Design of multiplierless 2-D sharp wideband filters using FRM and GSA

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Abstract - One of the efficient and most popular technique for designing sharp 1-D linear phase FIR filters is the Frequency Response Masking (FRM) approach. It is an effective method for the design of high speed, low power, sharp FIR digital filters with a small number of non-zero coefficients. Very recently, a modified McClellan transformation (T1 and T2) is proposed (Jie-Cherng Liu and Yang-Lung Tai, 2011) for converting 1-D linear phase FIR digital filter to 2-D digital filter, in which the transformation is completely multiplierless. So the resulting 2-D filter contains the same number of multipliers as the 1-D digital filter. In this paper, our aim is to design a 2-D linear phase FIR filter which is completely multiplierless, by designing a multiplier free 1-D linear phase FRM FIR filter and using multiplierless transformation.

Keywords: Frequency Response Masking, T1 and T2 Transformations, Canonic Signed-Digit (CSD), Gravitational Search Algorithm (GSA), Two-Dimensional (2-D) Filter.

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Abstract - One of the efficient and most popular technique for designing sharp 1-D linear phase FIR filters is the Frequency Response Masking (FRM) approach. It is an effective method for the design of high speed, low power, sharp FIR digital filters with a small number of non-zero coefficients. Very recently, a modified McClellan transformation (T1 and T2) is proposed (Jie-Cheng Liu and Yang-Lung Tai, 2011) for converting 1-D linear phase FIR digital filter to 2-D digital filter, in which the transformation is completely multiplierless. So the resulting 2-D filter contains the same number of multipliers as the 1-D digital filter. In this paper, our aim is to design a 2-D linear phase FIR filter which is completely multiplierless, by designing a multiplier free 1-D linear phase FRM FIR filter and using multiplierless transformation. This paper presents a novel population-based optimization algorithm called Gravitational Search Algorithm (GSA) (Rashedi, 2009) for the design and optimization of FRM FIR digital filter whose coefficients are synthesized in the conventional Canonical Signed-Digit (CSD) format. Simulation results show that GSA gives a better performance than the Genetic Algorithm (GA).

Keywords: Frequency Response Masking, T1 and T2 Transformations, Canonical Signed-Digit (CSD), Gravitational Search Algorithm (GSA), Two-Dimensional (2-D) Filter.

I. Introduction

The field of the two dimensional filters and their design methods have been investigated by many researchers for more than three decades and have been deployed in a variety of application scenarios. Different techniques exist for the design of 2-D linear phase FIR filters which include windowing, frequency sampling, linear programming and Chebyshev techniques (Lim, 1990). These techniques produce a better approximation to an ideal response for a given filter, but the design of the filters requires large amount of computation and it becomes complex for higher order filters. Another method called Frequency transformation method (Lim, 1990) for the design of 2-D linear phase FIR filter from a 1-D linear phase FIR filter, is simple and has high computational efficiency. As the time required by the transformation method is less, it helps to design higher order filters with modest computation time, meeting the filter specifications closely. For the implementation of a filter whose impulse response is (N×N) point, N2 multiplications per output value are required using direct convolution, but a filter obtained by McClellan transformation can be implemented with a number of multiplications per output value which is proportional to N (Mersereau, 1976). Very recently Liu and Tai (Jie-Cheng Liu and Yang-Lung Tai, 2011) have proposed two multiplierless transformation (T1 and T2) which are capable of designing a 2-D filter with circular contour even at wideband radius. This is bestowed with the feature that, using a single transformation, a 1-D filter can be converted to its 2-D equivalent without any optimization procedures or complicated computations.

In this paper, we propose the design of a sharp multiplierless 2-D circularly symmetric, wideband filter using the transformations proposed in (Jie-Cheng Liu and Yang-Lung Tai, 2011). Sharpness is achieved by using FRM for the design of the 1-D filter. FRM technique provides a cost - effective way for the design of high speed, low power FIR digital filters, which leads to very low hardware complexity, round off noise and coefficient sensitivity (Y. C. Lim, 1986). The 1-D FRM filter is made multiplierless by representing it in the Canonical Signed Digit (CSD) space. The T1 and T2 transformations are completely multiplierless. When the digital filter coefficients are quantized to the Signed-Power-Of-Two space (SPT), multipliers can be replaced by a series of shift and add operations (R.Hartley, 1996) during the implementation. Among the various SPT forms, the CSD representation is a minimal one. The advantages of CSD representation are that it decreases the number of additions/subtraction needed and handle negative multipliers (R.Hartley, 1996). After the quantization of the infinite-precision multiplier coefficient values, the resulting 1-D FRM FIR digital filter will no longer meet the initial design specifications. As a result, optimization methods have to be introduced to obtain finite precision digital filters that satisfy the design specifications closely. Over the last decades, there has been a growing interest in algorithms inspired by the behavior of natural phenomena (D.H. Kim, 2007), (K.S. Tang, 1996), (M. Dorigo, 1996). There are different heuristic algorithms in the literature which resemble various physical and biological processes, such as Genetic Algorithm, Simulated Annealing (S. Kirkpatrick, 1983), Artificial Immune System (J.D. Farmer, 1986), Ant Colony Search Algorithm, Particle Swarm Optimization (J. Kennedy, 1995) etc. for solving different optimization problems. In this paper, a new population based algorithm named Gravitational Search Algorithm (GSA) (Rashedi, 2009) has been used, which is based on Newtonian law of gravity and law of motion. We propose a discrete optimization based on modified GSA. This algorithm is modified in such a way that during the
process of optimization, the candidate solution turns out to be integers. This multiplierless 1-D filter is in-turn converted to a 2-D multiplierless filter by using multiplierless transformation like T1 or T2. It is found that the magnitude response specifications using this algorithm are better than those obtained with other optimization algorithms like integer coded GA (Manoj, 2009). The paper is organized as follows. Section II gives an overview of frequency response masking. In Section III, the T1 and T2 transformation is briefed. Section IV gives an overview of the GSA algorithm. The design of 1-D multiplierless FRM linear phase filter is discussed in Section V. Section VI illustrates the proposed design of multiplier-less 2-D FRM filter using the modified GSA algorithm. The results and discussions are done in Section VII and Section VIII gives the conclusions.

II. Frequency Response Masking

As the filter length is inversely proportional to the width of the transition band, higher order filters are needed for the implementation of narrow transition width FIR filters. Frequency response masking technique is an effective method for the design of high speed, low power, sharp FIR digital filters. It is suitable for implementing linear phase, arbitrary passband sharp FIR filters (Y. C. Lim, 1986) with a few number of non-zero coefficients. The computational complexity of the FRM is considerably small compared with the complexity of the filter designed using the traditional minimax approach having equivalent frequency response. Since multipliers are the most power consuming elements in a filter, reducing the number of multipliers is equivalent to reducing the power consumption and chip area. Due to these advantages, FRM has been deployed in a wide range of applications like FPGA, audio processing, beam-forming etc (Lu, W.S, Hinamoto, 2008). The basic block diagram of the overall FRM filter using several subfilters is shown in Fig(1).

The narrow transition width of FRM results from the interpolated version of prototype filter Fa(zM), derived by replacing each delay element of Fa(z) by M delay elements and Fc(zM) is its complementary version obtained by subtracting the output of Fa(zM) from a suitably delayed version of the input. There are two parallel branches each of which is composed of an interpolated model filter in cascade with masking filters FMa(z) and FMc(z) respectively. Interpolation leads to the imaging of the frequency response along with reduction of the passband and transition band by a factor of M. Masking filters are used to select the useful part of Fa(zM) and Fc(zM). Addition of two masked responses gives the response of a sharp wideband FIR filter.
For T2 transformation
\[ f(\omega_1, \omega_2) = g_1(\omega_1, \omega_2) \times f_2(\omega_1, \omega_2) - 1 \]

where
\[ f_2(\omega_1, \omega_2) = \cos^2(\omega_1/2) + \cos^2(\omega_2/2) \]

Besides, the deployment of k-th order T1 and T2 transformations, permits the reduction in the order of the 1-D prototype filter by a factor \( \sqrt{k} \) and \( \sqrt{k + \frac{1}{2}} \) respectively. Fig.2 and 3 show the frequency mapping of T1 and T2 transformations respectively.

**Fig.2**: Frequency mapping of T1 transformation.

**Fig.3**: Frequency mapping of T2 transformation.

**IV. Gravitational Search Algorithm**

Rashedi, proposed a new heuristic optimization algorithm named GSA in 2009. GSA is based on Newtonian Law of gravity and motion. GSA can be considered as an artificial world of masses, where every mass represents a solution of the problem. In this method, agents are considered as masses and every mass attracts each other by the gravity force and this force causes a movement of all objects towards the object with heavier mass which is the optimum solution. Exploration and exploitation phase are carried out using the rules of gravity and mass interaction. The members of a population-based search algorithm undergo three steps in each iteration to realize the concepts of exploration and exploitation: self-adaptation, cooperation, and competition. In the self-adaptation step, each member (agent) improves its performance. In the cooperation step, members collaborate with each other by information transferring. Finally, in the competition step, members compete to survive. The heavy masses which correspond to a good solution move more slowly than the lighter ones which guarantee the exploitation.

In GSA, each mass has four specifications: position in \( d \)-th dimension, inertial mass, active gravitational mass, and passive gravitational mass. The position of a mass corresponds to the solution of the optimization problem, and its gravitational and inertial masses are determined by the fitness function. Each mass represents a solution and the algorithm is navigated by properly adjusting gravitational and inertial masses. As the algorithm proceeds, the masses will be attracted by the heaviest mass which gives an optimum solution in the search space. GSA provides a good optimum solution for the problem in a higher dimensional search space.

**V. Proposed Design of 1-D Multiplierless FRM FIR Filter**

a) **Problem Statement**

The final objective of the work is to design a fully multiplierless 2D sharp filter. To this end, 1-D FRM
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filter in the CSD space is mapped to the 2-D scenario using T1 and T2 transformations. First of all the 1-D continuous coefficient FRM filter is to be designed. The advantage of using FRM for the design of sharp FIR filter is the enormous saving in the computational complexity. If all sub-filters have linear phase responses with Na and Nc either both even or both odd and (N-1)/M even, where N is the order of prototype filter and M is the interpolation factor, then the FRM filter has linear phase response. These conditions stated in (Lu, W.S. Hinamoto 2003) have been used in our work to design the FRM filter with the linear phase. Once the band edges of the 1-D filter are obtained from the 2-D specifications, the continuous coefficient FRM filter is designed. The various sub-filters Ha(z), Hma(z) and Hmc(z) are designed using Parks-McClellan algorithm. The optimum interpolation factor is found in such a way that the total number of multipliers for the realization of the FRM filter is minimized. Direct rounding to the CSD space with restricted SPT terms causes the degradation of the frequency response specification of the 1-D FRM filter. This calls for the use of a discrete optimization. In this paper, we propose a new discrete optimization approach using the modified GSA. GSA has emerged as a good optimization tool and it offers relatively better performance compared to similar meta-heuristic algorithm. GSA is modified in such a way that during the exploitation and exploration phase, the candidate solution turns out to be integers. The design of 1-D FRM filter is modeled as a minimization problem as given in (M. Manuel, E. Elias, 2012). Here the objective function is defined as the approximation error as defined below

$$F(x) = \|H_d(\omega) - H(\omega, x)\|_2$$

where $H_d(\omega)$ represents the frequency response of the infinite precision FRM filter and $H(\omega, x)$ is the response of the optimized filter. $x$ is obtained by concatenating the filter coefficients of the various sub-filters. To further reduce the computational complexity of the resulting discrete FRM filter, a constraint is also added to reduce the total number of SPT terms. The constraint is given by $n(x) \leq n_b$, where $n(x)$ denotes the average number of sub-filter coefficients, and $n_b$ represents the upper bound.

**b) Encoding of Filter Coefficients**

Since multipliers are the most power consuming elements in digital filters, multiplierless implementation of filters will lead to enormous saving in power and hardware complexity. Multiplierless implementation of linear phase FIR filters are possible by representing the filter coefficients in the CSD format. The redundancy in the multiplier coefficient representation caused by the non-unique nature of the SPT representation is removed by the use of CSD number system, which represents the multiplier coefficient values uniquely by reducing the number of non-zero digits. The multipliers can be represented by a series of shifts and additions or subtractions.

An infinite precision multiplier coefficient $x$ can be represented in CSD format as:

$$x = \sum_{i=1}^{B} b_i 2^{R-i}$$

where $B$ represents the wordlength of the CSD number and $R$ represents a radix-point in the range $0 < R < B$. The CSD number obeys the following constraints

$$b_i \in \{1,-1,0\}$$

$$b_i \times b_i + 1 = 0$$

In our problem, the filter coefficients are encoded as signed integer indices of the look up table (LUT) locations of the nearest CSD equivalent as done in (M. Manuel, E. Elias, 2012). For this purpose, look up table is created as per the details provided in (M. Manuel, E. Elias, 2012). There are four fields for the LUT, namely CSD representation, decimal equivalent, index and the number of SPT terms. 2 bits are allocated for the integer part and 12 bits are provided for the fractional part. If a filter coefficient is negative, it is encoded as the negative of the index of its positive counterpart. Thus the candidate solution in the optimization problem turns out to be integers. In this work, a variable number of SPT terms have been used for obtaining the optimized filter. This allocation has a significant advantage compared to that using fixed terms (Lim, 1999). The look up table approach avoids the use of restoration algorithm (Ashrafzadeh, 1997) as needed in the ternary encoding of the CSD filter coefficients in the CSD space.

c) Proposed Modified GSA algorithm for the design of 1-D multiplierless FRM FIR filter

In our work, GSA has been tailor made to be suitable for the discrete optimization problem proposed. The various steps are briefed below.

**Step 1: Initialization of the agents**

An agent is constituted by concatenating the CSD encoded filter coefficients of the sub-filters. Let $N$ number of agents constitute a GSA system. Initialize the position of these agents by randomly perturbing the CSD encoded filter coefficients. Consider the position of the $i$-th agent

$$x_i = (x_i^1, x_i^2, \ldots, x_i^d, \ldots, x_i^n)$$

for $i=1,2,\ldots,N$. $i$ indicates the position of the $i$-th agent in the $d$-th dimension and $n$ is the dimension of the search space. In our proposed design, each represents a typical encoded filter coefficient in the CSD space.
Step 2: Fitness evaluation and the best fitness computation

In our problem, the fitness function is identified with the approximation error as given by eq.(9). Compute the fitness for all agents in each iteration and also find the best and worst fitnesses at each iteration as given below. Since our optimization problem is a minimization type, we have

\[ \text{best}(t) = \min_{j \in \{1,2,\ldots,N\}} \text{fit}_j(t) \]  
\[ \text{worst}(t) = \max_{j \in \{1,2,\ldots,N\}} \text{fit}_j(t) \]  

Step 3: Compute the gravitational constant G

Due to the effect of the decrease in the gravity, the true value of the gravitational constant depends on the age of the universe and there is a decrease in the gravitational constant G with the age. Gravitational constant G at each iteration ‘t’ is computed by the following equation (R. Mansouri, 1999)

\[ G(t) = G_0 e^{-\frac{\alpha t}{T}} \]  

where \( G_0 \) is set to 100, \( \alpha \) is taken as 20 and \( T \) is the total number of iterations.

Step 4: Calculate the mass of the agents

For each filter coefficient, the gravitational and inertial masses are calculated at each iteration by the following equations. Consider \( M_{ai} = M_{pi} = M_{ii} \), for \( i = 1,2,\ldots,N \)

\[ m_i(t) = \frac{\text{fit}_i(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)} \]  
\[ M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \]  

where \( M_{ai}, M_{pi}, \) and \( M_{ii} \) represents the active gravitational mass, passive gravitational mass and inertia mass respectively of the \( i \)-th agent (Rashedi, 2009).

Step 5: Compute the acceleration of the agents

According to the law of motion, the acceleration of the \( i \)-th agent at time \( t \) in the \( d \)-th dimension is given by

\[ a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} \]  

\( F_i^d(t) \) is the total force acting on agent ‘\( i \)’ in a dimension of \( d \). To give a stochastic nature to the algorithm, it can be expressed as a randomly weighted sum of the \( d \)-th components of the forces exerted from other agents.

\[ F_i^d(t) = \sum_{j=1, i \neq j}^{N} \text{rand}_j F_{ij}^d(t) \]  

\( \text{rand}_j \) is a random number in the interval \([0,1] \).

For controlling the exploration and exploitation, which decreases the performance of GSA, ‘Kbest’ agents can be selected which attract each other. ‘Kbest’ is the set of the first \( k \) agents with the best fitness value and the biggest mass. \( F_{ij}^d(t) \) is the force acting on mass ‘\( i \)’ from mass ‘\( j \)’ at time \( t \) in the \( d \)-th dimension.

\[ F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{ai}(t)}{R_{ij}(t)^{d+1}} \left( x_i^d(t) - x_j^d(t) \right) \]  

\( R_{ij}(t) \) is the Euclidian distance between two agents \( i \) and \( j \), \( \varepsilon \) is a small constant.

Step 6: Update the velocity and position of the agents

The velocity of agent in the next iteration \((t+1)\) can be represented as a fraction of its current velocity added to its acceleration. The new position and velocity of the agents can be calculated as

\[ v_i^d(t+1) = \text{rand}_i \times v_i^d(t) + a_i^d(t) \]  
\[ x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \]  
\[ x_i^d(t+1) = \lfloor x_i^d(t+1) \rfloor \]  

\( \lfloor \cdot \rfloor \) corresponds to the rounding to lower value.

This operation ensures that the new candidate solution turns out to be integers. Yet another modification is done to the new position so that any encoded filter coefficient falls within the boundary of the CSD look up table. If \( x_i^d(t+1) > v_u \) then \( x_i^d(t+1) = v_u \) and if \( x_i^d(t+1) < v_l \) then \( x_i^d(t+1) = v_l \) where \( v_u \) and \( v_l \) represent the upper and lower bound of the CSD look up table respectively.

Step 7: Repeat step 2-6 until the iterations reach its limit. The best fitness is obtained and the position of the corresponding agent is the global solution. Obtain the best solution and it corresponds to the solution with the least approximation error. The best solution is decoded using the look up table to obtain the optimal FRM filter in the CSD space.

VI. Proposed Design of 2-D Multiplierless Filter

In this paper, the design of sharp wideband multiplierless 2-D linear phase filter is proposed. The block diagram of the proposed design is shown in Fig.5. From the specification of the required 2-D filter using inverse mapping, obtain the band-edges of the 1-D FRM filter. Then the continuous coefficient 1-D FRM filter...
is designed. The continuous coefficient FRM filter is converted to the CSD space using the proposed modified GSA algorithm as mentioned in section V.C. This 1-D sharp multiplierless filter is converted to the 2-D scenario using the frequency transformation named T1 and T2. T1 and T2 transformations provide an efficient method for the design of 2-D filter with circular contour and ensures better circularity as the order of the transformation increases. As the 1-D linear phase FRM FIR filter is made multiplierless, the transformations result in a 2-D filter which is multiplier free. The realization of a 2-D filter is given in Fig.4. In the realization h(n) represents the 1-D filter coefficient in the CSD space and f(ω₁, ω₂) corresponds to either the T1 or T2 transformation. Thus the realization is totally multiplierless.

Case-1

By using T1 transformation with k=1, ωₚ=0.8 and ωₛ=0.81. The bandedges of the 1D prototype filter to be designed are found as Ωₚ=0.7944, Ωₛ=0.8013. Proposed GSA was used to design the 1-D multiplierless FRM filter and the maximum number of iterations and the number of agents are taken to be 100 and 50 respectively. GA (Manoj, 2009) was also used for the above design for comparison purpose and parameters are Popkeep fraction= 0.2, MuteRate= 0.01, Elite count=5 and Iterations=100. Fig.6 shows the magnitude response of the continuous coefficient 1-D FRM filter, the magnitude response of the 1-D filter before and after GSA optimization are shown in Fig.7. The magnitude response and contour of the 2-D multiplierless lowpass filter using T1 transformation (k=1) is shown in Fig.8 and 9 respectively.

VII. Simulation Results

The proposed method was used to design a 2-D sharp wideband lowpass filter whose design specifications are given below:

\[ H(\omega₁, \omega₂) = \begin{cases} 1 \pm \delta_p, & 0 \leq \sqrt{\omega₁² + \omega₂²} \leq 0.8\pi \\ \pm \delta_s, & 0.81\pi \leq \sqrt{\omega₁² + \omega₂²} \leq \pi \end{cases} \]

where \( \delta_p=\delta_s=0.01. \)

Comparison of the performance in terms of maximum passband ripple and minimum stopband attenuation are done in Table.1 for the two optimization techniques GSA and GA. It is found that our modified GSA gives better results than the GA.
Table 1: Performance Comparison of optimized FRM filter

<table>
<thead>
<tr>
<th>Parameters of Infinite-precision</th>
<th>CSD rounded</th>
<th>GSA</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRM filter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum pass-band ripple</td>
<td>0.0904</td>
<td>0.1237</td>
<td>0.0939</td>
</tr>
<tr>
<td>Minimum stop-band attenuation</td>
<td>42.64</td>
<td>33.7</td>
<td>40.68</td>
</tr>
</tbody>
</table>

Fig.8: Magnitude response of the 2-D low-pass filter using T1 transformation

Fig.9: Contour plot of the 2-D low-pass filter

Case 2

T2 transformation was also applied with the same specifications as above. The corresponding 1-D prototype filter specifications are $Q_p = 0.8387\pi$.

Fig.10: Magnitude response of the 2-D low-pass filter using T1 transformation

Fig.11: Magnitude response of the 2-D low-pass filter using T2 transformation (k=1)

Table 2: Performance Comparison of optimized FRM filter

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>FRM filter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum pass-band ripple</td>
<td>0.0992</td>
<td>0.1341</td>
<td>0.0962</td>
</tr>
<tr>
<td>Minimum stop-band attenuation</td>
<td>42.48</td>
<td>36.26</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Fig.10: Magnitude response of the continuous coefficient FRM filter

Fig.11: Magnitude response of FRM filter direct rounding to CSD and GSA optimized

Table 2: Performance Comparison of optimized FRM filter

<table>
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<td>36.26</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Performance comparison in terms of passband ripple and stopband attenuation for T2 transformation are done in Table II.
A new approach for the design of 2-D multiplierless sharp FIR filter is proposed. First of all a 1-D sharp FIR filter is designed using FRM technique. It results in a 1-D filter with sparse coefficients. The resulting 1-D filter is converted to the CSD space using a new discrete optimization. This optimization is based on a modified GSA. GSA has been modified in such a way that during the course of optimization the candidate solution turns out to be integers. This multiplierless 1-D filter is in-turn transformed to 2-D domain using the recently proposed T1 and T2 transformations. The resulting approach for the design of 2-D multiplierless filter is bestowed with the features of reduced computational complexity and computational time.

Fig.12: Magnitude response of the 2-D low-pass filter using T2 transformation

Fig.13: Contour plot of the 2-D low-pass filter

VIII. Conclusion

REFERENCES Références Referencias


19. Jae S. Lim (1990),“Two dimensional Signal and image processing”, Prentice - Hall, New Jersey, USA.
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