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Robust H-infinity (H∞) Stabilization of Uncertain Wheeled Mobile Robots

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A.Dolly Mary [α], Abraham T. Mathew [σ], Jeevamma Jacob [ρ]

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I. Introduction

Wheeled Mobile Robots (WMR) can be considered as an isolated system or as a basic unit or a building block of a multi linked articulated vehicle. Alternatively, WMR can be considered an autonomous vehicle in disguise. The development of control strategies for long haul articulated vehicles shall begin by benchmarking the control methods for a single unit of WMR. Non holonomic constraints for control arise out of the kinematics and the uncertainty is consequent to the nonlinearities and parameter variations. Imperfections in the measurements, lead to inaccuracies or noise in the measurement output. So, even if we consider a single unit of WMR, the control issues evolve as a sophisticated problem that needs elaborate treatment. These could be the reasons for active research in the control, stabilization and tracking of WMR for the last several decades.

Non holonomic property of WMR can be attributed to systems using velocity inputs. Such systems cannot autonomously produce a velocity which is transversal to the axle of their wheels. This limitation appears as the non holonomic constraint on the velocity of the system. In order to pursue the modeling, stabilization and tracking of these systems, it is assumed that a wheel has only two degrees of freedom with no sideway slip. This assumption which imposes non holonomic constraint on the motion of the vehicle is called the ideal rolling condition. Unlike robot manipulators, their constraint equations are not integrable. Thus the coordinates cannot be eliminated making the system description with a larger number of coordinates than their degrees of freedom. So our attempt shall be to design a robust controller for such non holonomic WMR systems.

Various control schemes and analysis had been proposed for the WMR system based on the research with the model kinematics. In this kinematic framework there exists no continuous state feedback for WMR as proven by Brockett (1983). Hence research had been focused on other control methods like Lyapunov stability and Feedback linearization. In (Godhavn &.Egeland 1997; Deng &Brady 1993; Sik Shim & Gyeoung Sung 2003; Saso Blazic 2011) investigations were made on the WMR system to design a control that yields asymptotic stabilization of the closed loop for these kinematic models, in the Lyapunov stability analysis framework. The control law achieved global asymptotic stability based on the usual requirement for reference velocity. A drawback of this method was that the stabilizing controllers do not guarantee performance. One of the common methods in the design of controllers for the WMR, employs the dynamic feedback linearization (Giuseppa Oriolo, Alessandra De Luca & Marilena Vendittelli 2002). The control of non linear WMR systems via feedback linearization was achieved, provided the feedback is able to cancel out the nonlinearities. If so, trajectory tracking and setpoint regulation problems can be solved in the case of motion control of WMR. But since it is designed in the kinematic framework, the controller does not take care of uncertainties and input saturations. Uncertainties in the parameters of the system model can invariably affect the stability and performance of the closed loop system and hence have to be addressed. Uncertainty modeling, either structured or unstructured, form an integral part of the robust control design. In the research work as in (Wenjie Dong 2000; Jinbo WU, Guohua XU 2009; Mohammad Ali Sadrnia, and Atiyeh Haji Jafari 2007; Doyle 1985) the need of uncertainty modelling was stressed and the robust controllers were designed for the uncertain systems by adaptive or H-infinity (H_∞) control. Robust approaches for control design renders a fixed controller .

H_∞ has played an important role in the study and analysis of control theory, since its original formulation in an input output setting (Doyle 1989). It is well known that , though conservative , they provide better response in the presence of disturbance than H_2

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optimal techniques. The control requirement is that the response of error to the disturbances with the stabilization controller should be brought to less than γ, where γ is the suboptimal H_∞ limits with $0 < \gamma \leq 1$. For the standard H_∞ problem, a controller exists if and only if unique positive definite solutions to the two Riccati equations exists (John C. Doyle 1989). State space formulas can be derived for all controllers such that the H_∞ norm of the closed loop transfer function is less than γ. Based on the above control criteria, investigations on the WMR were carried out as in (Hong Chen et al 2009; Carlos De Souza, Minyue Fu & Lihua Xie 1993; Zhijian Ji 2006; Wenan Sun and Jun Zhao 2005) for different preset conditions aiming at robust stabilization, performance and tracking. In Hong Chen et al (2009) a moving horizon H_∞ control algorithm for WMR was computed in presence of external disturbances and control constraints. The results showed that the H_∞ controller can reduce tracking errors with desired performances. A H_∞ control as applied to a discrete time systems with time-varying uncertainties was synthesised in Carlos De Souza, Minyue Fu & Lihua Xie (1993).The H_∞ problem was converted to a scaled form for quadratic stability analysis. In Wenan Sun and Jun Zhao (2005) a sufficient condition for hybrid output feedback H_∞ control was derived for uncertainties in the state and input matrices. The controller satisfied guaranteed cost with H_∞ disturbance attenuation γ. Thus the above works highlights the possibility that robust stability and performance measure could be achieved by H_∞ control.

In this paper, a robust H_∞ design is addressed for the uncertain, linear WMR system, accounting for disturbances and model uncertainties. Remaining part of the paper is organised as follows. Section II deals about the H_∞ problem and its necessary robustness criteria for WMR implementation. The robust H_∞ control objective and its design procedure are stated. The procedure aims at a controller such that the closed loop system is internally stable and the effect of disturbance inputs on some of the outputs is attenuated. The norm of system sensitivity which is the effect of the disturbance on the output of the system is to be less than unity. In Section III the nominal modelling of WMR is carried out by incorporating the kinematics and the dynamics of the system. The disturbances along with the control input are assumed to act on the system. In Section IV, the uncertain WMR plant is modelled assuming low frequency disturbance inputs to act on the system and outputs. In Section V the H_∞ controller obtained is evaluated to verify its effectiveness. The closed loop response, asymptotic stability plots and stability and performance margins are plotted.

II. H∞ Problem Statement

The output feedback structure for the proposed H_∞ control scheme is shown in Fig. 1. The system consists of a linear time invariant augmented system P which belongs to the class P of uncertain system. P comprises the nominal model and weighting functions corresponding to model uncertainties and disturbances. The inputs of the system are w and u which represent exogenous and control inputs respectively; z and y would represent the controlled and measured outputs. Considering the input-output relations that are depicted by P, the relation between $[z, y]$ and $[w, u]$ could be written as given in (1)

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{1}$$

Considering the invariance of the plant P and the separability of the variables for the linear setting, the system has been partitioned into the state space for as given in (2)

$$\dot{x}(t) = A\,x(t) + B_1 w(t) + B_2 u(t) \tag{2a}$$
$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \tag{2b}$$
$$y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t) \quad , \tag{2c}$$

where $x \in R^n$ is the state, $u \in R^r$ is the control input, $y \in R^m$ is the observed output, $z \in R^q$ is the controlled output and $w \in R^p$ is the disturbance.

- (A, B_1) is stabilizable and (A, C_1) is detectable.
- (A, B_2) is controllable and (A, C_2) is observable,

The control problem is: with a feedback $u = K(s)y$, find an admissible internally stabilizing control K which would be attenuating disturbances such that the norm of the stable closed loop system from the disturbances to the controlled outputs is less than γ (γ is equal to 1 for optimal and slightly greater than 1 for suboptimal control). The control objective is stated in the mathematical form in (3)

$$\| F_l(P,K) \|_\infty = sup_{\omega \epsilon R} / F_l(P,K)(j\omega) / \ < \gamma \tag{3}$$

The H_∞ solution involves two Hamiltonian matrices (Doyle 1989),

$$H_\infty := \begin{bmatrix} A & \gamma^{-2} B_1 B_1' - B_1 B_1' \\ -C_1' C_1 & -A' \end{bmatrix}$$

$$J_\infty := \begin{bmatrix} A' & \gamma^{-2} C_1' C_1 - C_2' C_2 \\ -B_1 B_1' & -A \end{bmatrix}$$

Theorem (Doyle 1989): There exists an admissible controller such that $\|T_{zw}\|_\infty < \gamma$ iff the following three conditions hold.

i. $H_\infty \in dom(Ric)$ and $X_\infty := Ric(H_\infty) \geq 0$.
ii. $J_\infty \in dom(Ric)$ and $Y_\infty := Ric(J_\infty) \geq 0$.
iii. $\rho(X_\infty Y_\infty) < \gamma^2$

The proof of this theorem is seen in (Doyle 1989). For these conditions being satisfied the suboptimal controller is

$$K_{sub}(s) = \left[\begin{array}{c|c} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{array}\right] \quad (4)$$

where $\hat{A}_\infty := A + \gamma^{-2}B_1B_1'X_\infty + B_2F_\infty + Z_\infty L_\infty C_2$
$F_\infty := -B_2'X_\infty$, $L_\infty := -Y_\infty C_2'$, $Z_\infty := (I-\gamma^{-2}Y_\infty X_\infty)^{-1}$

The resulting closed loop system with a feedback $u = K(s)\,y$ obtained by Linear Fractional Transformation is brought to the form

$$\dot{x}_c(t) = A_C x_c(t) + B_1 w(t) \quad (5)$$
$$z(t) = C_1 x_c(t)$$

where x_c indicates the states of the closed loop system.

Definition: Given a scalar $\gamma > 0$, system (5) is said to be stable with disturbance attenuation γ if it satisfies the following conditions

1. A_C is a stable matrix
2. The transfer function from disturbance w to the controlled output z satisfies,

$$||C_1(sI - A_C)^{-1}B_1||_\infty < \gamma \quad (6)$$

Lemma: Let $\gamma > 0$ be given. The system (5) is stable with disturbance attenuation γ if and only if there exists a symmetric matrix $X_\infty > 0$ such that

$$A_C'X_\infty + X_\infty A_C + \gamma^{-2}X_\infty B_1B_1'X_\infty + C_1'C_1 < 0 \quad (7)$$

Proof:

Let the Lyapunov function for the system be $V(x_c(t)) = x_c(t)'X_\infty x_c(t)$.Making substitutions from (5) ,

$$\dot{V}(x_c(t)) = x_c{}'(t)[A_C{}'X_\infty + X_\infty A_C]x_c(t) + x_c{}'(t)X_\infty B_1 w(t) + w(t)'B_1'X_\infty x_c(t)$$

Let the performance measure corresponding to disturbance attenuation be

$$J = \int_0^\infty [z(t)'z(t) - \gamma^2 w(t)'\,w(t)]\,dt$$
$$J \le \int_0^\infty [z(t)'z(t) - \gamma^2 w(t)'\,w(t) + \dot{V}(x_c(t))]\,dt$$
$$J \le \int_0^\infty \{x_c{}'(t)C_1'C_1x_c(t) - \gamma^2 w(t)'\,w(t) + x_c{}'(t)[A_C{}'X_\infty + X_\infty A_C]x_c(t) + x_c{}'(t)X_\infty B_1 w(t) + w(t)'B_1'X_\infty x_c(t)\}\,dt$$

With

$$w(t)'(B_1'X_\infty x_c(t)) + (B_1'X_\infty x_c(t))'\,w(t) \le \gamma^2 w(t)'w(t) + \frac{x_c{}'(t)X_\infty B_1B_1'X_\infty x_c(t)}{\gamma^2}$$

Since $J \le 0$ to be satisfied,

$$A_C'X_\infty + X_\infty A_C + \gamma^{-2}X_\infty B_1B_1'X_\infty + C_1'C_1 < 0$$

III. Modeling of WMR

A schematic of the WMR in the {X-O-Y} coordinate axis is shown in Fig. 2. The mobile robotic system consists of two rear wheels (A1-A2) which are driving wheels. The driving wheels are driven by two independent DC motors which are the actuators of the left and right wheels. Along with the driving wheels, there is pasive wheel (C). The movement of the WMR is brought about by the differential control of velocity of the wheels A1 and A2. The linear and rotational motions of WMR are facilitated by the control of A1 and A2. F is the projection of mass centre of the robot and P the centre of two front wheels of the robot. l_f is the distance between point P and point F. θ_p is the heading angle of the robot. v_p is the speed at P. ω is the angular speed of WMR.

Let (x_P, y_P) be the coordinates at P and (x_F, y_F) coordinates at F. The x and y axis speed at P which represent the kinematics of the system can be derived as in (8).

$$\dot{x}_P = v_p cos\theta_p \quad (8a)$$
$$\dot{y}_P = v_p sin\theta_p \quad (8b)$$

also P and F can be related as given in (9).

$$x_F = x_p - l_f cos\theta_p \quad (9a)$$
$$y_F = y_p - l_f sin\theta_p \quad (9b)$$

The non holonomic constraint is written in the form of (10). This equation is not integrable, so the fesible trajectory is limited.

$$\dot{x}_F sin\theta_P - \dot{y}_F cos\theta_P - l_f\dot{\theta}_P = 0 \quad (10)$$

Differentiating (9) twice results in (11).

$$\ddot{x}_F = \ddot{x}_P + l_f\dot{\omega}\,sin\theta_p + l_f\omega^2 cos\theta_p \quad (11a)$$
$$\ddot{y}_F = \ddot{y}_P - l_f\dot{\omega}\,cos\theta_p + l_f\omega^2 sin\theta_p \quad (11b)$$

where $\omega = \frac{d\theta_P}{dt}$ and $\dot{\omega} = \frac{d\omega}{dt} = \frac{d^2\theta_P}{dt^2}$

Let T_1 and T_2 be the driving torques on left and right front wheels of the differential WMR. Then $u_1 = T_1 + T_2$; $u_2 = T_1 - T_2$. The dynamics of the WMR are represented in (12).The accelerations of the system depend not only on the inputs but are acted upon by disturbances. Let $u(t) = [u_1\, u_2]'$ be the control input vectors, $w_0(t) = [w_1\, w_2]'$ the input disturbances representing external forces and torques in the direction of v_p and ω.

$$\dot{v}_p = \beta_1 u_1 + \beta_3 w_1 \quad (12a)$$
$$\dot{\omega} = \beta_2 u_2 + \beta_4 w_2 \quad (12b)$$

Here $\beta_1 = \frac{1}{rM}$, $\beta_2 = \frac{b}{rI}$, $\beta_3 = \frac{1}{M}$, $\beta_4 = \frac{1}{I}$ with r representing the radius of the wheel, M the mass of the WMR, I the moment of inertia with respect to the centre of mass F of the WMR and $2b$ the length of wheel axis.

The state space equations of the dynamical systems can be derived as in Wenping Jaing (2008) by the substitution of (12) and derivative of (8) in (11). The state variables of the system are taken as $x = [\theta_p\ \dot{x}_F\ \dot{y}_F\ \dot{\theta}_P]^T$ angular displacement, x axis speed at F, y axis speed at F and angular speed. Table 1. shows the nominal values of the WMR parameters.

Table 1 : Parameters of the WMR system

Parameter	Nominal value
r (m)	0.105
M (kg)	25
I (kg m^2)	0.5512
2b (m)	0.41
l_f(m)	0.09

Thus the state space representation of the nominal open loop system is in the form of (13)

$$\dot{x} = A\,x + B_1 w_o + B_2 u \qquad (13a)$$

$$y = C_2 x \qquad (13b)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -0.0175 & -0.01 \\ 0 & 0.0175 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0.04 & 0 \\ 0 & -0.163 \\ 0 & 1.814 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 & 0 \\ 0.381 & 0 \\ 0 & -0.319 \\ 0 & 3.542 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The nominal open loop system is unstable with all the poles on the imaginary axis

IV. WMR H∞ Control Scheme

The nominal model in (13) do not provide an exact representation of the WMR system. Modelling errors, parameter uncertainties and nonlinearities of the system which were not taken into account do not guarantee a robust controller. For the WMR system, the robot's parameters particularly the mass and inertia are subjected to variations and hence uncertainties arise in system modeling. Also the outputs of the system are subjected to disturbances. Hence uncertainties should be taken into account when designing a robust controller, so that we get satisfactory control over a wider range of the operating variables. It is noted that the input disturbance w_0 was modeled along with the control inputs of the system. To accommodate the parameter variations and nonlinearities w_0 is acted on by unstructured uncertainty weighting W_W. The H_∞ control to the WMR as in Fig. 3 shows the feedback structure of the uncertain plant. Disturbances inputs can act on the model and the output. Let it be $w_i(t)$ and $w_y(t)$ respectively. These disturbances act on the plant in the low frequency range. Let W_W and W_P be the low frequency disturbances weightings for the above inputs.

Then

$$w_o(t) = W_W w_i(t) \qquad (14)$$

With uncertainty model weighting being selected as

$$W_W = \frac{6(s+60)}{(s+0.3)} = C_W(sI - A_W)^{-1}B_W + D_W \qquad (15)$$

$[A_W, B_W, C_W, D_W]$ which are the state space matrices of the transfer function (15) can be obtained.

The output disturbance weighting W_P is selected based on the maximum steady state error, bandwidth etc of (16). Fig. 4. shows the inverse of performance weighting that is acting on the system.

$$W_P = \frac{s/_M + \omega_B}{s + \omega_B A} \qquad (16)$$

ω_B is the bandwidth frequency, A is the maximum steady state tracking error and M the maximum peak magnitude of sensitivity. For the WMR design performance weighting W_P is selected as in (17).

$$W_P = \frac{(s+77)}{(s+24)} = C_P(sI - A_P)^{-1}B_P \qquad (17)$$

$[A_P, B_P, C_P, D_P]$ are the state space matrices of the transfer function (17).

Next we consider the regulated or controlled output as shown by z_1 and z_2. Stabilization and regulation of the state variables corresponds to z_1 and error on u corresponds to z_2.

$$z_1 = \varphi x \qquad (18)$$

φ is a 3 by 3 diagonal matrix corresponding to first 3 state variables to be regulated.

$$z_2 = \rho u \qquad (19)$$

ρ is a 2 by 2 diagonal weighting matrix on u.

Let x_w and x_p be the states of the low frequency disturbances W_W and W_P.

Then $$w_o = C_W x_W + D_W w_i \qquad (20a)$$

and $$\dot{x}_W = A_W x_W + B_W w_i \qquad (20b)$$

$$\dot{x}_P = A_P x_P + B_P w_y \qquad (20c)$$

Combining the nominal model (13) and the above uncertain states, the augmented model is brought to the form of (2).The state space representation is shown in (21), the states of the uncertain plant are $[x^T\ x_w^T\ x_p^T]$, $w = [w_i^T\ w_y^T]$ and $z = [z_1^T\ z_2^T]$.

$$\begin{bmatrix} \dot{x} \\ \dot{x}_W \\ \dot{x}_P \end{bmatrix} = \begin{bmatrix} A & B_1 C_W & 0 \\ 0 & A_W & 0 \\ 0 & 0 & A_P \end{bmatrix} \begin{bmatrix} x \\ x_W \\ x_P \end{bmatrix} + \begin{bmatrix} B_1 D_W & 0 & B_2 \\ B_W & 0 & 0 \\ 0 & B_P & 0 \end{bmatrix} \begin{bmatrix} w_i \\ w_y \\ u \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = \begin{bmatrix} \varphi & 0 & 0 \\ 0 & 0 & 0 \\ -C_2 & 0 & -C_P \end{bmatrix} \begin{bmatrix} x \\ x_W \\ x_P \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \rho \\ 0 & -D_P & 0 \end{bmatrix} \begin{bmatrix} w_i \\ w_y \\ u \end{bmatrix}$$

The regulated matrix corresponding to the states and input are selected as

$$\varphi = \begin{bmatrix} .1e^{-3} & 0 & 0 \\ 0 & .4e^{-3} & 0 \\ 0 & 0 & 1e^{-4} \end{bmatrix} \quad , \quad \rho = \begin{bmatrix} 2e^{-2} & 0 \\ 0 & .5e^{-2} \end{bmatrix}.$$

It is noted that (A,B_1), (A,B_2) are controllable and (A,C_2) observable. Hence the uncertain plant that represents the WMR also satisfies the requirement for the existence of the controller.

V. Controller Evaluation

With the uncertain plant in the general form of (2) and the controllability and observability conditions being satisfied, the controller was simulated in MATLAB ® as per the theorem in Section II. The suboptimal H∞ controller was obtained in the form of (4) with a γ value of 1.09. The Linear Fractional Transformation of the augmented system and the controller yields a closed loop system in the form of (5). This system with inputs as disturbances w and outputs z represent the robust closed loop system. Fig. 5. shows the magnitude of the uncertain closed loop plant for varying frequencies. The maximum bound of the closed loop system which indicates the sensitivity of the system guarantees robustness for values less than unity. The maximum value of the WMR system was around 0.7.

Fig. 6. shows the impulse response of the uncertain plant from output disturbances w_y to z_1 . The variation in the output z_1 because of the disturbance was negligible ascertaining that the control design is robust. To verify the internal stability with the H∞ controller, the asymptotic stability of the output of the closed loop system was plotted as in Fig. 7. Both the outputs tend to zero in minimum time thus proving that the controller stabilizes the WMR system. To further evaluate the robustness of the above control design, the stability and performance margins for the closed loop system were calculated subject to variation in the parameters of the robotic system. The mass M of the system was varied between [20 : 30] and moment of inertia I between [0.6 : 0.7]. The stability analysis showed that the uncertain system is robustly stable to modeled uncertainty. It can tolerate up to 500% of the modeled uncertainty. The sensitivity with respect to uncertain element I is 67% and M 129% for robust stability. The robust performance margin for the WMR was calculated subject to the above variations. The uncertain system achieves a robust performance margin of 1.32. A model uncertainty exists of size 131% resulting in a performance margin of 0.761 at 0.0169 rad/sec. For robust performance, the sensitivity with respect to uncertain element I is 17% and M 14% . The above results clearly indicated that the controller design is robust.

VI. Conclusion

In this paper the robust control design of a single unit Wheeled Mobile Robot was presented. Modeling of the WMR system was done taking into account the kinematics and dynamics to achieve a linearized model for these systems. The exogenous inputs in the form of model uncertainties and output disturbances were assumed to act on the system and the augmented plant was formulated. The robust internally stabilizing H∞ controller was obtained. The above design rendered a controller such that the closed loop system was internally stable and the effect of disturbances and model uncertainties on some of the outputs was attenuated. The asymptotic stability plot, disturbance response and Robust Stability and Performance margins were found to be satisfactory.

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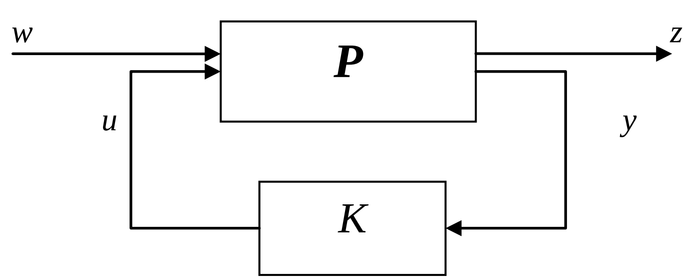

Fig. 1 : H_∞ Feedback control

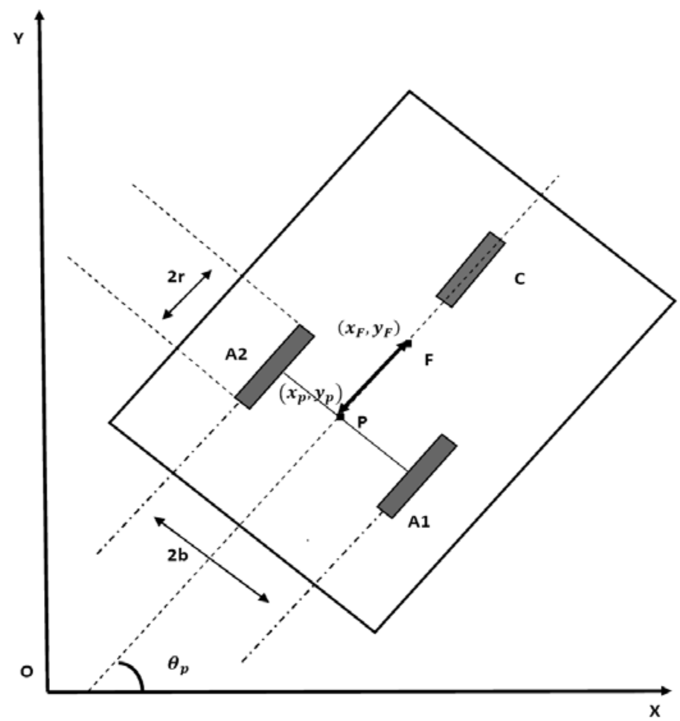

Fig. 2 : Model of a Wheeled Mobile Robot

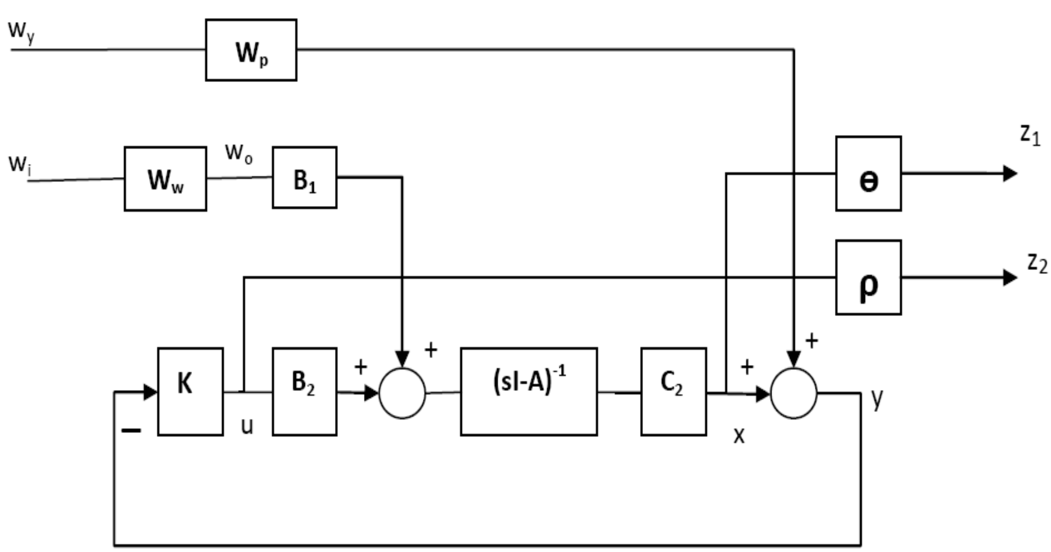

Fig. 3 : Augmented structure of uncertain plant

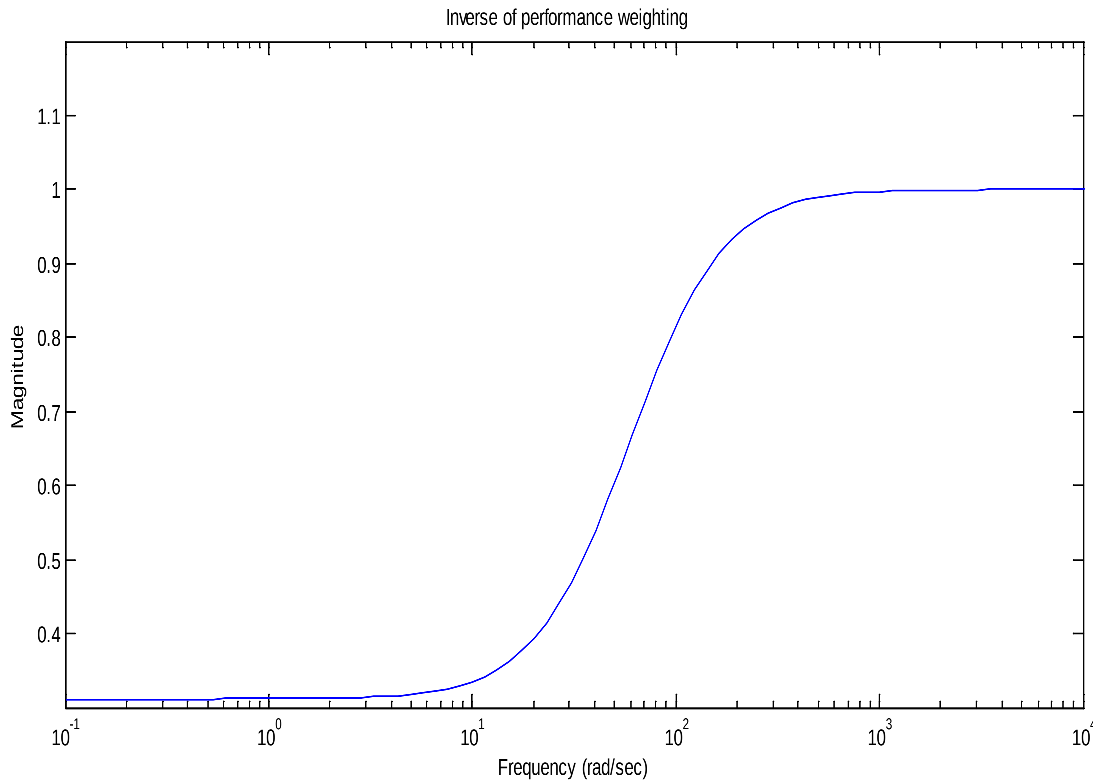

Fig. 4 : Inverse of performance weighting W_P

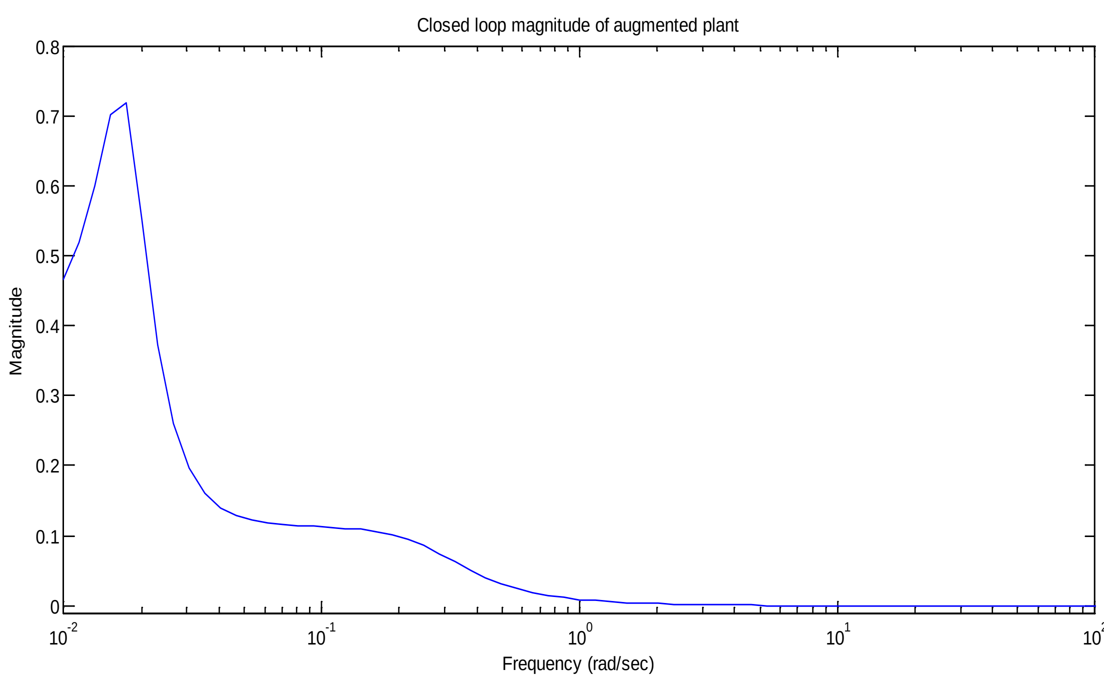

Fig. 5 : Closed loop magnitude of the uncertain plant

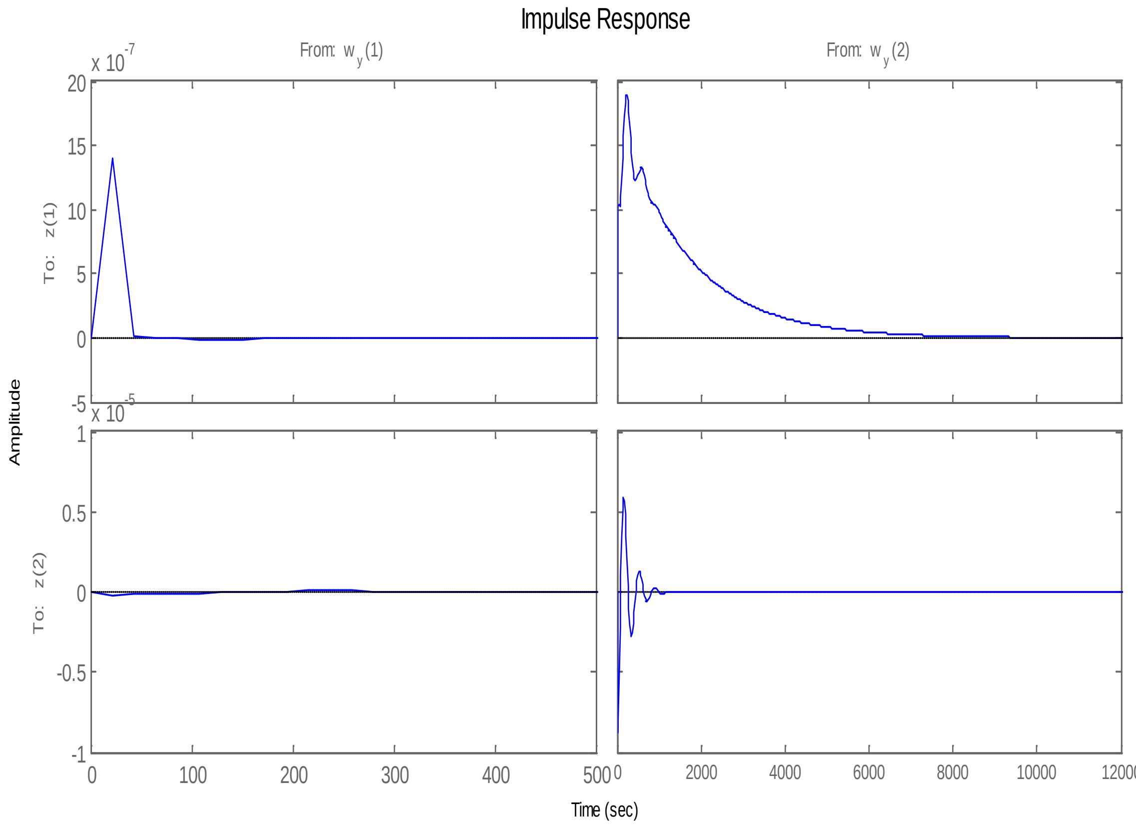

Fig. 6 : Impulse response of the uncertain plant

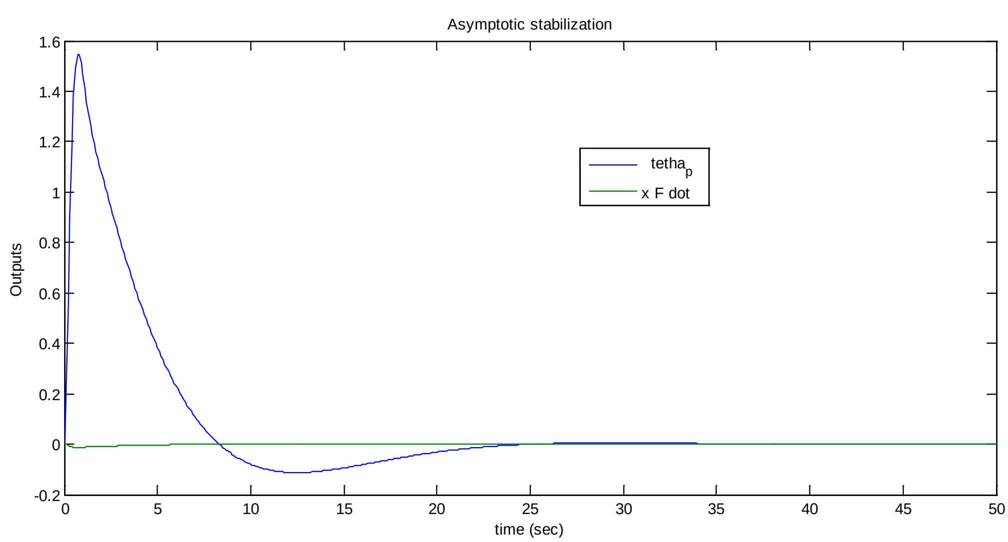

Fig. 7 : Asymptotic stabilization of the closed loop system