggesting a log normal distribution.

Author α : Department of Petroleum Engineering, Covenant University, Ota, Nigeria.
E-mail : favour.makinde@covenantuniversity.edu.ng
Author σ : Petroleum Engineering Department, Covenant University, Ota, Nigeria.

Arps was able to derive and establish the various basic equations for Decline Curve Analysis, but unfortunately, he could not establish this transition point from hyperbolic to exponential decline.

To tackle this problem, in early 1980s, Fetkovich[2] designed Type Curve models from different studied wells and fields based on Arps' empirical equations and known reservoir flow properties from which the decline curve derived from the production data of analysis is compared to look for a perfect match of which the well automatically assumes the Type Curve properties, whose values are then used for the forecasting of the well of interest.

Long and Davis[3] (1988) further added to the work of Arps. They tried to look at how the problem of the transition point could be surmounted. In their pursuits, they developed a log-rate-versus-time overlay to cope with this problem. They introduced the use of Type Curve Matching method, where the curve from a well production history is matched with already developed curves from different wells based on different reservoir-well properties for correlation.

Robertson[4] (1988) also developed a production rate equation that is hyperbolic initially but asymptotically exponential with time. He introduced a dimensionless constant; its value ranges from 0 to 1 and is related to the abandonment pressure, and the rock and fluid properties.

Long and Davis, and Robertson ignored determining the precise transition point between the hyperbolic and exponential declines, they assumed the exponential decline rate was known based on experience with analogous wells or experience with particular reservoirs. This method proposed by them is limited in application because:

- Different wells situated in the same reservoir at times exhibit different reservoir-well properties such as the skin effect along their production history, thus, making them to exhibit different productivity indices, effective permeabilities etc, thereby drastically varying their production decline rates. This limits the dependability on the value.
- Also, in a situation when the well to be analyzed is the first producing well on the field or probably the other wells on the field are in a hyperbolic decline state, a dependable exponential decline rate (constant) that could have served as a reference for comparison with the decline rate value of the well-in-question is no more readily available.

Due to the fact that the hyperbolic decline rate at which exponential decline starts is the rate that is used to forecast the various production rates and cumulative production at any time or point after this transition point, conciseness is necessary in determining this value for improved accuracy in any estimated forecast made after this point (that is in the exponential decline state of the well productions).

Khaled[5] from King Saud University, Saudi Arabia, in his publication "Predicting Production Performance using a Simplified Model", looked at combining hyperbolic and exponential decline models empirically together to predict production performance using the available well production history data. In his method, he stated that at a point in the production life of a well, the rate of decline of the decline rates, D with production time will behave as a constant (that is $dD/_{dt} = C$). The time, at which this occurs, he described as the transition point from hyperbolic to exponential declines, from where production forecast can now be made exponentially. In that publication, Khaled assumed the value of this constant, based on the behaviour of the curve gotten from the constant-time graph he plotted. Thus, he employed a deterministic approach in estimating the transition time.

II. Methodology

In this work, a set of empirical models are developed for estimating the cumulative production at the economic limit as follows:

Model for Cumulative Production Forecast

Cumulative Production, Q_{cum} till any production rate, q_o in the exponential decline period:

$$Q_{cum} = Q_{hyp} + Q_{exp} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.0)$$

From the general hyperbolic decline equation for cumulative production,

$$Q_{hyp} = \frac{q_{oi}^{b} * \left(q_{oi}^{(1-b)} - q_{iexp}^{(1-b)}\right)}{D_i(1-b)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.1)$$

Global Journal of Researches in Engineering (J) Volume XII Issue II Version I May 2012

$$but \quad q_{iexp} = q_{oi}\left(1 + bD_i t_{exp}\right)^{-\frac{1}{b}} \text{ ....(xx)}$$

$$t_{exp} = b^{-1}\left(D_{exp}^{-1} - D_i^{-1}\right) \text{ ....(xxx)}$$

$$Therefore:$$

$$Q_{hyp} = \frac{q_{oi}^{b} * \left(q_{oi}^{(1-b)} - q_{oi}^{(1-b)}\left(1 + bD_i t_{exp}\right)^{-\frac{1}{b}*(1-b)}\right)}{D_i(1-b)}$$

$$Q_{hyp} = \frac{q_{oi}^{b} * \left(q_{oi}^{(1-b)} - q_{oi}^{(1-b)}\left(1 + bD_i t_{exp}\right)^{\frac{b-1}{b}}\right)}{D_i(1-b)}$$

$$Q_{hyp} = \frac{q_{oi}^{b} * q_{oi}^{(1-b)}\left[1 - \left(1 + bD_i t_{exp}\right)^{\frac{b-1}{b}}\right]}{D_i(1-b)}$$

$$Q_{hyp} = \frac{q_{oi} * \left[1 - \left(1 + bD_i t_{exp}\right)^{\frac{b-1}{b}}\right]}{D_i(1-b)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

From the general exponential equation for cumulative production, the cumulative production (from the beginning of the exponential decline model, t_{exp} to any production time with a production rate, q_o), Q_{exp} in the exponential decline period is:

$$Q_{exp} = \frac{q_{iexp} - q_o}{D_{exp}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2.3)$$

$$Where \quad D_{exp} = \frac{D_i}{1 + bD_i t_{exp}}$$

$$Therefore:$$

$$Q_{exp} = \frac{\left(q_{iexp} - q_o\right)\left(1 + bD_i t_{exp}\right)}{D_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

$$Q_{exp} = \frac{\left(q_{oi}\left(1 + bD_i t_{exp}\right)^{-\frac{1}{b}} - q_o\right)\left(1 + bD_i t_{exp}\right)}{D_i}$$

Global Journal of Researches in Engineering (J) Volume XII Issue II Version I May 2012

$$Q_{exp} = \frac{\left[q_{oi}\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right] - q_o\left(1+bD_i t_{exp}\right)}{D_i} \quad \dots\dots\dots\dots\dots\dots\dots\dots (2.5)$$

Substituting (2.2) *and* (2.5) *into* (2.0),

$$Q_{cum} = \frac{q_{oi} * \left[1-\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right]}{D_i(1-b)} + \frac{\left[q_{oi}\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right] - q_o\left(1+bD_i t_{exp}\right)}{D_i}$$

Therefore:

$$Q_{cum} = \frac{q_{oi} * \left[1-\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right) - q_o\left(1+bD_i t_{exp}\right)\right](1-b)}{D_i(1-b)}$$

The Cumulative production to the economic limit production rate (q_{el}), Q_{el} *is:*

$$Q_{el} = \frac{q_{oi} * \left[1-\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\left(1+bD_i t_{exp}\right)^{\frac{b-1}{b}}\right) - q_{el}\left(1+bD_i t_{exp}\right)\right](1-b)}{D_i(1-b)}$$

In terms of a constant β *, Cumulative Production,* Q_{cum} *till any production rate* (q_o) *in the exponential decline period can be expressed as:*

$$Q_{cum} = \frac{q_{oi} * \left[1-\beta^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\beta^{\frac{b-1}{b}}\right) - q_o\beta\right](1-b)}{D_i(1-b)} \quad \dots\dots\dots\dots\dots\dots\dots (2.6)$$

while Cumulative production to the economic limit production rate (q_{el}), Q_{el} *is*

$$Q_{el} = \frac{q_{oi} * \left[1-\beta^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\beta^{\frac{b-1}{b}}\right) - q_{el}\beta\right](1-b)}{D_i(1-b)} \quad \dots\dots\dots\dots\dots\dots\dots (2.7)$$

$$\text{where:} \quad \beta = 1 + bD_i t_{exp} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.8)$$

Hence, the residual oil reserve in the reservoir at the point of economic limit assuming q_{oi} *is the first production rate in the production history of the producing well,* $Q_{residual,el}$ *is:*

$$Q_{residual,el} = OIIP - Q_{el} \quad \dots\dots\dots\dots\dots\dots\dots\dots (2.9)$$

If the well has produced certain amount of oil, Q_{oi} *before the analysed production decline period begins, either as a result of well's transient production period, production downtime, or any other reasons, the residual oil reserve,* $Q_{residual,el}$ *is:*

$$Q_{residual,el} = OIIP - (Q_{oi} + Q_{el}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

In addition, the ultimate recovery through the producing well concerned, from the onset of the production decline period to the economic limit (that is within the analysed production decline period), E_R *is:*

$$E_R = \frac{Q_{el}}{OIIP} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

While the ultimate recovery through the producing well assuming there has been a period of production prior to the production decline period being analysed, E_R *is given as:*

$$E_R = \frac{Q_{oi} + Q_{el}}{OIIP} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12)$$

NOTE: For all the cumulative production models above, if any of the decline rates, D, D_i *or* D_{exp} *is in per year* (yr^{-1})*, and the associated production rate,* q_o *is in* $\frac{STB}{day}$*, the respective cumulative production expression must be multiplied by a factor of 365 to express it in STB.*

Also, if D is expressed in $month^{-1}$*, and* q_o *is in* $\frac{STB}{day}$*, the cumulative production expression must be multiplied by a factor of 30.4 to express it in STB. The cumulative production models stand the way they are expressed ONLY if the involved declines, D are in* day^{-1}*, and production rates,* q_o *are in* $\frac{STB}{day}$*, or the declines, D are in* $month^{-1}$*, and production rates,* q_o *are in* $\frac{STB}{month}$*, or the declines, D are in* yr^{-1}*, and production rates,* q_o *are in* $\frac{STB}{yr}$*.*

All these are just to correct for the varying units of the decline rate, D.

a) Estimation of Required Parameters

From the developed models, the required parameters for forecasting the performance of a well in a reservoir, and subsequently estimating the cumulative production and residual reserve at a set or predetermined economic limit production rate are:

- t_{exp} , the time at which the hyperbolic decline models turn to exponential decline models along the production history of the well,
- D_i , the initial decline rate at the onset of the hyperbolic decline behavior of the well's production rates, and
- b , the hyperbolic decline constant or exponent.

b) Estimation of t_{exp}

t_{exp} is an important parameter needed for forecasting a well's production and residual reserve using a combination of hyperbolic and exponential decline models.

There are three methods which could be used in the estimation of t_{exp} . Each method has its own applicability and conditions of usage. They include:

- Graphical method,
- Comparison with already producing well in the same field, and
- Company's Production Policy.

c) Solving For the Hyperbolic Decline Constant/ Exponent, b

The value of the hyperbolic constant b is very crucial in any hyperbolic DCA forecast. The shape of the hyperbolic curve is controlled by both the hyperbolic constant b , and the initial decline D_i . The higher the b factor, the lower the decline in production rate, the longer the life of the well to the economic limit and the higher the ultimate recovery is.

Over the years, engineers calculate the hyperbolic constant using simple and approximate methods. Apart from the various empirical methods developed for calculating the various parameters needed in hyperbolic decline models, various DCA softwares such as the Oilfield Manager (OFM) are available in the industry for estimating them.

Empirically, the following methods can be employed in solving for the hyperbolic decline constant, b.

They are:

- The three-point method, and
- The initial rate and decline method.

d) Calculation Of The Initial Decline Rate, D_i

The initial decline rate D_i plays an important role in a hyperbolic-exponential decline models as seen above. Apart from the widespread DCA computer softwares available presently in the oil and gas industry for estimating the various hyperbolic decline constants, this paper identifies two simplified and empirical methods for achieving the task of calculating D_i. These are:

- The tangential method, and
- The two-rate method.

III. Model Validation and Discussion of Results

There are various methods of estimating the forecast parameters as highlighted above but for the sake of simplicity and time, quite a number of computer softwares have been developed nowadays to ease seemingly complex calculations such as estimation of the hyperbolic decline exponent, the hyperbolic initial decline rate, and the initial decline rate.

In this work, the Schlumberger Oilfield Manager (OFM) was used in carrying out the various estimations. Shown in Figure 1 in the Appendix A is a Microsoft Excel format of the semi-log plot obtained from the OFM software showing both hyperbolic and exponential decline models fit for the region of best historical decline behaviour and their respective decline parameters as well as their regression coefficients.

It is understood that most naturally-producing wells start their production history with a hyperbolic decline behaviour, which does occur for just some periods after production commences, before finally exponential decline behaviour sets in. This understanding aided the development of the models in this paper. The result of the decline rates, Di above clearly shows that the hyperbolic initial decline rate is higher than that of the exponential decline rate, therefore permitting the hyperbolic initial decline to reduce to the exponential decline rate, from where the decline rate remains constant till the end of the forecast. This is a good pointer to establishing the intent of this work.

The forecast starts with a hyperbolic rate, then the time at which this rate declines to the exponential rate is calculated from one of the models (equation xxx), hence, from this time, the exponential decline is employed to forecast the remaining future production time behaviour of the well till the economic limit is reached assuming no work-over activities are being carried out on the well.

After estimating all these parameters (for Hyperbolic Decline):

$$D_i = 0.129655\ month^{-1}$$

$$b = 0.192333$$

$$D_{exp} = 0.101351\ month^{-1}$$

Then, estimating the time at which the

hyperbolic behaviour changes to the exponential behaviour:

$$From\ Equation\ (XX): T_{exp} = b^{-1}(D_{exp}^{-1} - D_i^{-1})$$

$$Therefore: t_{exp} \approx 11.2\ months$$

Note that this parameter t_{exp} is the most important tool in forecasting using combined models of two decline behaviours. Its value is what mostly determines how valid our forecast result is and this is not known to have been determined with any known software yet. It is best solved empirically by analyzing the historical production decline behaviour of the well. The method used here in calculating it is employed because we have a well that is showing a decline behaviour that is very close to both hyperbolic and exponential declines. The graphical method cited under the methodology would have been the only applicable method assuming there is a great difference between the hyperbolic and exponential decline fits.

Having estimated t_{exp} and the other required parameters for forecast with the developed models, the cumulative production forecast is done using the developed models.

The validity of the models developed and used here is authenticated with the data set analyzed.

a) Cumulative Production Forecast

Setting the last historical oil production rate (Table 1) of the well as the initial production rate for the forecast:

$$q_{last\ historical} = q_{oi} = 77.667\ bbls/day$$

Assuming that no work-over activities like perforation interval plugging, pressure build-up and drawdown tests etc, as well as no secondary or enhanced recovery mechanisms are to be employed on the field till the bench-marked economic limit rate is reached, then the developed models for cumulative forecast could be used in forecasting the cumulative production increments through the well from the field, from the last historical date/rate to any forecasted production rate, q_o at various production times. For this forecast, the minimum forecasted production rate is set at: $q_o = 0.1\ bbl/day$.

For the cumulative production forecast,

$$equation(2.2): Q_{hyp} = \frac{q_{oi} * \left[1 - (1 + bD_i t_{exp})^{\frac{b-1}{b}}\right]}{D_i(1-b)}$$

is used to estimate the cumulative production increments at different forecasted production times ranging from the last historical date where t=0 to when $t = t_{exp}$, and

$$equation(2.6): Q_{cum} = \frac{q_{oi} * \left[1 - \beta^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\beta^{\frac{b-1}{b}}\right) - q_o\beta\right](1-b)}{D_i(1-b)}$$

is used to estimate the cumulative production increments from t_{exp} to t_o where $q_o = 0.1\ bbl/day$.

Also, the constant $\beta \approx 1.279294$, *estimated from equation (2.8):* $\beta = 1 + bD_i t_{exp}$

Calculating the various cumulative production increments at the various forecasted times using the above models at designated regions would generate Table 2 in Appendix A.

A normal plot of these forecasted cumulative production increments against the respective production times is shown in Figure 2.

The forecast in Figure 2 is very useful in analyzing at what production time a particular cumulative production increment will be achieved in the future production of the well. It is also applicable when determining the total recovery from a well at any forecasted production time in future.

On the other hand, the relationship between the forecasted rates and the forecasted cumulative production increments can be studied. This is very important in analyzing how the decrease in production rate with time is affecting the cumulative increments in production. It could also be used in predicting the forecasted production rate at which no economically viable increase in cumulative production will be experienced. This helps in using the forecast to decide when there will be need for a production-enhancement programme or a post-primary recovery mechanism (secondary or enhanced) to be adopted for further oil recovery from the field. This is based on the estimated reserves remaining in the field at this forecasted production rate.

The forecasted rate-cumulative production increment data are graphically shown in Figure 3:

ANOTHER POSSIBLE FORECAST

Forecasting Total Cumulative Production Increment Till Economic Limit

To forecast the total possible cumulative incremental production till economic production rate, i.e till q_{el} is reached,

$$equation\ (3.7): Q_{el} = \frac{q_{oi} * \left[1 - \beta^{\frac{b-1}{b}}\right] + \left[\left(q_{oi}\beta^{\frac{b-1}{b}}\right) - q_{el}\beta\right](1-b)}{D_i(1-b)}$$

is used.

In this forecast, setting the economic limit production rate at $q_{el} = 1.0\ bbl/day$, and already estimated $\beta \approx 1.279294$ above, the forecasted cumulative production increment, is $Q_{el} \approx 20,705\ bbls$.

b) Model Validation

Here comparison is done between the use of a hyperbolic-exponential combined model in predicting or forecasting the performance of a naturally-depleted well, and that of using either a full hyperbolic decline model

Global Journal of Researches in Engineering (J) Volume XII Issue II Version I May 2012

or an exponential model.

In doing this, the essential forecast done so far with the developed models is carried out again using a full hyperbolic decline model as well as a full exponential model.

Now, forecasting the cumulative production increment using the three models and creating a comparison between them shows how valid the developed models in this work in predicting a well's performance, as well as the lapses involved in using either a hyperbolic or exponential decline type for the full forecast.

A comparison using a normal plot of each model's forecasted cumulative production increment results against time is presented in Figure 4.

In addition, results obtained from the other forecasts (like cumulative production) when compared with the ones that hyperbolic and exponential declines would have generated if they were to be separately used to forecast these parameters also show that neither hyperbolic nor exponential decline alone could adequately forecast the performance of a naturally-flowing well, hence the need for a combined model as the one developed, analyzed and used in this work. It is either one decline model over-estimates the reserves and the other under-estimates the reserves or vice versa. But in most cases, the hyperbolic over-estimates while the exponential model under-estimates. This is as a result of the hyperbolic decline having its decline rate reducing with time, thereby experiencing a decreasing rate of production decline with time. Exponential decline in its own case has a constant decline rate with time, thereby making the production decline with time to be constant throughout the forecast.

The forecast results have satisfactorily validated the reason for the development of the models in this paper, which is showing that in most well performance prediction or forecast involving the use of Decline Curve Analysis (DCA), only one model cannot just be employed for the forecast because most naturally producing wells exhibit mostly the combination of two models. Despite that the hyperbolic decline behaviour that they display at the onset of their production history does not really last for long before exponential decline sets in, still if this period of hyperbolic decline is not considered (or probably taken as exponential), or the remaining exponential decline period is taken to be hyperbolic, it seriously affects the forecast results as shown above with feasible space or shift between the different models' results.

Table 3 in Appendix A shows in summary the effectiveness of the developed model on cumulative production increment forecast. This is done by applying the developed model in making a cumulative production forecast using an already-established set of production data employed by Long and Davis[3] and Khaled[5] in their works. This is just to show the level of deviation of the developed model's result from the results estimated by Long and Davis[3], as well as Khaled[5]. It significantly shows that the result generated by the developed model (for total cumulative production forecast) in this paper correlates well with the results from the works of Long and Davis[3] and Khaled[5].

IV. Conclusion and Recommendation

a) *Conclusions*

The use of either hyperbolic or exponential decline model has been discovered to produce unrealistic results in predicting or forecasting the production performance of a naturally-flowing well. This serves as the platform on which this work was based.

New models are developed to forecast the production performance of a well in terms of cumulative production increment. These new models combined the conventional hyperbolic and exponential decline models. The transition time from hyperbolic to exponential decline along the production life of such well was estimated by a deterministic approach.

It indeed showed in the results generated from the analysis of the data used in this study that neither the hyperbolic nor the exponential decline model could predict or forecast the performance of a naturally-flowing well accurately. The results obtained vividly showed that the two models, (hyperbolic and exponential) either over-estimate the forecasted performance (rate, time and cumulative production increment) or under-estimate it.

b) *Acknowledgments*

The authors appreciate and acknowledge the faculty and staff of the Department of Petroleum Engineering, Covenant University for the conducive atmosphere provided for this study.

c) *Nomenclature*

$$q_o - oil\ production\ rate\ at\ any\ point\ in\ time, \frac{STB}{day}$$

$$q_{oi} - initial\ oil\ production\ rate\ at\ the\ onset\ of\ production\ decline, \frac{STB}{day}$$

$$q_{iexp} - \text{ oil production rate at the end of the hyperbolic decline model or onset}, \frac{STB}{day}$$

$$q_{el} - \text{set oil production rate at the economic limit}, \frac{STB}{day}$$

$$q_1 - \text{oil production rate at the end of the first year of hyperbolic decline behaviour}, \frac{STB}{day}$$

$$q_{01} - \text{ selected oil production rate at time } t_1 \text{ during the hyperbolic decline behaviour}, \frac{STB}{day}$$

$$q_{02} - \text{selected oil production rate at time } t_2 \text{ during the hyperbolic decline model}, \frac{STB}{day}$$

$$q_{03} - \text{selected oil production rate at time } t_3 \text{ during the hyperbolic decline model}, \frac{STB}{day}$$

$$b - \text{hyperbolic decline constant or exponent}$$

$$D_i - \text{initial decline rate, in } yr^{-1}, month^{-1}, \text{or } day^{-1}$$

$$t - \text{any time along the production history of the well, in years, months or days}$$

$$t_{exp} \text{ or } t_o - \text{time along the production history at which exponential decline model sets in}$$

$$t_{el} - \text{forecasted production period of the well from } q_{oi} \text{ till economic limit rate}, q_{el}$$

$$t' - \text{a defined production time parameter, in years, months or days}$$

$$D - \text{decline rate at any production time in the hyperbolic model, in } yr^{-1}, month^{-1}, \text{or } day^{-1}$$

$$C - \text{a defined constant}$$

$$D_{exp} = \text{constant exponential decline rate, in } yr^{-1}, month^{-1}, \text{or } day^{-1}$$

$$\beta = \text{a defined constant}$$

$$Q_{hyp} - \text{cumulative oil production till the end of the hyperbolic decline behaviour}, STB$$

$$Q_{exp} - \text{cumulative production from the onset of the exponential model to any time}, STB$$

$$Q_{cum} - \text{cumulative production till any time in the exponential model}, STB$$

$$Q_{residual,el} - \text{residual oil in the reservoir at the economic limit of production}, STB$$

$$Q_{el} - cumulative\ production\ till\ economic\ limit, STB$$

$$Q_{oi} - cumulative\ production\ from\ the\ producing\ well\ prior\ to\ q_{oi}, STB$$

$$Q_{23} - cumulative\ production\ from\ time, t_2 to\ t_3\ along\ the\ hyperbolic\ decline\ period, STB$$

$$OIIP - Oil\ Initially\ In\ Place, STB$$

$$E_R - Ultimate\ Recovery, \%$$

References Références Referencias

1. Arnold, "Manual for the Oil and Gas Industry", 1919.
2. Cutler, W.W., "Estimation of Underground Oil Reserves by Oil-Well Production Curves", 1924.
3. Arps, J.J., "Analysis of Decline Curves", Trans., AIME, Vol. 160, 1945, pp. 228-247.
4. Fetkovich, M.J., "Decline Curve Analysis Using Type Curves", SPE Paper-4629 presented at the SPE 48th Annual Fall Meeting, Las Vegas, Nev., Sept. 30-Oct. 3, 1973.
5. Brons, F., "On the Use and Misuse of Production Decline Curves", Producers Monthly, 1963.
6. Society of Petroleum Engineers, "Petroleum Engineering Handbook", 1987.
7. Long, D.R., and Davis, M.J., "A New Approach to the Hyperbolic Curve", Journal of Petroleum Technology, Vol. 40, 1988, pp. 909-912.
8. Robertson, S., "Generalized Hyperbolic Equation", Unsolicited, SPE 18731, 1988.
9. Khaled, A.F., "Predicting Production Performance Using a Simplified Model", Production Technology-World Oil, April, 2006.
10. Bailey, W., "Optimized Hyperbolic Decline Curve Analysis of Gas Wells", Oil & Gas Journal, Feb. 15, 1982.
11. McNulty, R.R., and Knapp, R.M., "Statistical Decline Curve Analysis", Paper SPE 10279, Presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Oct. 5-7, 1981.
12. Thompson, R.S., Wright, J.D., and Digert, S.A., "The Error in Estimating Reserves Using Decline Curves", Paper SPE 16925, Presented at the SPE Hydrocarbon Economics and Evaluation Symposium, Texas, March 2-3, 1987.
13. Gentry, R.W., "Decline-Curve Analysis", SPE Paper-3356, 1972.
14. Ladipo, A. O. 2010, Predicting the Production Performance of an Oil Well Using Simplified "Hyperbolic-Exponential" Decline Models, B.Sc. Project, Covenant University.

Appendix

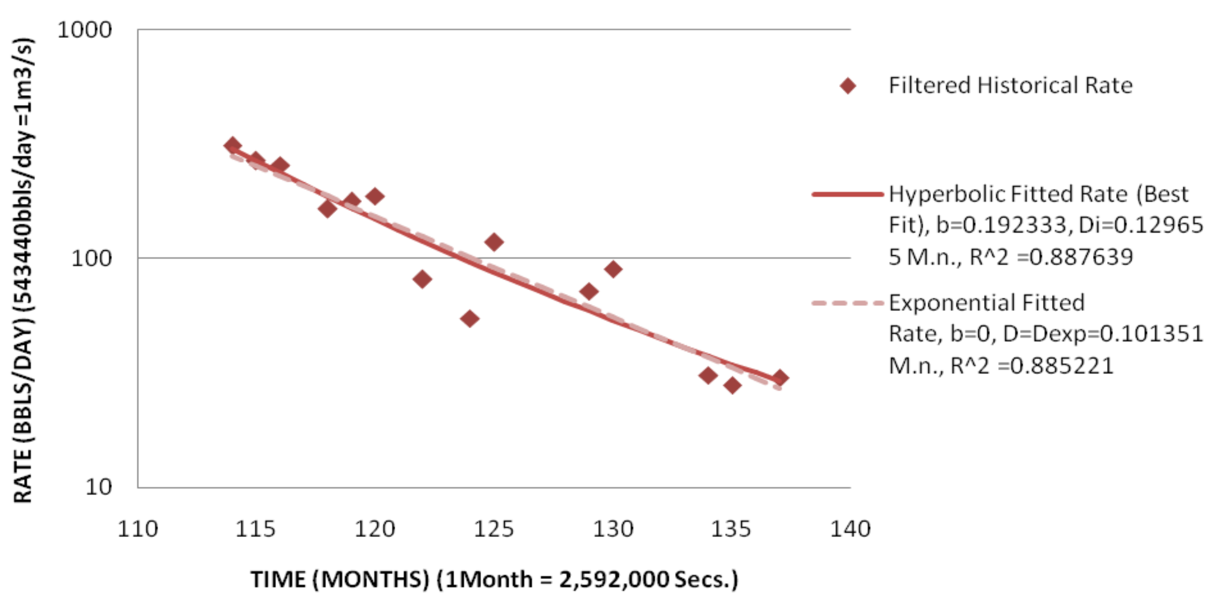

Fig.1: EXCEL FORMAT OF THE OFM DECLINE FITS

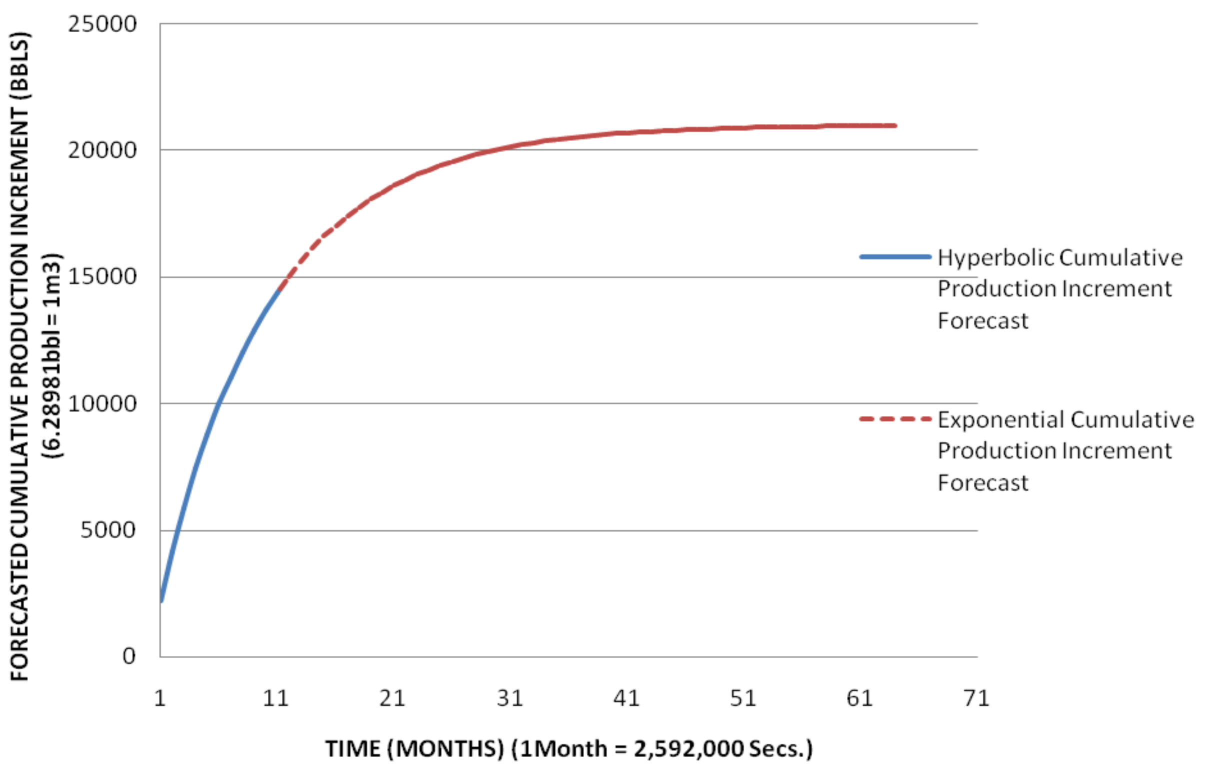

Fig.2: FORECASTED CUMULATIVE PRODUCTION INCREMENT-TIME PLOT FROM THE LAST HISTORICAL PRODUCTION TIME

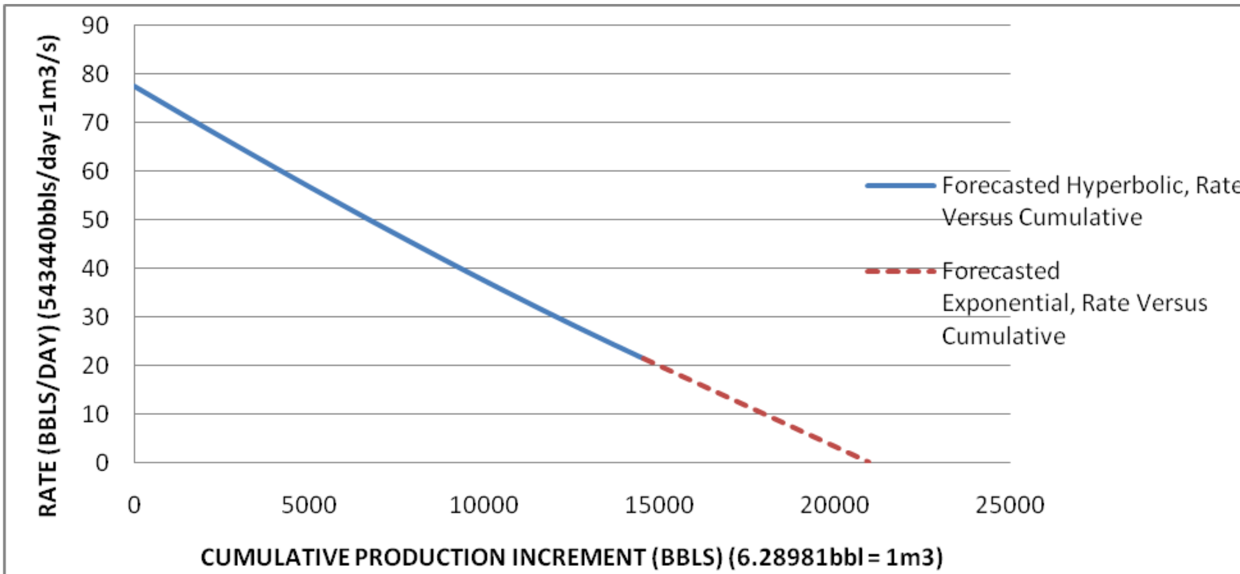

Fig. 3: FORECASTED RATE-CUMULATIVE PRODUCTION INCREMENT RELATIONSHIP

Fig.4: COMPARISON OF CUMULATIVE PRODUCTION INCREMENTS' RESULTS AMONG THE THREE MODELS

Table 1 : Well's Historical Production Data

DATE	TIME (MONTHS) (1month = 2,592,000secs)	HISTORICAL CUMULATIVE PRODUCTION (BBLS) (6.28981bbls = $1m^3$)	HISTORICAL PRODUCTION RATE (BBLS/DAY) ($1m^3/s$=543440bbls/day)
7/15/1975	0	5.09	164.19
8/15/1975	1	24.09	612.9
9/15/1975	2	47.6	783.67
9/15/1976	14	314.54	620.33
10/15/1976	15	333.84	622.58
11/15/1976	16	350.83	566.33
7/15/1977	24	449.82	310.97
8/15/1977	25	459.04	297.42
2/14/1978	31	509.88	257.5
3/15/1978	32	517.69	251.94
4/15/1979	45	616.02	232
6/15/1979	47	631.67	249.33
7/15/1980	60	728.55	174.84
8/15/1980	61	729.88	42.9
3/15/1981	68	736.4	210.32
4/15/1981	69	746.34	331.33
6/15/1982	83	777.74	10
7/15/1982	84	778.03	9.35
1/15/1985	90	778.19	5.16
8/15/1985	97	820.13	122.9

1/15/1986	102	821.35	39.35
2/14/1986	103	827.92	234.64
12/15/1987	125	905.36	117.74
12/15/1988	137	916.05	30.32
1/15/1989	138	916.56	16.45
5/15/1990	142	918.19	52.58
6/15/1990	143	920.04	61.67
1/15/1991	150	949.59	126.77
2/14/1994	175	1016.13	210
12/15/1994	185	1073.15	215.81
1/15/1995	186	1080.29	230.32
7/15/1996	192	1106.02	133.55
8/15/1996	193	1110.5	144.52
9/15/1996	194	1115	150
10/15/1996	195	1119.99	160.97
11/15/1996	196	1122.32	77.67

Table 2 : Forecasted Cumulative Production Data from the Last Historical Rate

FORECASTED TIME (MONTHS AFTER LAST HISTORICAL RATE) (1month = 2,592,000secs)	FORECASTED CUMULATIVE PRODUCTION (BBLS) (6.28981bbls = 1m^3)
0	0
1	2215.552893
2	4167.752185
3	5893.083292
4	7422.290562
5	8781.38399
6	9992.452056
7	11074.3212
8	12043.09341
9	12912.58633
10	13694.69521
11	14399.69167
11.2	14532.24207
12	15036.25694
13	15611.58971
14	16131.46949
15	16601.24108

Table 3 : Comparison of the Developed Model with Long and Davis[3] and Khaled[5] Data Results for Total Cumulative Production Increment

ESTIMATED PARAMETER @ $q_{el} = 100 bbls/month = 6.13377e^{-6} m/s$	LONG AND DAVIS[3]	DEVELOPED MODEL USING LONG AND DAVIS[3] DATA	KHALED[5]	DEVELOPED MODEL USING KHALED'S DATA
FORECASTED CUMULATIVE PRODUCTION INCREMENT, Q_{el} (BBLS) **(6.28981bbls = 1m^3)**	142,010 22,577.8m^3	142,014 22,578.4m^3	142,014 22,578.4m^3	142,224 22,611.8m^3