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Numerical Methods

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Highlights

- Quintic B-spline Collocation
- Polytrophic and Isothermal sphere
- Fractional Differential Equations
- Threshold Acceptance Algorithm

Unmatched Sample of Geometry
Louvre Pyramid

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Quintic B-spline Collocation Method For Sixth Order Boundary Value Problems

By K.N.S. Kasi Viswanadham & Y. Showri Raju
National Institute of Technology, Warangal, India

Abstract - A finite element method involving collocation method with quintic B-splines as basis functions have been developed to solve sixth order boundary value problems. The sixth order and fifth order derivatives for the dependent variable are approximated by the central differences of fourth order derivatives. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain. The proposed method is tested on several linear and non-linear boundary value problems. The solution of a non-linear boundary value problem has been obtained as the limit of a sequence of solutions of linear boundary value problems generated by quasilinearization technique. Numerical results obtained by the present method are in good agreement with the exact solutions or numerical solutions available in the literature.

Keywords : collocation method, quintic B-spline, basis function, sixth order boundary value problem, absolute error.

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Quintic B-spline Collocation Method For Sixth Order Boundary Value Problems

K.N.S. Kasi Viswanadham [α] & Y. Showri Raju [σ]

Abstract - A finite element method involving collocation method with quintic B-splines as basis functions have been developed to solve sixth order boundary value problems. The sixth order and fifth order derivatives for the dependent variable are approximated by the central differences of fourth order derivatives. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain. The proposed method is tested on several linear and non-linear boundary value problems. The solution of a non-linear boundary value problem has been obtained as the limit of a sequence of solutions of linear boundary value problems generated by quasilinearization technique. Numerical results obtained by the present method are in good agreement with the exact solutions or numerical solutions available in the literature.

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I. Introduction

In this paper, we developed a collocation method with quintic B-splines as basis functions for getting the numerical solution of a general sixth order linear boundary value problem, which is in the form

$$a_0(x)y^{(6)}(x) + a_1(x)y^{(5)}(x) + a_2(x)y^{(4)}(x) + a_3(x)y'''(x) + a_4(x)y''(x) + a_5(x)y'(x) + a_6(x)y(x) = b(x), \quad c < x < d \tag{1}$$

subject to the boundary conditions

$$\begin{aligned} y(c) &= A_0, & y(d) &= B_0 \\ y'(c) &= A_1, & y'(d) &= B_1 \\ y''(c) &= A_2, & y''(d) &= B_2. \end{aligned} \tag{2}$$

where A_0, B_0, A_1, B_1, A_2, B_2 are finite real constants and $a_0(x)$, $a_1(x)$, $a_2(x)$, $a_3(x)$, $a_4(x)$, $a_5(x)$, $a_6(x)$ and $b(x)$ are all continuous functions defined on the interval $[c, d]$.

Generally, these types of differential equations have special significance in astrophysics. The dynamo action in some stars may be modeled by sixth order boundary value problems [1].

Author α : Department of Mathematics, National Institute of Technology, Warangal – 506004, India. Telephone: +91 9866028867
E-mail : kasi_nitw@yahoo.co.in
Author σ: Department of Mathematics, National Institute of Technology, Warangal – 506004, India. Telephone: +91 7893555752
E-mail : showri_y@rediffmail.com

The narrow convecting layers bounded by stable layers, which are believed to surround A-type stars, may be modeled by sixth-order boundary value problems [2]. Moreover, when an infinite horizontal layer of fluid is heated from below and is subjected to the action of rotation, instability sets in. When this instability is an ordinary convection, the ordinary differential equation is a sixth-order ordinary differential equation. For further discussion of sixth-order boundary value problems, see [3,4,5].

The existence and uniqueness of solution of such type of boundary value problems can be found in the book written by Agarwal [6]. El-Gamel et al. [7] used Sinc-Galerkin method to solve sixth order boundary value problems. Akram and Siddiqi [8] solved the boundary value problem of type (1)-(2) with non-polynomial spline technique. Siddiqi et al.[9] solved the same boundary value problems using quintic splines. Also Siddiqi and Akram [10] used septic splines to solve the boundary value problems of type (1)-(2). Lamini et al. [11] used spline collocation method to solve the sixth-order boundary value problems.

Further, decomposition methods [12], Ritz's method based on the variational theory [13], and the homotopy perturbation method [14] have been applied for the solution of sixth-order boundary value problems. Variational iteration method for solving sixth-order boundary value problems have been developed by Noor et al.[15]. Siraj-ul-Islam et al. [16,17] developed non-polynomial splines approach to the solution of sixth-order and fourth-order boundary-value problems. Viswanadham et al. [18,19] used sixth order and septic B-splines to solve sixth order boundary value problems.

In this paper, we try to present a simple collocation method using quintic B-splines as basis functions to solve the sixth order boundary value problem of the type (1)-(2).

In section 2 of this paper, the justification for using the collocation method has been mentioned. In section 3, the definition of quintic B-splines has been described. In section 4, description of the collocation method with quintic B-splines as basis functions has been presented and in section 5, solution procedure to find the nodal parameters is presented. In section 6, numerical examples of both linear and non-linear boundary value problems are presented. The solution of a nonlinear boundary value problem has been obtained as the limit of a sequence of solutions of linear boundary value problems generated by quasilinearization

technique [20]. Finally, the last section is dealt with conclusions of the paper.

II. Justification for Using Collocation Method

In finite element method (FEM) the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration. FEM involves variational methods such as Ritz's approach, Galerkin's approach, least squares method and collocation method etc. The collocation method seeks an approximate solution by requiring the residual of the differential equation to be identically zero at N selected points in the given space variable domain where N is the number of basis functions in the basis [21]. That means, to get an accurate solution by the collocation method, one needs a set of basis functions which in number match with the number of collocation points selected in the given space variable domain. Further, the collocation method is the easiest to implement among the variational methods of FEM. When a differential equation is approximated by m^{th} order B-splines, it yields $(m+1)^{th}$ order accurate results [22]. Hence this motivated us to use the collocation method to solve a sixth order boundary value problem of type (1)-(2) with quintic B-splines.

III. Definition of Quintic B-Splines

The cubic B-splines are defined in [23,24]. In a similar analogue, the existence of the quintic spline interpolate $s(x)$ to a function in a closed interval $[c,d]$ for spaced knots (need not be evenly spaced)

$$c = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = d$$

is established by constructing it. The construction of $s(x)$ is done with the help of the quintic B-Splines. Introduce ten additional knots $x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ and x_{n+5} such that

$$x_{-5} < x_{-4} < x_{-3} < x_{-2} < x_{-1} < x_0$$

$$\text{and} \quad x_n < x_{n+1} < x_{n+2} < x_{n+3} < x_{n+4} < x_{n+5}.$$

Now the quintic B-splines $B_i(x)$ are defined by

$$B_i(x) = \begin{cases} \sum_{r=i-3}^{i+3} \frac{(x_r - x)_+^5}{\pi'(x_r)}, & x \epsilon [x_{i-3}, x_{i+3}] \\ 0, & \text{otherwise} \end{cases}$$

where

$$(x_r - x)_+^5 = \begin{cases} (x_r - x)^5, & \text{if } x_r \geq x \\ 0, & \text{if } x_r \leq x \end{cases}$$

and

$$\pi(x) = \prod_{r=i-3}^{i+3} (x - x_r).$$

It can be shown that the set $\{B_{-2}(x), B_{-1}(x), B_0(x), \cdots, B_n(x), B_{n+1}(x), B_{n+2}(x)\}$ forms a basis for the space $S_5(\pi)$ of quintic polynomial splines. The quintic B-splines are the unique nonzero splines of smallest compact support with knots at

$$x_{-5} < x_{-4} < x_{-3} < x_{-2} < x_{-1} < x_0 < \cdots$$
$$< x_n < x_{n+1} < x_{n+2} < x_{n+3} < x_{n+4} < x_{n+5}$$

IV. Description of the Method

To solve the boundary value problem (1)-(2) by the collocation method with quintic B- splines as basis functions, we define the approximation for $y(x)$ as

$$y(x) = \sum_{j=-2}^{n+2} \alpha_j B_j(x) \qquad (3)$$

where α_j's are the nodal parameters to be determined. In the present method, the internal mesh points are selected as the collocation points. In collocation method, the number of basis functions in the approximation should match with the number of selected collocation points [21]. Here the number of basis functions in the approximation (3) is n+5, where as the number of selected collocation points is n-1. So, there is a need to redefine the basis functions into a new set of basis functions which in number match with the number of selected collocation points. The procedure for redefining the basis functions is as follows:

Using the quintic B-splines described in section 3 and the Dirichlet boundary conditions of (2), we get the approximate solution at the boundary points as

$$y(c) = y(x_0) = \sum_{j=-2}^{2} \alpha_j B_j(x_0) = A_0 \qquad (4)$$

$$y(d) = y(x_n) = \sum_{j=n-2}^{n+2} \alpha_j B_j(x_n) = B_0. \qquad (5)$$

Eliminating α_{-2} and α_{n+2} from the equations (3),(4), and (5) we get the approximation for $y(x)$ as

$$y(x) = w_1(x) + \sum_{j=-1}^{n+1} \alpha_j P_j(x) \qquad (6)$$

where

$$w_1(x) = \frac{A_0}{B_{-2}(x_0)} B_{-2}(x) + \frac{B_0}{B_{n+2}(x_n)} B_{n+2}(x)$$

and $P_j(x) =$

$$\begin{cases} B_j(x) - \frac{B_j(x_0)}{B_{-2}(x_0)} B_{-2}(x), & for \quad j = -1,0,1,2 \\ B_j(x), & for \quad j = 3,4,\ldots,n-3 \\ B_j(x) - \frac{B_j(x_n)}{B_{n+2}(x_n)} B_{n+2}(x), & for \quad j = n-2, n-1, n, n+1 \end{cases}$$

Using the Neumann boundary conditions of (2) to the approximate solution $y(x)$ in (6), we get

$$y'(c) = y'(x_0) = w_1'(x_0) + \alpha_{-1}P_{-1}'(x_0) + \alpha_0 P_0'(x_0) + \alpha_1 P_1'(x_0) + \alpha_2 P_2'(x_0) = A_1 \quad (7)$$

$$y'(d) = y'(x_n) = w_1'(x_n) + \alpha_{n-2}P_{n-2}'(x_n) + \alpha_{n-1}P_{n-1}'(x_n) + \alpha_n P_n'(x_n) + \alpha_{n+1}P_{n+1}'(x_n) = B_1. \quad (8)$$

Now, eliminating α_{-1} and α_{n+1} from the equations (6),(7) ,and (8), we get the approximation for $y(x)$ as

$$y(x) = w_2(x) + \sum_{j=0}^{n} \alpha_j Q_j(x) \quad (9)$$

where

$$w_2(x) = w_1(x) + \frac{A_1 - w_1'(x_o)}{P_{-1}'(x_o)} P_{-1}(x) + \frac{B_1 - w_{.1}(x_n)}{P_{n+1}'(x_n)} P_{n+1}(x)$$

and $Q_j(x)$ =

$$\begin{cases} P_j(x) - \dfrac{P_j'(x_o)}{P_{-1}'(x_o)} P_{-1}(x), & \text{for } j = 0,1,2 \\ P_j(x), & \text{for } j = 3,4,\ldots,n-3 \\ P_j(x) - \dfrac{P_j'(x_n)}{P_{n+1}'(x_n)} P_{n+1}(x), & \text{for } j=n-2,n-1,n. \end{cases}$$

Using the boundary conditions $y''(c) = A_2$ and $y''(d) = B_2$ of (20) to the approximate solution $y(x)$ in (9), we get

$$y''(c) = y''(x_0) = w_2''(x_0) + \alpha_0 Q_0''(x_0) + \alpha_1 Q_1''(x_0) + \alpha_2 Q_2''(x_0) = A_2 \quad (10)$$

$$y''(d) = y''(x_n) = w_2''(x_n) + \alpha_{n-2}Q_{n-2}'' + \alpha_{n-1}Q_{n-1}''(x_n) + \alpha_n Q_n''(x_n) = B_2. \quad (11)$$

Now, eliminating α_0 and α_n from the equations (9),(10), and (11) we get the approximation for $y(x)$ as

$$y(x) = w(x) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j(x) \quad (12)$$

where

$$w(x) = w_2(x) + \frac{A_2 - w_2''(x_0)}{Q_0''(x_0)} Q_0(x) + \frac{B_2 - w_2''(x_n)}{Q_n''(x_n)} Q_n(x)$$

and $\tilde{B}_j(x)$

$$\begin{cases} Q_j(x) - \dfrac{Q_j''(x_o)}{Q_o''(x_o)} Q_o(x), & \text{for } j = 1,2 \\ Q_j(x), & \text{for } j = 3,4,\ldots,n-3 \\ Q_j(x) - \dfrac{Q_j''(x_n)}{Q_n''(x_n)} Q_n(x), & \text{for } j= n-2,n-1, \end{cases}$$

Now the new basis functions for the approximation $y(x)$ are $\{\tilde{B}_j(x), j = 1,2,\ldots,n-1\}$ and they are in number matching with the number of selected collocation points. Since the approximation for $y(x)$ in (12) is a quintic approximation, let us approximate $y^{(5)}$ and $y^{(6)}$ at the selected collocation points with central differences as

$$y_i^{(5)} = \frac{y_{i+1}^{(4)} - y_{i-1}^{(4)}}{2h} \text{ and } y_i^{(6)} = \frac{y_{i+1}^{(4)} - 2y_i^{(4)} + y_{i-1}^{(4)}}{h^2} \quad (13)$$

where

$$y_i = y(x_i) = w(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j(x_i) \quad (14)$$

Now applying collocation method to (1), we get

$$a_0(x_i)y_i^{(6)} + a_1(x_i)y_i^{(5)} + a_2(x_i)y_i^{(4)} + a_3(x_i)y_i''' + a_4(x_i)y_i'' + a_5(x_i)y_i' + a_6(x_i)y_i = b(x_i) \quad \text{for } i=1,2,\ldots,n-1. \quad (15)$$

Using (13) and (14) in (15), we get

$$\frac{a_0(x_i)}{h^2}\left[w^{(4)}(x_{i+1}) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_{i+1}) - 2\left\{w^{(4)}(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_i)\right\} + w^{(4)}(x_{i-1}) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_{i-1}) \right]$$

$$+\frac{a_1(x_i)}{2h}\left[w^{(4)}(x_{i+1}) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_{i+1}) - \left\{w^{(4)}(x_{i-1}) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_{i-1})\right\} \right]$$

$$+a_2(x_i)\left[w^{(4)}(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j^{(4)}(x_i)\right] + a_3(x_i)\left[w'''(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j'''(x_i)\right]$$

$$+a_4(x_i)\left[w''(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j''(x_i)\right] + a_5(x_i)\left[w'(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j'(x_i)\right]$$

$$+a_6(x_i)\left[w(x_i) + \sum_{j=1}^{n-1} \alpha_j \tilde{B}_j(x_i)\right]$$

$$= b(x_i) \qquad \text{for} \quad i = 1,2,\ldots,n-1. \tag{16}$$

Rearranging the terms and writing the system of equations (16) in matrix form, we get

$$A\alpha = B \tag{17}$$

where

$$A = [a_{ij}];$$

$$a_{ij} = \tilde{B}_j^{(4)}(x_{i-1})\left(\frac{a_0(x_i)}{h^2} - \frac{a_1(x_i)}{2h}\right)$$
$$+\tilde{B}_j^{(4)}(x_i)\left(-2\frac{a_0(x_i)}{h^2}\right)$$
$$+\tilde{B}_j^{(4)}(x_{i+1})\left(\frac{a_0(x_i)}{h^2} + \frac{a_1(x_i)}{2h}\right)$$
$$+\tilde{B}_j^{(4)}(x_i)a_2(x_i)$$
$$+\tilde{B}_j'''(x_i)a_3(x_i) + \tilde{B}_j''(x_i)a_4(x_i)$$
$$+\tilde{B}_j'(x_i)a_5(x_i) + \tilde{B}_j(x_i)a_6(x_i)$$
$$\text{for} \quad i = 1,2,\ldots,n-1, \quad j = 1,2,\ldots,n-1. \tag{18}$$

$$B = [b_i];$$

$$b_i = b(x_i) - \left[\; w^{(4)}(x_{i-1})\left(\frac{a_0(x_i)}{h^2} - \frac{a_1(x_i)}{2h}\right)\right.$$
$$+w^{(4)}(x_i)\left(-2\frac{a_0(x_i)}{h^2}\right)$$
$$+w^{(4)}(x_{i+1})\left(\frac{a_0(x_i)}{h^2} + \frac{a_1(x_i)}{2h}\right)$$
$$+w^{(4)}(x_i)a_2(x_i) + w'''(x_i)a_3(x_i)$$
$$+w''(x_i)a_4(x_i) + w'(x_i)a_5(x_i)$$
$$\left.+w(x_i)a_6(x_i)\;\right]$$
$$\text{for} \quad i = 1,2,\ldots,n-1. \tag{19}$$

and

$$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{n-1}]^T.$$

V. Solution Procedure to Find the Nodal Parameters

The basis function $\tilde{B}_i(x)$ is defined only in the interval $[x_{i-3}, x_{i+3}]$ and outside of this interval it is zero. Also at the end points of the interval $[x_{i-3}, x_{i+3}]$ the basis function $\tilde{B}_i(x)$ vanishes. Therefore, $\tilde{B}_i(x)$ is having non-vanishing values at the mesh points x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2} and zero at the other mesh points. The first four derivatives of $\tilde{B}_i(x)$ also have the same nature at the mesh points as in the case of $\tilde{B}_i(x)$. Using these facts, we can say that the matrix A defined in (18) is a seven diagonal band matrix. Therefore, the system of equations (17) is a seven diagonal band system in α_i's. The nodal parameters α_i's can be obtained by using band matrix solution package. We have used the FORTRAN-90 programming to solve the boundary value problem (1)-(2) by the proposed method.

VI. Numerical Examples

To demonstrate the applicability of the proposed method for solving the sixth order boundary value problems of type (1)-(2) we considered seven examples of which four are linear and three are non linear boundary value problems. Numerical results for each problem are presented in tabular forms and compared with the exact solutions or numerical solutions available in the literature.

Example 1 : Consider the linear boundary value problem

$$y^{(6)} + xy = -(24 + 11x + x^3)e^x, \quad 0 < x < 1 \tag{20}$$

subject to

$$y(0) = 0, \; y(1) = 0,$$
$$y'(0) = 1, \; y'(1) = -e, \tag{21}$$
$$y''(0) = 0, \; y''(1) = -4e.$$

The exact solution for the above problem is given by $y(x) = x(1-x)e^x$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 1. The maximum absolute error obtained by the proposed method is 7.361174×10^{-6}

Table 1 : Numerical results for Example 1

x	Exact Solution	Absolute error by proposed method
0.1	9.946539E-02	4.619360E-07
0.2	1.954244E-01	1.847744E-06
0.3	2.834704E-01	3.874302E-06
0.4	3.580379E-01	6.288290E-06
0.5	4.121803E-01	7.361174E-06
0.6	4.373085E-01	7.241964E-06
0.7	4.228881E-01	6.496906E-06
0.8	3.560865E-01	4.649162E-06
0.9	2.213642E-01	2.324581E-06

Example 2: Consider the linear boundary value problem

$$y_{(6)} + e^{-x}y = -720 + (x - x^2)^3 e^{-x}, \quad 0<x<1 \tag{22}$$

subject to

$$y(0) = 0, y(1) = 0,$$
$$y'(0) = 0, y'(1) = 0,$$
$$y''(0) = y''(1) = 0 \quad (23)$$

The exact solution for the above problem is given by $y(x) = x^3(1-x)^3$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 2. The maximum absolute error obtained by the proposed method is 2.942979×10^{-6}.

Table 2 : Numerical results for Example 2

x	Exact solution	Absolute error by proposed method
0.1	7.290000E-04	3.894675E-07
0.2	4.096000E-03	1.216307E-06
0.3	9.261001E-03	2.081506E-06
0.4	1.382400E-02	2.698973E-06
0.5	1.562500E-02	2.942979E-06
0.6	1.382400E-02	2.727844E-06
0.7	9.261001E-03	2.096407E-06
0.8	4.096000E-03	1.219101E-06
0.9	7.289993E-04	3.807945E-07

Example 3 : Consider the linear boundary value problem

$$y^{(6)} + y''' + y'' - y =$$
$$e^{-x}(-15x^2 + 78x - 114), \quad 0 < x < 1 \quad (24)$$

subject to

$$y(0) = 0, \ y(1) = \frac{1}{e},$$
$$y'(0) = 0, \ y'(1) = \frac{2}{e}, \quad (25)$$
$$y(0) = 0, \ y(1) = \frac{1}{e},$$

The exact solution for the above problem is given by $y(x) = x^3e^{-x}$. The proposed method is tested on this problem where the domain [0,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 3. The maximum absolute error obtained by the proposed method is 4.939735×10^{-6}

Table 3 : Numerical results for Example 3

x	Exact solution	Absolute error by proposed method
0.1	9.048374E-04	1.808512E-07
0.2	6.549846E-03	9.508803E-07
0.3	2.000210E-02	2.233312E-06
0.4	4.290048E-02	3.498048E-06
0.5	7.581633E-02	4.529953E-06
0.6	1.185433E-01	4.939735E-06
0.7	1.703288E-01	4.217029E-06
0.8	2.300564E-01	2.712011E-06
0.9	2.963893E-01	1.430511E-06

Example 4 : Consider the linear boundary value problem

$$y^{(6)} + y = 6(2x\cos x + 5\sin x), \quad -1 < x < 1 \quad (26)$$

subject to

$$y(-1) = 0, \ y(1) = 0,$$
$$y'(-1) = 2\sin 1, \ y'(1) = 2\sin 1,$$
$$y''(-1) = -4\cos 1 - 2\sin 1, \quad (27)$$
$$y''(1) = 4\cos 1 + 2\sin 1.$$

The exact solution for the above problem is given by $y(x) = (x^2 - 1)\sin x$. The proposed method is tested on this problem where the domain [-1,1] is divided into 10 equal subintervals. Numerical results for this problem are shown in Table 4. The maximum absolute error obtained by the proposed method is 8.076429×10^{-6}.

Table 4 : Numerical results for Example 4

x	Exact solution	Absolute error by proposed method
-0.8	2.582482E-01	4.172325E-07
-0.6	3.613712E-01	1.251698E-06
-0.4	3.271114E-01	1.788139E-07
-0.2	1.907225E-01	2.488494E-06
0.0	0.000000	5.105962E-06
0.2	-1.907226E-01	8.076429E-06
0.4	-3.271114E-01	7.987022E-06
0.6	-3.613712E-01	6.377697E-06
0.8	-2.582482E-01	3.904104E-06

Example 5 : Consider the nonlinear boundary value problem

$$y^{(6)} - 20e^{-36y(x)} =$$
$$-40(1+x)^{-6}, \quad 0 < x < 1 \quad (28)$$

subject to

$$y(0) = 0, \ y(1) = \frac{1}{6}\ln 2,$$
$$y'(0) = \frac{1}{6}, \ y'(1) = \frac{1}{12}, \quad (29)$$
$$y''(0) = \frac{-1}{6}, \ y''(1) = \frac{-1}{24}.$$

The exact solution for the above problem is given by $y(x) = \frac{1}{6}\ln(1+x)$. This nonlinear boundary value problem is converted into a sequence of linear boundary value problems generated by quasilinearization technique [20] as

$$y^{(6)}_{(n+1)} + \left[720e^{-36y_{(n)}}\right]y_{(n+1)} =$$
$$720y_{(n)}e^{-36y_{(n)}} + 20e^{-36y_{(n)}}$$

$$-40(1+x)^{-6}, \quad \text{for} \quad n = 0,1,2,\ldots \tag{30}$$

subject to

$$\begin{aligned} y_{(n+1)}(0) &= 0, \quad y_{(n+1)}(1) = \frac{1}{6}\ln 2, \\ y'_{(n+1)}(0) &= \frac{1}{6}, \quad y'_{(n+1)}(1) = \frac{1}{12}, \\ y''_{(n+1)}(0) &= \frac{-1}{6}, \quad y''_{(n+1)}(1) = \frac{-1}{24}. \end{aligned} \tag{31}$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for y. The domain [0,1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of problems (30). Numerical results for this problem are presented in Table 5. The maximum absolute error obtained by the proposed method is 1.329929×10^{-6}.

Table 5 : Numerical results for Example 5

x	Exact solution	Absolute error by proposed method
0.1	1.588503E-02	1.303852E-07
0.2	3.038693E-02	5.215406E-07
0.3	4.372738E-02	9.723008E-07
0.4	5.607871E-02	1.329929E-06
0.5	6.757752E-02	1.259148E-06
0.6	7.833394E-02	8.419156E-07
0.7	8.843804E-02	4.023314E-07
0.8	9.796445E-02	8.195639E-08
0.9	1.069757E-01	1.713634E-07

Example 6 : Consider the nonlinear boundary value problem

$$y^{(6)} + e^{-x}y^2 = e^{-x} + e^{-3x}, \quad 0 < x < 1 \tag{32}$$

subject to

$$\begin{aligned} y(0) &= 1, \quad y(1) = \frac{1}{e}, \\ y'(0) &= -1, \quad y'(1) = \frac{-1}{e} \\ y''(0) &= 1, \quad y''(1) = \frac{1}{e}. \end{aligned} \tag{33}$$

The exact solution for the above problem is given by $y(x) = e^{-x}$. This nonlinear boundary value problem is converted into a sequence of linear boundary value problems generated by quasilinearization technique [20] as

$$y^{(6)}_{(n+1)} + [2e^{-x}y_{(n)}]y_{(n+1)} =$$

$$e^{-x}y^2{}_{(n)} + e^{-x} + e^{-3x}, \quad \text{for} \quad n = 0,1,2,\ldots \tag{34}$$

subject to

$$\begin{aligned} y_{(n+1)}(0) &= 1, \quad y_{(n+1)}(1) = \frac{1}{e}, \\ y'_{(n+1)}(0) &= -1, \quad y'_{(n+1)}(1) = \frac{-1}{e}, \\ y''_{(n+1)}(0) &= 1, \quad y''_{(n+1)}(1) = \frac{1}{e}. \end{aligned} \tag{35}$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for y. The domain [0,1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of problems (34). Numerical results for this problem are presented in Table 6. The maximum absolute error obtained by the proposed method is 1.913309×10^{-5}.

Table 6 : Numerical results for Example 6

x	Exact solution	Absolute error by proposed method
0.1	9.048374E-01	1.907349E-06
0.2	8.187308E-01	7.033348E-06
0.3	7.408182E-01	1.347065E-05
0.4	6.703200E-01	1.877546E-05
0.5	6.065307E-01	1.913309E-05
0.6	5.488116E-01	1.478195E-05
0.7	4.965853E-01	8.702278E-06
0.8	4.493290E-01	3.188848E-06
0.9	4.065697E-01	2.980232E-08

Example 7 : Consider the nonlinear boundary value problem

$$y^{(6)} = e^x y^3(x), \qquad 0 < x < 1 \tag{36}$$

subject to

$$\begin{aligned} y(0) &= 1, \quad y(1) = e^{\frac{-1}{2}}, \\ y'(0) &= \frac{-1}{2}, \quad y'(1) = \frac{-1}{2}e^{\frac{-1}{2}}, \\ y''(0) &= \frac{1}{4}, \quad y''(1) = \frac{1}{4}e^{\frac{-1}{2}}. \end{aligned} \tag{37}$$

This nonlinear boundary value problem is converted into a sequence of linear boundary value problems generated by quasilinearization technique [20] as

$$y^{(6)}_{(n+1)} + [-3e^x y^2_{(n)}]y_{(n+1)} =$$

$$-2e^x y^3_{(n)} \qquad \text{for} \quad n = 0,1,2,\ldots \tag{38}$$

subject to

$$y_{(n+1)}(0) = 1, \quad y_{(n+1)}(1) = e^{\frac{-1}{2}},$$

$$y'_{(n+1)}(0) = \frac{-1}{2}, \quad y'_{(n+1)}(1) = \frac{-1}{2}e^{\frac{-1}{2}}, \qquad (39)$$

$$y''_{(n+1)}(0) = \frac{1}{4}, \quad y''_{(n+1)}(1) = \frac{1}{4}e^{\frac{-1}{2}}.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for y. The domain [0,1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of problems [38]. The exact solution for the problem (36) and (37) is not available in the literature. The numerical solutions can be obtained for this problem by refining the mesh size. Hussin and Kilicman [25] obtained the numerical solutions for this problem by refining the mesh size. Numerical results obtained by the proposed method are compared with the numerical results obtained by Hussin and Kilicman [25], and the results are presented in Table 7.

Table 7 : Numerical results for Example 7

x	Numerical solutions obtained by Hussin and Kilicman [25]	Absolute error by the proposed method when compared with Hussin and Kilicman [25]
0.1	9.512295E-01	2.563000E-06
0.2	9.048374E-01	1.049042E-05
0.3	8.607080E-01	2.151728E-05
0.4	8.187308E-01	3.141165E-05
0.5	7.788008E-01	3.391504E-05
0.6	7.408182E-01	2.819300E-05
0.7	7.046881E-01	1.811981E-05
0.8	6.703200E-01	7.688999E-06
0.9	6.376281E-01	5.960464E-07

VII. Conclusion

In this paper, we have developed a collocation method with quintic B-splines as basis functions to solve sixth order boundary value problems. Here we have taken internal mesh points as the selected collocation points. The quintic B-spline basis set has been redefined into a new set of basis functions which in number match with the number of selected collocation points. The proposed method is applied to solve several number of linear and non-linear problems to test the efficiency of the method. The numerical results obtained by the proposed method are in good agreement with the exact solutions or numerical solutions available in the literature. The objective of this paper is to present a simple method to solve a sixth order boundary value problem and its easiness for implementation.

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An Application of Evolutionary Computational Technique to Non-Linear Singular System Arising in Polytrophic and Isothermal Sphere

By Junaid Ali Khan, Muhammad Asif Zahoor Raja & Ijaz Mansoor Qureshi
International Islamic University Islamabad, Pakistan

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Keywords : Singular non-linear systems; Evolutionary computational technique; Differential transform method; Optimal Homotopy asymptotic method; Artificial neural network.

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Junaid Ali Khan [α], Muhammad Asif Zahoor Raja [α] & Ijaz Mansoor Qureshi [σ]

Abstract - The paper presents a method to solve singular non-linear system representing polytrophic and isothermal sphere using neural network optimized by evolutionary computational approach. A trial solution of the system is written as a feed-forward neural network containing adaptive parameters (weights and biases). We prepare a fitness evaluation function defining unsupervised error. The optimization of the error defines the accuracy in the model that is highly stochastic in nature. Genetic algorithm is exploited as a tool for global convergence and active set algorithm as a rapid local search. The given scheme is tested on the model with polytrophic index $\lambda = 5$. A comparative study is made with exact and optimal Homtopy asymptotic method. The stability and reliability of the proposed scheme is investigated by a comprehensive statistical analysis. The proposed results are found to be in good agreement with exact solution as well as numerical solvers.

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I. Introduction

The singular phenomenon arises in the modeling of physical structures, random processes, control theory, networks synthesis and other areas of the applied sciences and engineering [1-2]. These singular non-linear systems have substantial significance in classical and modern science, as it defines the dynamics of a system [3]. The singular non-linear system of polytrophic and isothermal sphere is represented by the following non-linear second order homogenous equation:

$$\frac{d^2y(t)}{dt^2} = -\frac{2}{t}\left(\frac{dy(t)}{dt}\right) + y^{\lambda} \qquad 0 \le t \le 1, \qquad (1)$$

The tropic index is taken to be $\lambda = 5$, and subject to the following initial conditions

$$y(0) = 1,\ \dot{y}(0) = 0, \qquad (2)$$

It is essentially a Poisson's equation applied is the various modeling application like radioactively cooling, self-gravitating clouds and in the clusters of galaxies [4]. The analytic solution of the model given in equation(1) has been considered by the authors in the recent years by handling the singularity having the index λ. In the recent years, the merging of Homotopy idea with perturbation is applied for non-linear systems. [5]. Optimal Homotopy Asymptotic Method (OHAM) is established very recently by Marinca et al [6]. The OHAM have built in convergence criteria like HAM but is more flexible. From the contribution of Marinca et al [7-9] and Iqbal et al. [10] have proved generalization, effectiveness and reliability of the method to important engineering applications. The linear/nonlinear singular initial value problems have been solved by Iqbal et al. by using OHAM method. But no one yet tried to solve the singular non-linear system of Polytrophic and Isothermal sphere by using artificial neural network optimized by evolutionary computation. The applicability of artificial neural network (ANN) is remarkable for problem involving singularity and convex nature.In the last decay various scientists exploited ANN along with stochastic computational techniques, some of them are provided as a reference. Radial Bases functions neural networks were exploited to design a moving mass attitude control system [11] to control a vehicle with three axis stabilization in intra-atmospheric space. C. Monterola and C. Saloma [12-13] works on non-linear system represented by Schrodinger equation, this system is also formulated by FFNN. The flexibility of neural network and GA can be seen in the work carried out by Stackelberg [14] where Nash equilibrium was achieved by a hybrid intelligent algorithm in which fuzzy simulation, neural networks and genetic algorithms were integrated. Bumps and ruts on the roads cause the abrupt change in the rectilinear motion of the vehicles and adhesion coefficient. For a stable movement of the vehicle an appropriate breaking torque is needed. The optimization of this required torque in this problem can also be accomplished using the neural networks and genetic algorithms [15].

In this paper the mathematical modeling for singular non-linear system is performed by ANN with log-sigmoid as activation function. The optimization of the weights of the model is performed by genetic

Author α : Department of Electronic Engineering, International Islamic University Islamabad, Pakistan.
E-mails : Junaid.phdee17@iiu.edu.pk, asif.phdee10@iiu.edu.pk
Author σ : Department of Electrical Engineering, Air University, Islamabad, Pakistan. E-mail : imqureshi@mail.au.edu.pk

algorithm. The result of the GA is provided as a start point for rapid local search. The optimal Homtopy asymptotic method and exact solution of the problem are analyzed with the proposed scheme. The scheme is run for 100 independent runs to get a comprehensive statistical analysis based on the minimum fitness achieved, maximum fitness and spread in the results on the based of mean and standard deviation. The timing analysis is provided to see the computational complexity of the method along with the level of accuracy. According to best of the author knowledge this is the first article in which stochastic methods are incorporated with NN to optimize the non-linear singular systems of Polytrophic and Isothermal sphere.

The remainder of the paper is organized in the following way. The section 2 describes the importance of evolutionary techniques in optimization .In section 3 a mathematical modeling that contains neural network architecture of the singular system is explained. The numerical results along with discussion on the results are revealed in section 4. Finally section 5 presents some concluding remarks on the results along with directions to the future research.

II. Evolutionary Computation

The term evolutionary computation (EC), used vigorously for all evolutionary algorithms (EAs) this describes field of investigations. The major advantage of these techniques can be seen in practical difficult optimization problems. The major benefits are multifold, simplicity of the concept, robustness in changing circumstances, flexibility and other facets [16]. By this the EC has received special interest in the researchers for its applications in science and engineering. Because it is conceptually simple so no gradient information needs to be presented to the algorithms. The domain of EC is for all those problems that can have a function optimization task. The phenomenon of EC depends upon a data structure to represent solutions, the index of performance to evaluate solutions, and operators to generate new solutions from old [17]. The operator should takes care a behavioral link between parents and offspring's. A disjoint state space for possible solutions is formed which encompass infeasible regions, and time varying index or a function of competing solutions in the population [18]. The procedure of applicability is same for problems like, continuous-valued parameter optimization problems, discrete combinatorial problems, mixed-integer problems, and so forth [19].

a) Genetic Algorithms

The modern researches in genetic algorithms (GAs) has outline that the initial proposals were incapable of solving hard problems in a robust and efficient way. In large-scale Optimization problems, the execution time of first-generation GAs increases dramatically whereas solution quality decreases. Moreover the things such as encoding schemes, selection procedures, and self-adaptive and knowledge-based operators play a key role in the optimization of highly convex and stochastic ion nature problems. The birth of the GAs is also for optimization of various fields of interest [20-21]. Over and above the problems in which optimization itself is the final goal, it is also a way for achieving modeling, forecasting, control, simulation, and so forth. Traditional optimization techniques begin with a single candidate and search iteratively for the optimal solution by applying static heuristics. On the other hand, the GA approach uses a population of candidates to search several areas of a solution space, simultaneously and adaptively.The most popular methods that go beyond simple local search are GAs [22], simulated annealing (SA) [23], and tabu search (TS) [24]. Genetic algorithms operate on a population of individuals. Each individual is a potential solution to a given problem and is typically encoded as a fixed-length binary string, which is an analogy with an actual chromosome. After an initial population is randomly or heuristically generated, the algorithm evolves population through sequential and iterative application of three operators: selection, crossover, and mutation. A new generation is formed at the end of each iteration. The strongest aspect of the GA is, it does not get stuck in local minimum.

III. Neural Network Mathematical Modeling

The linear combination of log-sigmoid functions can be used a mathematical model of feed-forward ANN. The log-sigmoid is used as universal function approximator [25-26] as it has the tendency to model the non-linear systems effectively and efficiently in diverse fields of engineering [27-29]. Any network suitably trained to approximate a mapping satisfying some ODE will have an output function that will also approximate the DE [30]. In this feed-forward ANN, the input and output layers used linear function as activation while log-sigmoid is used for hidden layers. The following continuous mapping is employed for the function, its first and second derivatives respectively,

$$\hat{y}(t) = \sum_{i=1}^{m} \alpha_i \varphi(w_i t + b_i), \tag{3}$$

$$\frac{d\hat{y}(t)}{dt} = \sum_{i=1}^{m} \alpha_i \frac{d}{dt} \varphi(w_i t + b_i) \tag{4}$$

$$\frac{d^2\hat{y}(t)}{dt^2} = \sum_{i=1}^{m} \alpha_i \frac{d^2}{dt^2} \varphi(w_i t + b_i), \tag{5}$$

where the activation function is considered to be log-sigmoid and is given in expression (5)

$$\varphi(t) = \frac{1}{1+e^{-t}} \tag{6}$$

Therefore the error function formed by the neural networks given (3) to (5) is formed as

$$\varepsilon_j = \varepsilon_j^1 + \varepsilon_j^2 \quad j = 1,2,3\cdots\cdots, \tag{7}$$

where j is the number of generations.

$$\varepsilon_j^1 = \frac{1}{s}\sum_{i=1}^{s}\left(\frac{d^2\hat{y}(t_i)}{dt^2} + \frac{2}{t_i}\frac{d\hat{y}(t_i)}{dt} - \hat{y}(t_i)^5\right)^2\Bigg|_j \tag{8}$$

while the error subject to initial condition is given in (9).

$$\varepsilon_j^2 = \frac{1}{2}\left\{(\hat{y}(0)-1)^2 + \left(\frac{d}{dt}\hat{y}(0)\right)^2\right\} \tag{9}$$

The linear combinations of networks from (3) to (5) can approximately model the system given in (1). It is named as differential equation neural (DEN) network, whose architecture is given in the Fig. 1; the activation function used in this is log-sigmoid. The learning procedure adopted in GA and basic flow chart is provided in one of our last article [31].

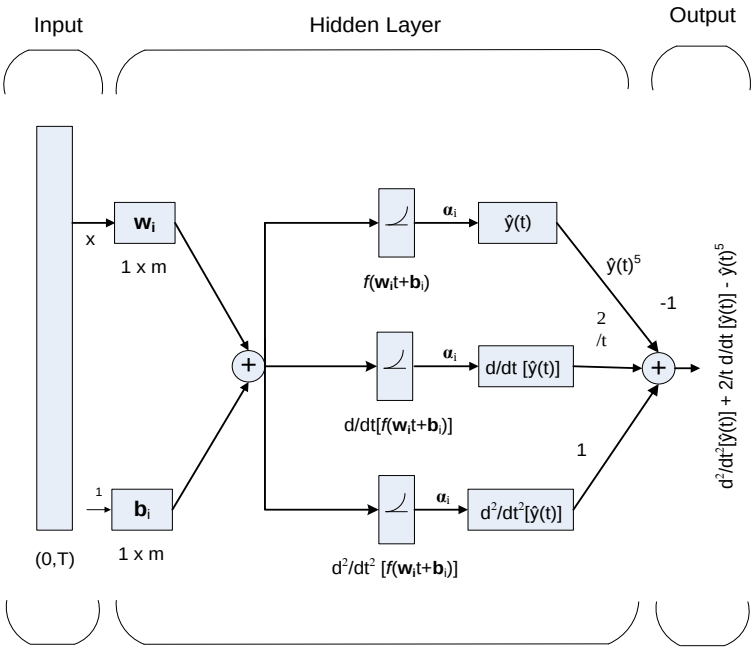

Fig.1 : The DEN network of the Singular system

IV. Results and Discussion

In this section, we shell consider the solution of (1) by the proposed scheme. In order to prove the applicability and effectiveness of the proposed scheme for singular system the solution of OHAM and exact solution is compared with the results attained by the given approach. However, the statistical analysis is also carried out for the solvers to see the reliability and depth of the algorithm. Moreover, the time complexity of the scheme is also provided in the discussion.

The exact solution of expression (1) is [32]:

$$y(t) = \left(1+\frac{t^2}{3}\right)^{-\frac{1}{2}}. \tag{10}$$

The OHAM method series solution is generated by taking zero-order, first-order and second-order solution respectively [10], and the final reported expression is as follow:

$$y_{OHAM}(t) = 1 - 0.16252076779054878\,x^2 + 0.028096429522343383\,x^4 \tag{11}$$

Now, the singular non-linear system of Polytrophic and Isothermal sphere is solved by the proposed scheme as well, the number of neurons in each hidden layer of the DEN network are taken to be m = 10 that results in 30 unknown adaptive weights (α_i, w_i and b_i). The optimization of these weights is carried

out using built-in function for GA in MATLAB. The parameter setting used for the execution of the algorithm is given in Table 1.

Table 1 : Parameters setting of the algorithms

GA		ASA	
Parameters	*Setting*	*Parameters*	*Setting*
Population Size	240	Start Point	Weights from GA
Chromosome size	30	Chromosome size	30
No. of runs	1500	Number of iterations	1500
Selection	Stochastic uniform	Maximum function evaluation	150000
Scaling function	Rank	Function tolerance	1e-12
Reproduction	Elite Count of 3 crossover fraction 0.6	Non-Linear constraints tolerance	1e-20
Mutation	Adaptive feasible	Derivative type	Central difference
Crossover	Scattered	X-Tolerance	1e-06
Migration interval	15	Bound	(-30 30)
Hybridization	ASA	Minimum fitness value	1e-12

The input of the training set is taken from time $t \in (0, 1)$ with a step size of 0.1. It means that the total time steps $m = 11$ so the fitness function is formulated as:

$$\varepsilon_j = \frac{1}{11}\sum_{i=1}^{11}\left(\frac{d^2\hat{y}(t_i)}{dt^2} + \frac{2}{t_i}\frac{d\hat{y}(t_i)}{dt} - \hat{y}(t_i)^5\right)^2 + \frac{1}{2}\left\{(\hat{y}(0)-1)^2 + \left(\frac{d}{dt}\hat{y}(0)\right)^2\right\}\Bigg|_j, \quad j = 1,2,3\cdots \tag{12}$$

where j be the iteration index, $\frac{d^2\hat{y}(t)}{dt^2}$, $\frac{d\hat{y}(t)}{dt}$ and $\hat{y}(t)$ are the networks given in (3) to (5) respectively. The scheme runs iteratively to find the minimum of fitness function ej, with stoppage criteria as 3000 number of runs or fitness value $e_j \leq 10^{-9}$ whichever comes earlier. One of the unknown weights learned by given scheme with fitness value *7.2039e-11* is provided in Table 2. These weights can be used in equation (3) to obtain the solution of the equation for any input time t between *0* and *1*.

Table 2 : Adaptive Parameters Obtained by DE-NN Networks

index	w_i	α_i	β_i
1	0.629640981406070	0.734362550675546	-0.200888390149721
2	-0.385947956618289	-0.442473051469874	-0.994494465387963
3	0.988325374583115	1.944524168475940	1.252751222355890
4	-0.428119414616133	0.497504101676858	0.686299892190351
5	0.066289997417983	-1.203306475712070	0.096988771307929
6	0.960650440363807	-0.791782203756771	0.081407828807695
7	-1.806500080437780	-0.904970658665490	-0.489752157826344
8	-1.434684547506530	1.372140554331030	-0.153510274865171
9	-1.861314323302590	-0.066247775539287	0.335975293394860
10	0.803834370819629	-0.892632934401436	-0.875741872774712

Table 3 : Comparison of the Results with exact and numerical method

t	y_{exact}	y_{ohtm}	y_{GA-AsA}	$\lvert y_{exact} - y_{ohtm} \rvert$	$\lvert y_{exact} - y_{GA-ASA} \rvert$
0.0	1.00000000	1.00000000	0.99999987	0.00000E+00	1.27568E-07
0.1	0.99833749	0.99837760	0.99833660	4.01135E-05	8.84555E-07
0.2	0.99339927	0.99354412	0.99339754	1.44856E-04	1.73096E-06
0.3	0.98532928	0.98560071	0.98532741	2.71434E-04	1.87168E-06
0.4	0.97435470	0.97471595	0.97435312	3.61242E-04	1.58756E-06
0.5	0.96076892	0.96112583	0.96076766	3.56912E-04	1.26070E-06
0.6	0.94491118	0.94513382	0.94491012	2.22638E-04	1.06611E-06
0.7	0.92714554	0.92711078	0.92714455	3.47643E-05	9.88417E-07
0.8	0.90784130	0.90749501	0.90784036	3.46293E-04	9.37937E-07
0.9	0.88735651	0.88679225	0.88735566	5.64264E-04	8.50186E-07
1.0	0.86602540	0.86557566	0.86602468	4.49742E-04	7.25942E-07

The comparison of the results is made with OHAM for the same ranges of the inputs as taken for stochastic numerical method. The results are summarized in Table 3 in comparison of the exact solution. It is clear from the results that the accuracy of the given method is in a good agreement with exact solution and also comparatively excellent with OHAM method. The absolute error of the OHAM is in the range *1.0e-5 to 1.0e-4* while the proposed scheme has the error from *1.0e-7 to1.0e-6.*

The derivative of the system representing in expression (1) is also approximated by the given scheme to check the depth in the method. The results are summarized in Table 4 upto twelve decimal places. It is quite evident from the table that the results obtained by neural network optimized by evolutionary computation is cable to find the derivative of the systems as well by using the same weights as given in Table 2.

Moreover, the reliability of the stochastic algorithm is being tested by a comprehensive statistical analysis. The analysis is performed in the complete range of the time between *0* to *1* and the results are narrated in table 5. The references of the analysis are the mean, standard deviation, best and worst values of the absolute error of proposed method with exact solution. It is quite evident from the table 5, that the Best and Mean absolute error of both the system is in the range *1e-04* to *1e-06* and *1e-06* respectively. The value of the worst of the absolute error is *1e-04* that is even remarkable.

Table 4 : Comparison of the Results with exact for the derivative of the system

t	y'_{exact}	y'_{GA-AsA}	$\lvert y'_{exact} - y'_{GA-ASA} \rvert$
0.0	0.000000000000	0.000001840886	-0.000001840886
0.1	-0.033277916282	-0.033178401419	-0.000099514863
0.2	-0.066226617853	-0.065360105484	-0.000866512369
0.3	-0.098532927816	-0.095661581246	-0.002871346570
0.4	-0.129913960492	-0.123332491326	-0.006581469167
0.5	-0.160128153805	-0.147807897045	-0.012320256761
0.6	-0.188982236505	-0.168732906775	-0.020249329729
0.7	-0.216333959525	-0.185959935851	-0.030374023674
0.8	-0.242091013068	-0.199524924233	-0.042566088834
0.9	-0.266206952825	-0.209610660651	-0.056596292174
1.0	-0.288675134595	-0.216505060841	-0.072170073754

Table 5 : Statistical Analysis of the solution by Proposed Scheme

t	Best	Worst	Mean	STD
0.0	-6.342636E-06	4.594012E-04	9.394565E-06	5.692623E-05
0.1	-9.995311E-05	3.793471E-04	7.351355E-06	5.810525E-05
0.2	-1.253426E-04	3.782469E-04	4.657521E-06	5.879943E-05
0.3	-1.147922E-04	3.337455E-04	3.722699E-06	5.432348E-05
0.4	-1.180902E-04	3.074559E-04	4.684137E-06	5.170384E-05
0.5	-1.197289E-04	3.298016E-04	6.264152E-06	5.263826E-05

0.6	-1.072861E-04	3.398575E-04	7.059596E-06	5.351095E-05
0.7	-8.684856E-05	3.364792E-04	6.459919E-06	5.092706E-05
0.8	-7.119308E-05	3.116579E-04	4.779545E-06	4.380955E-05
0.9	-6.673061E-05	2.635342E-04	2.851818E-06	3.402397E-05
1.0	-6.732281E-05	2.155496E-04	1.478341E-06	2.624317E-05

The value of the fitness functions are computed for 100 independent runs to have a close look on the optimization behavior of various input times. The value of the functions for some of the input times are drawn the descending order. The results are plotted on the semi log scale as the difference between the results for various inputs times are merely negligible. The optimization behavior is drawn in the Fig.2 in descending order for 100 independent runs. Finally it has been concluded from the figure that the convergence capability of the given scheme is 100% for all input times between *0* and *1*. Moreover, the absolute error for 12% of the independent runs is in the range 10^{-03} to 10^{-05} while 88% lies in the range 10^{-05} to 10^{-09}.

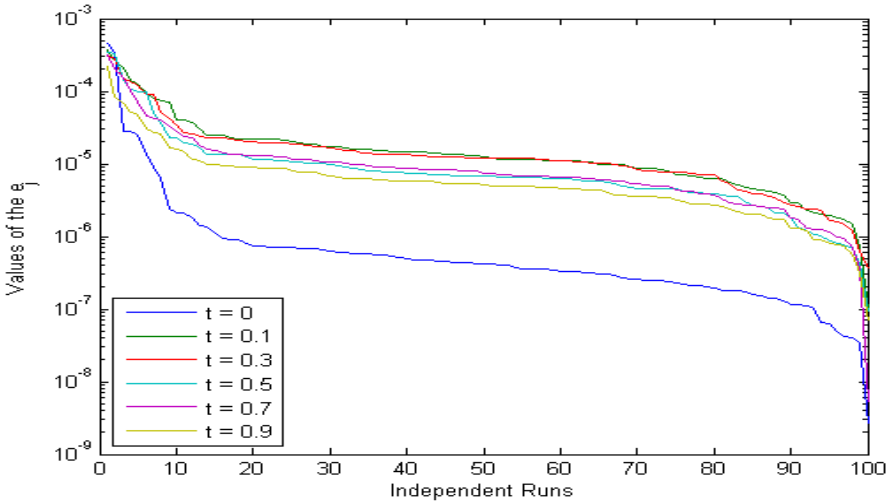

Fig 2 : The behavior of the optimization error for 100 independent runs

V. Conclusions and Future Remarks

On the basis of simulation and results provided in the last section it can be concluded that:

The stochastic solvers based on DEN networks optimized with hybridized genetic algorithm can effectively provides the solution of the Non-Linear Singular System of Polytrophic and Isothermal sphere model. The mean of the absolute error lies in range of 10^{-06}.

The reliability and effectiveness of proposed artificial intelligence techniques are validated from statistical analysis base on 100 independent runs. It is found that the confidence interval for the convergence of the given approach 100% to get an approximate solution in a acceptable error range.

It has been observed that the proposed scheme show the supremacy on the optimal Homotopy asymptotic method in comparison with the exact solution. Moreover, the proposed scheme can readily provide the solution on the continuous grid of time. Thus this provides an alternate approach to researchers to apply the solver to complex real life problems in engineering.

In future, one can look for application of other artificial intelligence techniques base on neural networks optimized with ant/bee colony optimization, genetic programming, particle swarm optimization and differential evolution etc. for solving such vast applications.

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Homotopy Analysis Method to Solve the Multi-Order Fractional Differential Equations

By V.G.Gupta & Pramod Kumar
University of Rajasthan, Jaipur, India

Abstract - This paper applies the homotopoy analysis method (HAM) to obtain the solution of multi-order fractional differential equation. The fractional derivative is described in Caputo sense. Some test examples have been present.

Keywords : *Fractional differential equation, Caputo's fractional derivative, Riemann-Liouville fractional derivative, homotopy analysis method.*

GJRE-I Classification : *FOR Code: 010102, 010302, 010109*

Strictly as per the compliance and regulations of:

Homotopy Analysis Method to Solve the Multi-Order Fractional Differential Equations

V.G.Gupta [α] & Pramod Kumar [σ]

Abstract - This paper applies the homotopoy analysis method (HAM) to obtain the solution of multi-order fractional differential equation. The fractional derivative is described in Caputo sense. Some test examples have been present.

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I. Introduction

In recent years, fractional differential equations have attracted many researchers [7-9] due to their very important applications in Physics, Science and Engineering such as damping law, rheology, diffusion process, description of fractional random walk and so on. Most fractional differential equations do not have exact solutions, so approximation and numerical techniques must be used, such as Laplace transform method [10], Adomian decomposition method [6,11,12], Variational iteration method [13,14], Homotopy perturbation method [15,16], Hamotopy analysis method [1,17,18] and so on. The homotopy analysis method (HAM) was first proposed by Liao [1] in his Ph.D. Thesis. This method (HAM) given in Liao [17] also provides a systematic and an effective procedure for explicit and numerical solutions of a wide and general class of differential equations system representing real physical and engineering problems.

In this paper, the homotopy analysis method (HAM) Liao [1] is applied to solve multi-order fractional differential equations studied by Diethelm and Ford [2]. We also present an algorithm to convert the multi-order fractional differential equation into a system of fractional differential equations without putting any of the restrictions. This algorithm is valid in the most general case and yields fewer number of equations in a system compared to those in Diethelm-Ford algorithm. In last the solutions of the system of FDE have been obtained by applying the Homotopy analysis method.

II. Some Basic Definitions

Definition 2.1:

A real function $F(x)$, $x > 0$ is said to be in space c_μ, $\mu \in R$ is there exists a real number p ($> \mu$) such that $F(x) = x^p F1(x)$ where $F1(x)$ $c[0,\infty]$ and it is said to be in the space C_μ^m iff $F^{(m)} \in c_\mu, m \in N$.

Definition 2.2:

The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$ of a function $F \in c_\mu$, $\mu \geq$, is defined as

$$J^\alpha F(x) = \frac{1}{\Gamma\alpha}\int_0^x (x-t)^{\alpha-1} F(t)\,dt\ ;\ \alpha > 0, x > 0 \tag{1}$$

$$J^0 F(x) = F(x)$$

Properties of the operator J^α can be found in [10,19] we mentioned only the following

$$(i)\quad J^\alpha J^\beta = J^{\alpha+\beta} \qquad (ii)\ J^\alpha J^\beta = J^\beta J^\alpha \tag{2}$$

$$(iii)\quad J^\alpha x^\gamma = \frac{\Gamma\gamma+1}{\Gamma\alpha+\gamma+1} x^{\alpha+\gamma}$$

For $F \in c_\mu$, $\mu \geq -1$, $\alpha,\beta \geq 0$ and $\gamma > -1$.

Definition 2.3 :

The Fractional derivative of F(x) in the Caputo sense is defined as

$$D^\alpha F(x) = J^{n-\alpha} D^n F(x) = \frac{1}{\Gamma n-\alpha}\int_0^x (x-t)^{n-\alpha-1} F^{(n)}(t)\,dt \tag{3}$$

For

$$x > 0,\ n-1<\alpha<n,\ n \in N,\ x > 0,\ F \in c_{-1}^\mu \tag{4}$$

Caputo's fractional derivative has a useful property [19]

$$J^\alpha [D^\alpha F(x)] = F(x) - \sum_{k=0}^{n-1} F^{(k)}(0^+)\frac{x^k}{k!} \qquad (n-1<\alpha<n) \tag{5}$$

Author α : Department of Mathematics, University of Rajasthan, Jaipur-302004, India. E-mail : guptavguor@rediffmail.com

Author σ : Department of Mathematics, Jaipur National University, Jaipur-302025, India. E-mail : pramodgupta472@gmail.com

For the Caputo's derivative

$$D^{\alpha}(c) = 0$$

$$D^{\alpha} x^{\beta} = \begin{cases} 0 & ; \ \beta < \alpha - 1 \\ \dfrac{\Gamma\beta + 1}{\Gamma\beta - \alpha - 1} x^{\beta-\alpha} & ; \ \beta > \alpha - 1 \end{cases} \quad (6)$$

$$D^{\beta} J^{\alpha} F(x) = \begin{cases} J^{\alpha-\beta} F(x) & ; \ \alpha > \beta \\ F(x) & ; \ \alpha = \beta \\ D^{\beta-\alpha} F(x) & ; \ \alpha < \beta \end{cases} \quad (7)$$

Caputo's fractional derivative is linear operator, similar to integer order derivative

$$D^{\alpha}[a\, f(x) + b\, g(x)] = a\, D^{\alpha} f(x) + b\, D^{\alpha} g(x) \quad (8)$$

where a and b are constants. Also this operator satisfies the so-called Leibnitz rule.

$$D^{\alpha}[g(x)\, f(x)] = \sum_{k=0}^{\infty} \binom{\alpha}{k} g^{(k)}(x)\, D^{\alpha-k} f(x) \quad (9)$$

For n to be the smallest integer that exceedsα, the Caputo space fractional derivative operator of order $\alpha > 0$ is defined as

$$D_t^{\alpha} u(x,t) = \frac{\partial^{\alpha} u(x,t)}{dt^{\alpha}} = \begin{cases} \dfrac{1}{\Gamma n - \alpha} \displaystyle\int_0^t (t-\tau)^{n-\alpha-1} \frac{\partial^n u(x,\tau)}{\partial \tau^n} d\tau & n-1 < \alpha < n \\ \dfrac{\partial^n u(x,t)}{\partial t^n} & \alpha = n \in N, \end{cases} \quad (10)$$

For the purpose of this article, the Caputo's definition of fractional differentiation will be used.

Definition 2.4 : The Mittag-Leffler function $E_{\alpha}(z)$ with $\alpha > 0$ is defined the following series representation valid in the whole complex plane [3] .

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma\alpha n + 1}, \ \alpha > 0, \ z \in C.$$

Lemma 2.5. Diethelm and Ford [4]. Let $Y(t) \in c^k(o, t)$ for some $T > 0$ and $K \in IN$ and let $q \notin IN$ be such that $0 < q < k$ then $D_{\alpha}^{q} y(0) = 0.$

III. Algorithm to Convert the Multi-Order FDE into a System of FDE

Let the given fractional differential equation is

$$D_*^{n_k} y(x) = g(x, y(x), D_*^{n_1} y(x), D_*^{n_2} y(x), \ldots, D_*^{n_{k-1}} y(x)) \quad (11)$$

Subject to the initial conditions

$$y^j(0) = y_0^{(j)} \ ; \quad j = 0,1,2,\ldots,\lceil n_k \rceil - 1 \quad (12)$$

where $0 < n_1 < \ldots < n_{k-1} < n_k$, $n_i - n_{i-1} \le 1$ for all $i = 1,2,\ldots,k$ and $0 < n_i \le 1$, assume that $n_i \in Q$.

In Daftardor-Gejji and Jafari [5], Jafari, Das and Tajadodi [6] it was proved that the FDE (11) can be represented as a system of FDE, without any additional restrictions mentioned in equation (2). Here is above mentioned approach. Let us define

$$y_1(x) = y(x)$$

then

$$D_*^{n_1} y_1 = y_2 \quad (13)$$

Here two cases arise

Case (i) : If $m - 1 \le n_1 \le n_2 \le m$ then define

$$D_*^{n_2 - n_1} y_2(x) = y_3 = D_*^{n_2} y(x) \quad (14)$$

Case (ii) : Consider $m-1 \le n_1 < m \le n_2$. If $n_1 = m-1$ then define

$$D_*^{n_2-n_1} y_2(x) = y_3, \tag{15}$$

$$D_*^{n_2-n_1} y_2(x)\ D_*^{n_2-m+1} y_1^{(m-1)} = D_*^{n_2} y_1(x)$$

If $m-1 < n_1 < m \le n_2$ then define

$$D_*^{m-n_1} y_2(x) = y_3 = y^{(m)}. \quad \text{Further define:} \tag{16}$$

$$= D_*^{n_2-m}\, y_3(x) = y_4$$

and continuing similarly one can convert the initial value problem (11) into a system of FDE.

The following example will illustrate the method. Consider

$$D_*^{3.3}\, y(x) = F\left(x, y(x), D_*^{0.1} y(x), D_*^{1} y(x), D_*^{1.2} y(x), D_*^{1.5} y(x), D_*^{1.7} y(x), D_*^{2} y(x)\right.$$
$$\left. D_*^{2.2} y(x), D_*^{2.6} y(x), D_*^{3} y(x)\right) \tag{17}$$

where

$$y^{(j)}(0) = y_0^{(j)} : j = 0,1,2,3 \tag{18}$$

This initial value problem can be viewed as the following system of FDE.

Let $y_1(x) = y(x)$

$$D_*^{0.1} y_1(x) = y_2(x) \quad ; \ y_1(0) = y_0^{(0)} \qquad D_*^{0.2} y_5(x) = y_6(x) \quad ; \ y_5(0) = 0$$

$$D_*^{0.9} y_2(x) = y_3(x) \quad ; \ y_2(0) = 0 \qquad D_*^{0.3} y_6(x) = y_7(x) \quad ; \ y_6(0) = 0$$

$$D_*^{0.2} y_3(x) = y_4(x) \quad ; \ y_3(0) = y_0^{(1)} \qquad D_*^{0.2} y_7(x) = y_8(x) \quad ; \ y_7(0) = y_0^{(2)}$$

$$D_*^{0.3} y_4(x) = y_5(x) \quad ; y_4(0) = 0 \qquad D_*^{0.4} y_8(x) = y_9(x) \quad ; \ y_8(0) = 0$$

$$D_*^{0.4} y_9(x) = y_{10}(x) \quad ; \ y_9(0) = 0 \tag{19}$$

$$D_*^{0.3} y_{10}(x) = F(x, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9) ; \ y_{10}(0) = y_0^{(3)}.$$

This algorithm is valid in the most general case, because we do not impose any of the restriction on α and $n\ell$ as mentioned in equation (12).

IV. Basic Idea of Ham and a System of FDE

We can present the multi-order equation (11) as system of fractional differential equations:

$$D^{\alpha_\ell} y_\ell(x) = y_{\ell+1} \quad ; \ \ell = 1,2,\ldots,n-1$$

$$D^{\alpha_n} y_\ell(x) = F(x, y_1, y_2, \ldots, y_n)$$

$$y_\ell^{(k)}(0) = c_k^\ell, \ 0 \le k \le m_\ell, \ m_\ell < \alpha_\ell \le m_\ell + 1, \quad 1 \le \ell \le n. \tag{20}$$

According to the HAM, we construct the so-called zeroth order deformation equations

$$(1-q)D^{\alpha_\ell}[\phi_\ell(x:q)-y_{\ell 0}(x)]=q\,h_\ell H_\ell(x)[D^{\alpha_\ell}\phi_\ell(x;q)-\phi_{\ell+1}(x;q)]$$

$$\ell=1,2,\ldots,n-1$$

$$(1-q)D^{\alpha_n}[\phi_n(x;q)-y_{n0}(x)]=q\,h_n H_n(x)[D^{\alpha_n}\phi_n(x;q)-F(x,\phi_1,\phi_2,\ldots,\phi_n] \quad (21)$$

where $h_\ell \neq 0$ denotes an auxiliary parameter, $H_\ell(x)$ is an auxiliary function, $q\in[0,1]$ is an embedding parameter, $y_{\ell 0}(x)$ is initial guess of $y_\ell(x)$ and $\phi_\ell(x;q)$ unknown function of independent variables x and q.

Obviously, when q = 0 and q = 1 it holds

$$\phi_\ell(x;0)=y_{\ell 0}(x)$$

$$\phi_\ell(x;1)=y_\ell(x) \qquad \ell=1,2,\ldots,n. \quad (22)$$

Thus as q increases from 0 to 1 the solution $\phi_\ell(x;q)$ varies from the initial guess $y_{\ell 0}(x)$ to the solution $y_\ell(x)$. Expanding in Taylor's series with respect to q, we have

$$\phi_\ell(x;q)=y_{\ell 0}(x)+\sum_{m=1}^{\infty} y_{\ell m}(x)\,q^m \quad (23)$$

where

$$y_{\ell m}=\frac{1}{m!}\frac{\partial^m\phi_\ell(x;q)}{\partial q^m}\bigg|_{q=0} \qquad \ell=1,2,\ldots,n \quad (24)$$

If the auxiliary linear operator, initial guess, the auxiliary parameters $\hbar$ and the auxiliary function are so properly chosen the series (23) converges at q = 1, then

$$y_\ell(x)=y_{\ell 0}(x)+\sum_{m=1}^{\infty} y_{\ell m}(x) \;:\qquad \ell=1,2,,\ldots,n \quad (25)$$

Define the vector

$$\vec{y}_\ell(x)=\{y_{\ell 0}(x),\, y_{\ell 1}(x),\ldots,y_{\ell n}(x)\} \quad (26)$$

Differentiating equation (21) m times with respect to q and then putting setting q = 0 and finally dividing them by m! we obtain the m^{th} order deformation equation

$$D^{\alpha_\ell}[y_{\ell m}(x)-\chi_m y_{\ell m-1}(x)]=\hbar_\ell H_\ell(x)\,R_{\ell m}(\vec{y}_{1m-1},\ldots,\vec{y}_{nm-1},x)\;;\;\ell=1,2,\ldots,n-1$$

$$D^{\alpha_n}[y_{nm}(x)-\chi_m y_{nm-1}(x)]=\hbar_n H_n(x)\,R_{nm}(\vec{y}_{1m-1},\ldots,\vec{y}_{nm-1},x) \quad (27)$$

where

$$R_{\ell m}(\vec{y}_{1m-1},\ldots,\vec{y}_{nm-1},x)=\frac{1}{(m-1)!}\frac{\partial^{m-1}[D^{\alpha_\ell}\phi_\ell(x;q)-\phi_{\ell+1}(x;q)}{\partial q^{m-1}}\bigg|_{q=0}$$

$$R_{nm}(\vec{y}_{1m-1},...,\vec{y}_{nm-1},x)=\frac{1}{(m-1)!}\frac{\partial^{m-1}[D^{\alpha_n}\phi_n(x;q)-F(x,\phi_1,\phi_2,...,\phi_n)]}{\partial q^{m-1}}\Bigg|_{q=0} \quad (28)$$

and

$$\chi_m=\begin{cases}0\ , & m\le 1\\ 1\ , & m>1\end{cases} \quad (29)$$

Applying $J^{\alpha l}$ the inverse operator D^{α_ℓ} of on both sides of equation (27), we have

$$y_{\ell m}(x)=\chi_m y_{\ell m-1}(x)-\chi_m\sum_{j=0}^{m-1} y^{(j)}_{\ell m-1}(0^+)\frac{x^j}{j!}+\hbar_\ell H_\ell(x)\,J^{\alpha_\ell}R_{\ell m}(\vec{y}_{\ell m-1},...,\vec{y}_{nm-1},x)$$

$$y_{nm}(x)=\chi_m y_{nm-1}(x)-\chi_m\sum_{j=0}^{m-1} y^{(j)}_{nm-1}(0^+)\frac{x^j}{j!}+\hbar_n H_n(x)\,J^{\alpha_n}R_{nm}(\vec{y}_{1m-1},...,\vec{y}_{nm-1},x) \quad (30)$$

The m-th order deformation equations are linear and thus can be easily solved. We have

$$y_\ell(x)=\sum_{m=0}^{\infty} y_{\ell m}(x)\qquad \ell=1,2,...,n \quad (31)$$

when $M\to\infty$, we get an accurate approximation of original equation (11).

V. Test Examples

Example 1.

$$D^4y(x)+D^{3.5}\,y(x)+y^3(x)=x^9 \quad (32)$$

with the initial conditions

$$y(0)=y'(0)=y''(0)=0\ ,y'''(0)=6 \quad (33)$$

In view of the discussion in the last section the equation (32) can be viewed as the following system of FDE

$$y_1(x)=y(x)$$

then

$$D^{3.5}\,y_1(x)=y_2(x);\ \ y_1(0)=y_1'(0)$$
$$=y_1''(0)=0\ ,\ y_1'''(0)=6$$

and

$$D^{0.5}\,y_2(x)\ \ =-y_2-y_1^3+x^9,\ \ \ y_2(0)=0$$

Using equation ( ), we get the following scheme:

$$y_{10}=x^3\ ,\ y_{20}=0$$

$$y_{1m}=(\chi_m+h_1)[y_{1m\text{-}1}(x)-y_{1m-1}$$
$$(0)]+h_1\,J^{3.5}[-y_{2m-1}]$$

$$y_{11}=h\,J^{3.5}[-y_{20}]=0$$

$$y_{2m}=(\chi_m+h_2)[y_{2m-1}(x)-y_{2m-1}(0)]$$

$$+h_2J^{0.5}\Bigg[y_{2m-1}+\sum_{i=0}^{m-1}y_{1i}\sum_{j=0}^{m-1-i}y_{1j}y_{1m-1-i-j}$$
$$-(1-\chi_m)x^9\Bigg]$$

$$y_{21}=h_2\,J^{0.5}[y_{20}+y_{10}^3-x^9]$$

$$y_{21}=0$$

and hence

$$y_{1m}=0,y_{2m}=0\ ;\ m\ge 1$$

In view of above terms, we find $y_1(x)=x^3$, $y_2(x)=0$ so $y(x)$ x^3 is the required solution of the given equation.

Example 2: Consider the following initial value problem

$$D_*^2y(x)-D_*^{3/2}\,y(x)+y(x)=1+x \quad (34)$$

with the initial conditions

$$y(0)=y'(0)=1 \quad (35)$$

Equation (34) is equivalent to the following system of equations

$$y_1(x) = y(x)$$

$$D^{1.5}\, y_1(x) = y_2(x)\ ; \quad y_1(0) = y_1'(0) = 1$$

$$D^{0.5}\, y_2(x) = y_2(x) - y_1(x) + (1+x)$$

as the initial guess we assume $y_{10} = 1+x$, $y_{20} = 0$.

By HAM the m-th order deformation equations are given by

$$y_{1m}(x) = \chi_m y_{1m-1} + h_1\, J^{1.5}[-y_{2m-1}]$$

$$y_{2m}(x) = \chi_m\, y_{2m-1} + h_2[-y_{2m-1} + y_{1m-1} - (1-\chi_m)(1+x)]$$

$$= h_2\, jJ^{0.5}[-y_{20} + y_{10} - (1-\chi_1)(1+x)] = 0$$

and hence $y_{1m} = 0, y_{2m} = 0\ ; m \geq 1.$

In view of above, we get the exact solution $y(x) = 1+x$.

VI. Conclusion

This paper deals with the approximate solution of a class of multi-order fractional differential equations by Homotopy analysis method. Thus it has been demonstrated that Homotopy analysis method proves useful in solving linear as well as non-linear multi-order fractional differential equation by reducing them into a system of fractional differential equations.

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Comparative Analysis of Threshold Acceptance Algorithm, Simulated Annealing Algorithm and Genetic Algorithm for Function Optimization

By Tejas P. Patalia & Dr. G.R. Kulkarni
Singhania University, Rajasthan, India

Abstract - The goal of this study of threshold acceptance algorithm (TA), simulated annealing algorithm (SA) and genetic algorithm (GA) is to determine strength of Genetic Algorithm over other algorithm. It gives a clear idea of how genetic algorithm works. It gives the idea of various sub methods used in genetic algorithm to improve the results and outcome. Basically genetic algorithm and all traditional heuristic methods are used for optimization. Optimization problems are class NP complete problems. Genetic algorithm can be viewed as an optimization technique which exploits random search within a defined search space to solve a problem by some intelligence ideas of nature. In this work we have done Comparative analysis of Threshold Acceptance Algorithm, Simulated Annealing Algorithm and Genetic Algorithm by considering different test functions and its constraints to minimize the test functions.

Keywords : Heuristic methods, Genetic Algorithm, Chromosomes, Mutation, threshold acceptance algorithm, simulated annealing algorithm, function optimization.

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Comparative Analysis of Threshold Acceptance Algorithm, Simulated Annealing Algorithm and Genetic Algorithm for Function Optimization

Tejas P. Patalia [α] & Dr. G.R. Kulkarni [σ]

Abstract - The goal of this study of threshold acceptance algorithm (TA), simulated annealing algorithm (SA) and genetic algorithm (GA) is to determine strength of Genetic Algorithm over other algorithm. It gives a clear idea of how genetic algorithm works. It gives the idea of various sub methods used in genetic algorithm to improve the results and outcome. Basically genetic algorithm and all traditional heuristic methods are used for optimization. Optimization problems are class NP complete problems. Genetic algorithm can be viewed as an optimization technique which exploits random search within a defined search space to solve a problem by some intelligence ideas of nature. In this work we have done Comparative analysis of Threshold Acceptance Algorithm, Simulated Annealing Algorithm and Genetic Algorithm by considering different test functions and its constraints to minimize the test functions.

Keywords : Heuristic methods, Genetic Algorithm, Chromosomes, Mutation, threshold acceptance algorithm, simulated annealing algorithm, function optimization.

I. Introduction

Optimization is the process of finding absolutely best values of the variables so that value of an objective function becomes optimal. Optimization problems are a class of NP-Complete problems. This work contains overview of threshold acceptance algorithm, simulated annealing algorithm and brief introduction to Genetic algorithm. Genetic algorithm is probabilistic, heuristic, robust search algorithm premised on the evolutionary ideas of natural selection and genetic. Main idea behind the design of genetic algorithm is to achieve robustness and adaptiveness in real world complex problems. Genetic algorithm can be viewed as an Optimization technique, which exploits random search within a defined search space to solve a problem, by some intelligence ideas of nature.

II. Threshold Acceptance Algorithm

Threshold Accepting (TA) is a local search method and was first described by Dueck and Scheuer and Moscato and Fontanari.

Author α : PhD Student, Singhania University – Rajasthan & Sr. Lecturer, V.V.P. Engineering College, Rajkot – Gujarat, India.
E-mail : pataliatejas@rediffmail.com
Author σ : Principal, C.U. Shah College of Engineering & Technology, Wadhwan City - 363030 – Gujarat, India.

A classical local search starts with a random feasible solution and then explores its neighbourhood in the solution space by moving (usually randomly) from its current position, accepting a new solution if and only if it improves the objective function. TA overcomes the problem of stopping in local minima by also allowing uphill-moves that is TA also accepts new solutions which lead to higher objective function values.

To implement TA, three points need to be specified:

1. The objective function f: This function is generally given by the problem at hand.
2. The neighborhood definition (the function N): Given a candidate solution x^c, one needs to define how to move from this solution to an alternative, but 'close' solution x^n.
3. The thresholds: Given a neigbourhood definition, one needs to determine the magnitude of the deterioration in the objective function that the algorithm should still accept for a new solution.

The pseudo-code of TA can be given as follows:

1: Initialize ηRounds and ηSteps
2: Compute threshold sequence Tr
3: Randomly generate current solution $x^c \epsilon X$
4: for r = 1: ηRounds do
5: for i = 1: ηSteps do
6: Generate $xn \epsilon N(xc)$ and compute $\Delta = f(x^n) - f(x^c)$
7: if $\Delta < Tr$ then $x^c = x^n$
8: end for
9: end for
10: $x^{sol} = x^c$

Here, *f* is the objective function to be minimized. x^c denotes the current solution, x^n is the 'new' (or neighbor) solution, and X is the set of feasible solutions.

TA starts with a (random) feasible solution. Given a threshold sequence T of length ηRounds, one can see that TA always accepts a solution that improves the objective function f, but deteriorations are only accepted if they are not worse than a particular threshold, Tr. Over time, the threshold decreases to zero, thus TA turns into a classical local search.

III. Simulated Annealing Algorithm

Simulated Annealing (SA) was introduced by Kirkpatrick. Like other trajectory methods, it evolves a single solution over time. By changing this solution

gradually, the algorithm follows some path ('trajectory') through the search space.

SA starts with a random solution x^c and creates a new solution x^n by adding a small perturbation to x^c. If the new solution is better ($\Delta < 0$), it is accepted. In case it is worse, though, SA applies a stochastic acceptance criterion, thus there is still a chance that the new solution is accepted, albeit only with a certain probability. This probability is a decreasing function of both the order of magnitude of the deterioration and the time the algorithm has already run. The latter feature is controlled by the temperature parameter T which is reduced over time; hence impairments in the objective function become less likely to be accepted and eventually SA turns into classical local search. Here, the algorithm stops after a predefined number of iterations Rmax; of course, alternative stopping criteria are possible.

The pseudo-code of SA can be given as follows:

1: Generate initial solution x^c, initialize Rmax and T
2: for r = 1 to Rmax do
3: while stopping criteria not met do
4: Compute $x^n \epsilon N(x^c)$ (neighbour to current solution)
5: Compute $\Delta = f(x^n) - f(x^c)$ and generate u (uniform random variable)
6: if ($\Delta < 0$) or ($e^{-\Delta/T} > u$) then $x^c = x^n$
7: end while
8: Reduce T
9: end for

IV. Genetic Algorithm

What is Genetic Algorithm?

Genetic algorithms are probabilistic, robust and heuristic search algorithms premised on the evolutionary ideas of natural selection and genetic.

Darwin's Principle of Natural Selection

IF there are organisms that reproduce, and

IF offspring's inherit traits from their progenitors, and

IF there is variability of traits, and

IF the environment cannot support all members of a growing population,

THEN those members of the population with less adaptive traits will die out, and

THEN those members with more-adaptive traits will thrive.

Concept

The basic concept of genetic algorithms is designed to simulate the processes in natural system necessary in for evolution, specifically for those that follow the principle of *survival of the fittest*. They represent the intelligent exploitation of a random search within a defined search space to solve a problem. Genetic Algorithm is developed by John Holland and his students at Michigan University during 1965-1975.

Encoding

Encoding is the first step towards genetic algorithm. First the data is encoded with the help of some encoding technique. Then it is given to genetic algorithm. Selection of encoding technique depends upon the problem. Different types of encoding techniques are available. They are as follows

Binary Encoding

Most common method of encoding. Chromosomes are strings of 1s and 0s and each position in the chromosome represents a particular characteristic of the problem

Chromosome A 110011001110110

Chromosome B 100011001100111

Permutation Encoding

Useful in ordering problems such as the Traveling Salesman problem (TSP). In TSP every chromosome is a string of numbers, each of which represents a city to be visited.

Chromosome A 1 5 3 2 6 4 7 9 8

Chromosome B 8 5 6 7 2 3 1 4 9

Value Encoding

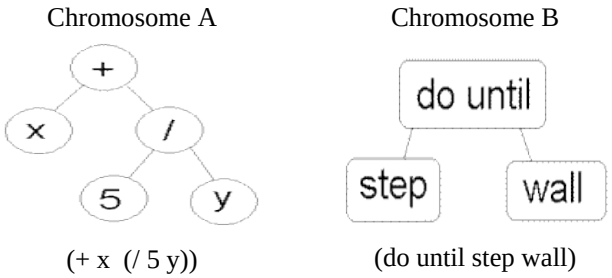
Used in problems where complicated values, such as real numbers, are used and where binary encoding would not suffice. Good for some problems, but often necessary to develop some specific crossover and mutation techniques for these chromosomes.

Chromosome A 1.235 5.323 0.454 2.321 2.454

Chromosome B (left), (back), (left), (right), (forward)

Tree Encoding

Tree encoding is used mainly for evolving programs or expressions. In the tree encoding every chromosome is a tree of some objects, such as functions or commands in programming language. Tree encoding is useful for evolving programs or any other structures that can be encoded in trees. Programming language LISP is often used for this purpose, since programs in LISP are represented directly in the form of tree and can be easily parsed as a tree, so the crossover and mutation can be relatively easily.



a) Components of Genetic Algorithm

i. *Chromosomes*

All living organisms are different then other

organisms of the same species as well as different species. Even twins have at least some minor differences. These differences are due to genetic structure, which is called chromosomes. Chromosomes or individuals are consisting of genes. Genes may contain different possible values depending on the environment, constraints and struggle to survive. Each gene is responsible for some part of solution but we cannot identify the role of each gene individually because they work collectively and their inter-relations are complex. The encoding process of solution as a chromosome is most difficult aspect of solving any problem using genetic algorithm. Encoding of solution as a chromosome is known as genotype and its equivalent physical representation is known as phenotype.

ii. *Fitness Function*

In the nature the organism's "fitness" can be measured by its ability to reproduce, to adapt and to survive. In genetic algorithm chromosomes should be measured by some technique to decide which chromosomes are good compared to other chromosomes. Fitness function is a objective or evaluation function which is used to measure how good a chromosome is. Fitness function assigns fitness value to each chromosome using genetic structure and relevant information of the chromosome. Fitness function plays a big role because subsequent genetic operators use fitness values to select chromosomes. Different fitness functions are used depending on type and solution vector of problem. For function optimization problems, fitness function may be the value of objective function.

iii. *Reproduction*

Reproduction or selection is based on the concept natural selection and it is one of the main three operators used in genetic algorithm. The main objective of reproduction operator is to emphasize good chromosomes in a population. Reproduction makes multiple copies of relatively good chromosomes at the cost of relatively bad chromosomes while keeping population size constant. The essential idea is that chromosomes having a higher fitness value have a higher probability of selection. The identification of good or bad chromosomes is done using fitness value of the chromosomes. Many selection methods are available, some of them make multiple copies of the chromosomes on the basis of probability, where as some make multiple copies deterministically.

1) Roulette Wheel Selection Method

In this method each chromosome in the population occupies an area of the roulette wheel proportional to its fitness value. Chromosomes with better fitness occupies large fraction of roulette wheel where as chromosomes with bad fitness occupies small fraction of roulette wheel. Then roulette wheel is spun as many times as population size. Each time roulette wheel pointer points one chromosome and that chromosome is placed in mating pool. A chromosome with a higher fitness is likely to receive more copies than a chromosome with a lower fitness. Roulette Wheel Selection method is widely used for the maximization problems but it has two main drawbacks:

i. It can handle only maximization problem so minimization problem must be converted into an equivalent maximization problem.
ii. If a population contains a chromosome having exceptionally better fitness compared to the rest of the chromosomes in the population then this chromosome occupies most of the roulette wheel area. Thus, almost all the spinning of the roulette wheel is likely to choose the same chromosome, this may result in the loss of genes diversity and population may coverage to local optima.

2) Rank Based Selection Method

Rank based selection uses fitness value of the chromosomes to sort chromosomes in to ascending or descending order depending on the minimization or maximization problem. Then it assigns reproduction probability and ranked fitness to each chromosome on the basis of only rank order of the chromosome in the current population. Rank based selection also assigns some probability to the worst chromosome so that it has some chance for getting selected.

3) Steady State Selection

In the steady state selection in every generation a few good chromosomes are selected for creating new offspring. Then some bad chromosomes are removed and the new offspring is placed. The rest of population survives to new generation.

4) Elitism

After crossover and mutation new population is generated. With the help of elitism we can store the best found chromosomes. The remaining chromosomes are delivered for the next generation. In this way we cannot lose best chromosomes.

5) Tournament Selection

Runs a “tournament” among a few individuals chosen at random from the population and selects the winner (the one with the best fitness) for crossover. Two entities are picked out of the pool, their fitness is compared, and the better is permitted to reproduce. Advantage is decreases computing time.

b) Operators of Genetic Algorithm

i. *Crossover*

Two parents chromosomes are selected randomly from the mating pool, few genes of the chromosomes are exchanged between these two parents and offspring are produced. In general crossover operator recombines two chromosomes so it is also known as recombination. Crossover is intelligent search operator that exploits the information acquired by

the parent chromosomes to generate new offspring. If both the parent has same genetic structure then offspring are just copies of the parent irrespective of cutting point but if parent have different genetic structure then offspring are different then parent. Thus, crossover is sampling process, which samples new points in search space. Generally crossover probability is very high like 1.00, 0.95, 0.90 etc.. Different types of Crossover methods can be used i.e. 1-point crossover, n-point crossover and uniform crossover.

ii. *Mutation*

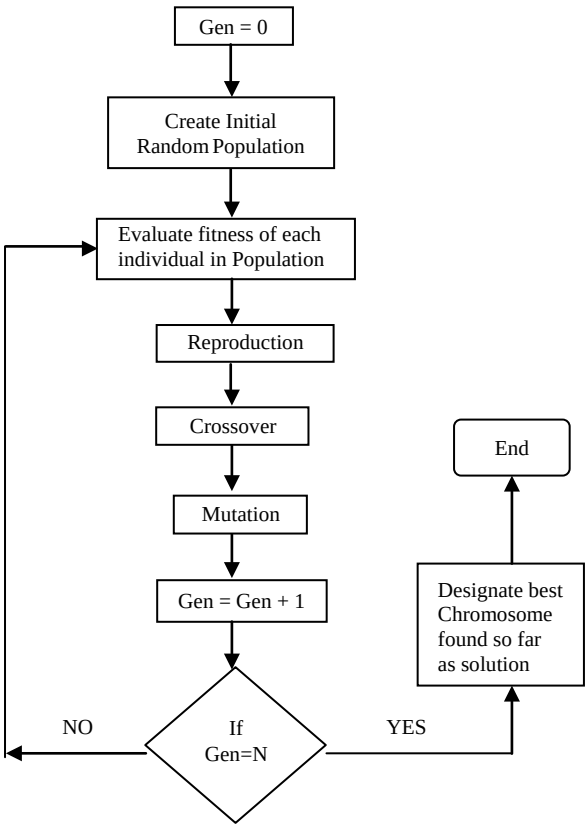
Mutation is secondary operator used in genetic algorithm to explore new points in the search space. In the latter stages of a run, the population may converge in wrong direction and stuck to local optima. The effect of mutation is to reintroduce divergence into a converging population. Mutation operator selects one chromosome randomly from the population, then selects some genes using mutation probability and flips that bit. So mutation is a random operator that randomly alters some value. Mutation either explores some new points in the search space and leads population to global optima direction or alters value of the best chromosome and losses knowledge acquired till now. So mutation should be used rarely. Generally per gene probability of mutation is 0.001, 0.01, 0.02 etc..

Flip a bit

Parent 110-011-010-001-101

Child 110-111-010-001-101

iii. *GA Flowchart*



The pseudo-code of GA can be given as follows:

1. Set the values of the parameters regarding population size, probability of crossover, probability of mutation, number of generations, and all the other parameters.
2. Generate random initial population of chromosomes.
3. Select two of the chromosomes as parents, with probability proportional to their fitness.
4. If crossover is used, combine the genes of these chromosomes using the crossover operator to form two children chromosomes. In the case no crossover is applied, the children chromosomes will be initially, just copies of the parent chromosomes.
5. Then apply the mutation operator to the children chromosomes, so that some (if any) random bits of the children chromosomes are inverted.
6. Repeat steps 4-6, until children chromosomes have been formed.
7. Repeat steps 3-7 until the specified number of generations have passed.

V. Test Functions

Different types of Nine Test Functions are considered as given below in Table 1. Threshold Acceptance Algorithm (TA), Simulated Annealing Algorithm (SA) and Genetic Algorithm (GA) are applied to these test functions.

Table I : List of 9 Test Functions

Function Name	9 Test Functions	Cons-traints	Function Value
F1	$f(x_1, x_2)=x_1 + x_2$	[0,1]	Minimize
F2	$f(x_1, x_2)=x_1^2 + x_2^2$	[0,1]	Minimize
F3	$f(x_1, x_2)=20 + x_1^2 + x_2^2$	[0,1]	Minimize
F4	$f(x_1, x_2)=x_1^4 + x_2^4 + (2 \times x_1 \times x_2)$	[0,1]	Minimize
F5	$f(x_1, x_2)=x_1^2 + x_2^2 + (20 \times x_1 \times x_2)$	[0,1]	Minimize
F6	$f(x_1, x_2, x_3, x_4)=x_1^2 + x_2^2 + x_3^2 + x_4^2$	[0,1]	Minimize
F7	$f(x_1, x_2)=(x_1 \times \sin\sqrt{x_1}) + (x_2 \times \sin\sqrt{x_2})$	[0,1]	Minimize
F8	$f(x_1, x_2)=x_1^2 - (10 \times \cos(2 \times \pi \times x_1)) + 10 + (10 \times \cos(2 \times \pi \times x_2)) + 10$	[0,1]	Minimize
F9	$f(x_1, x_2)=(4 \times x_1^2) + (2.1 \times x_1^4) + (x_1^6 / 3) + (4 \times x_1 \times x_2) + (4 \times x_2^2) + (4 \times x_2^4)$	[0,1]	Minimize

VI. Results by Comparative Analysis of TA, SA and GA

Function Name	No. of Iterations			Time in Seconds		
	TA	SA	GA	TA	SA	GA
F1	1506	5924	100	15	30	5
F2	1000	1637	51	9	12	2
F3	1000	1462	52	7	8	2
F4	1000	2620	51	7	13	2
F5	5918	5940	100	25	25	4
F6	2000	3483	64	17	19	3
F7	2220	1160	51	18	7	2
F8	1196	3079	51	9	12	2
F9	1000	1168	51	5	7	2

VII. Conclusion

Genetic algorithm is probabilistic, heuristic, robust search algorithm premised on the evolutionary ideas of natural selection and genetic. Main idea behind the design of genetic algorithm is to achieve robustness and adaptiveness in real world complex problems. From the above results we have concluded that genetic algorithm is more reliable, strong and robust than threshold acceptance algorithm and simulated annealing algorithm.

VIII. Acknowledgement

I take this opportunity to express my immense gratitude to my Research Guide Dr. G. R. Kulkarni. I am hearty grateful to him for his prolonged interest, excellent guidance and constant inspirations. The success of this work would not be possible without his uncompromising demand for quality, and his reviews of my work have helped me lot to complete this research paper

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23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be

sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to

shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic

principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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