Abstract - Orthogonal Frequency Division Multiplexing (OFDM) is a very attractive digital transmission method developed to meet the increasing demand for higher data rates in communications which can be used in both wired and wireless environments. OFDM consist of large number of independent subcarriers, as a result of which the amplitude of such a signal can have high peak values. High peak to average power ratio (PAPR) of the transmitted signals is one of the major drawbacks of OFDM. Several approaches exist to reduce PAPR of OFDM symbols. Precoding is a new method which is having less complexity compared to the other power reduction techniques and also it can reduce PAPR considerably and results in no distortion. In this paper, a combination of clipping and precoding technique is proposed to reduce PAPR. The reduction in PAPR of the OFDM signal is obtained through Discrete Cosine Transform-II (DCT) combined with clipping algorithm. The obtained results show that this precoding scheme is an attractive solution to the PAPR problem of OFDM signals. The proposed technique gives good result because the PAPR of precoded clipped signal would be less than that of single precoded signal.

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I. INTRODUCTION

Multicarrier communications is a technique that has recently seen rising popularity in wireless and wire line applications [1, 2, 3]. During the past 15 years, Orthogonal Frequency Division Multiplexing (OFDM) has been gaining year after year a well-deserved reputation, demonstrating its high data rate and robustness to wireless environments capabilities. In the multipath environment, broadband communications will suffer from frequency selective fading. However, wireless channels have some disadvantages, like multipath fading, that make them difficult to deal with. A modulation that efficiently deals with selective fading channels is orthogonal frequency division multiplexing (OFDM). OFDM is an attractive modulation scheme used in broadband wireless systems that encounters large delay spreads. The increasing interest in this technique can be ascribed to the advancing capabilities of digital signal processors. International standards making use of OFDM for wireless. For wireless applications, OFDM based systems can be of interest because they can provide a greater immunity to impulse noise and fast fades and eliminate the need for equalizers.

Fig. 1 shows the generation of an OFDM signal. To generate OFDM signal successfully the relationship between all the carriers must be carefully controlled to maintain the orthogonality between the carriers. OFDM is generated by first choosing the spectrum required, based on the input data and modulation scheme used. Each carrier to be produced is assigned some data to transmit.

A high-rate data stream is split into a number of lower rate streams to be transmitted simultaneously over a number of sub-carriers. Since the symbol duration increases for lower rate parallel sub-carriers, the amount of dispersion in time caused due to multipath delay is reduced. These carriers divide the available transmission bandwidth. The separation of the sub-carriers is such that there is a very compact spectral utilization and each being modulated at a low bit rate. In a conventional frequency division multiplex the carriers are individually filtered to ensure there is no spectral overlap.

OFDM offers many well documented advantages for multicarrier transmission at high data rates, particularly in mobile applications. One of the major drawbacks of OFDM systems is that the OFDM signal exhibits a high peak-to-average power ratio (PAPR). An OFDM signal consists of a number of independently modulated single carriers, which can give a large peak when added up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. A large PAPR brings disadvantages like an increased complexity converters and a reduced efficiency of the RF power amplifier that cause undesired distortion of the in-band and out-of-band signals. Such a high PAPR necessitates the linear amplifier to have large dynamic range which is difficult to accommodate.

To reduce the PAPR, several techniques have been proposed, which can be divided into three categories. First, there are signal distortion techniques, which reduce the peak amplitudes by nonlinearly distorting the OFDM signal at or around the peaks. Examples of distortion techniques are clipping [4], peak windowing [5], and peak cancellation. Second, there are coding techniques that use a special code set that excludes OFDM symbols with a large PAPR. Golay
complementary sequence [6], Reed Muller code [7], M-sequence [8], Hadamard code can be used in this approach [9]. The third technique scrambles each OFDM symbol with different scrambling sequences and selecting the sequence that gives the smallest PAPR such as selected mapping (SLM) [10,11] and partial transmit sequence (PTS) [12].

index and n is a subcarrier index. Then within the time interval T the following signal of the OFDM block period can be described as:

\[ x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} p_n(t - mT) \]  

Where, \( p_n(t) \) is a rectangular pulse applied to each subcarrier [13]

The precoding based techniques, show great promise as they are simple linear techniques to implement without the need of any side information. This combines the philosophy of pre-coding and peak clipping in place of clipping of entire symbol. This combination shows good results, because of pre-coding makes the envelope almost constant and then peak clipping further reduces the peak value. In this paper, the comparison of results obtained by proposed algorithm is made with results obtained by precoding technique, which clearly show the superiority of proposed method. The organization of this paper is as follow. Section ii formulates the problem of PAPR in OFDM systems. Section iii describes the precoding techniques. Clipping the existing PAPR reduction technique is described in section iv. Section v explains the proposed scheme. Simulations results will be presented in section vi and finally some conclusions are presented in section vii

II. PAPR IN OFDM SYSTEM

A multicarrier signal is the sum of many independent signals modulated on to subchannels of equal bandwidth. An OFDM symbol consists of N subcarriers by the frequency spacing of \( \Delta f \). Thus, the total bandwidth B will be divided into N equally spaced subcarriers and all the subcarriers are orthogonal to each other within a time interval of length \( T = 1/\Delta f \). Each subcarrier can be modulated independently with the complex modulation symbol \( X_{m,n} \), where \( m \) is a time and \( p_n(t) \) is defined as:

\[ p(t) = \begin{cases} \exp(j2\pi m \Delta f t), & 0 \leq t \leq T \\ 0, & \text{else} \end{cases} \]

The total continuous time signal \( x(t) \) consisting of all the OFDM block is given as:

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{M} \sum_{n=0}^{N-1} X_{m,n} p_n(t - mT) \]

For a single OFDM symbol consider \( m = 0 \) without loss of generality. This can be shown because there is no overlap between different OFDM symbols. Since \( m = 0 \), \( X_{m,n} \) can be replaced by \( X_n \). Then, the OFDM signal can be described as follows:

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t} \]

If the bandwidth of the OFDM signal is \( B = N \times \Delta f \) and the signal \( x(t) \) is sampled by the sampling time of \( \Delta t = \frac{1}{B} \), then the OFDM signal is in discrete time form and can be written as shown in:

\[ x_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_n e^{j2\pi k n N}, \quad k = 0, 1, ..., N-1 \]
Due to the presence of large number of independently modulated sub-carriers in an OFDM system the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as Peak-to-Average Power Ratio (PAPR). Coherent addition of N signals of same phase produces a peak which is N times the average signal. The PAPR of the transmitted signal is defined as by:

$$ PAPR[x(t)] = \frac{\max|x(t)|^2}{\frac{1}{NT} \int_{0}^{T} |x(t)|^2 dt} $$

(6)

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems. Also due to the large fluctuations in power output the high power amplifier (HPA) should have large dynamic range. This results in poor power efficiency.

In order to evaluate the performance of the proposed PAPR reduction scheme, the MATLAB simulator evaluates the complementary cumulative distribution function (CCDF) of the PAPR of the given OFDM signal. The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. The CDF of the amplitude of a signal sample is given by:

$$ F(z) = 1 - \exp(-z) $$

(7)

However, the complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. The CCDF of the PAPR of the data block is desired in our case to compare outputs of various reduction techniques. This is given as:

$$ P(PAPR > z) = 1 - F(z) $$

$$ = 1 - (1 - \exp(-z)) $$

(8)

III. DISCRETE COSINE TRANSFORM-II

The goal of precoding techniques is to obtain a signal with lower PAPR than in the case of OFDM without precoding techniques and to reduce the interference produced by multiple users. The PAPR reduction must compensate the non linearities of the HPA having as effect the reduction of the bit error rate (BER). The main characteristics of precoding based techniques are, no bandwidth expansion, no power increase, and no data rate loss, no BER degradation and distortion less. Some precoding techniques are presented in the following.

The discrete cosine transforms (DCT) are the members of a family of sinusoidal unitary transforms. The Discrete Cosine Transform (DCT) is a Fourier-like transform, which was first proposed by Ahmed et al. (1974) [14]. They are real, orthogonal, and separable with fast algorithms for its computation. They have a great relevance to data compression. PAPR in an OFDM signal, a DCT is applied to reduce the autocorrelation of the input sequence before the IFFT operation is applied [15]. DCT-II compresses the bandwidth of a wide range of signals and also reduces the bit-rate. The formal definition of DCT of length N is given by the following formula:

$$ X_c(b) = a(b) \sum_{n=0}^{N-1} x(n) \cos \left[ \frac{\pi}{N} \left( n + \frac{1}{2} \right) b \right] $$

(9)

where b = 0, 1,..., N-1

Similarly, the inverse transformation is defined as:

$$ x(n) = a(b) \sum_{b=0}^{N-1} X_c(b) \cos \left[ \frac{2\pi(n+1)b}{2N} \right] $$

(10)

DCT matrix P of size N-by-N can be created by using equation:

$$ a(k) = \begin{cases} \frac{1}{\sqrt{N}} & \text{if } i = 0, \ 0 \leq j \leq N - 1 \\ \frac{\sqrt{2}}{\sqrt{N}} \cos \left( \frac{\pi(2j+1)i}{N} \right) & \text{if } 1 \leq i \leq N - 1 \\ 0 & \text{if } 0 \leq j \leq N - 1 \end{cases} $$

(11)

IV. CLIPPING TECHNIQUE

Signal distortion techniques minimize high peak dramatically by distorting signal before amplification. One of the simple and effective PAPR reduction techniques is clipping [16], which cancels the signal components that exceed some unchanging amplitude called clip level. The clipping technique has low implementation complexity clipping simultaneously increases the average value and minimizes the peak value. The clipping operation is carried out at the transmitter. The following equation describes the method of amplitude clipping as:

$$ y[s] = \begin{cases} -L; & y[s] \leq -L \\ y[s]; & |y[s]| < L \\ L; & y[s] \geq L \end{cases} $$

(12)
where \( L \) is the pre-specified clipping level. After which the exceeded signal was clipped.

![OFDM Signal](image1)

**Figure 2 (a)**: Signal with high peak value

Fig 2(a) shows the plot of an OFDM signal having maximum amplitude of 1. The peak of the OFDM signal exceeds 0.8. This exceeded peak of the signal causes PAPR.

![Clipped Signal](image2)

**Figure 2 (b)**: Signal with clipped peak value

To reduce the peak a threshold value was assumed in Fig 2(b) the peak of the signal that exceeds the value of 0.6 and -0.6 was clipped off.

V. **Proposed Scheme**

The main idea of the proposed scheme is to use a combination of two appropriate methods. One is the DCT-II transform technique and the other is the clipping technique. The proposed combined peak clipping is different than normal peak clipping method because in normal peak clipping only one peak of OFDM symbol is clipped whereas in proposed method, clipping was combined with DCT-II precoding technique. The precoding has been considered as a best among all these techniques be as it improves PAPR without increasing much complexity and without destroying the orthogonality between subcarriers. The combined technique based OFDM system was shown in fig.3. DCT-II transform is used before conventional clipping processing unit in proposed scheme. At the transmitter end, the data stream is firstly transformed by DCT-II, and then the transformed data is processed by the clipping unit. If data block passed by DCT-II, before IFFT, the PAPR is reduced, then the PAPR of OFDM signal could be further reduced by clipping.

In these system, the kernel of the DCT-II acts as a precoding matrix \( P \) of dimension \( N=L \times L \) and it is applied to constellations symbols before the IFFT to reduce the correlation among the input sequence. In the proposed based systems baseband modulated data is passed through S/P converter which generates a complex vector of size \( L \) that can be written as \( X=[X_0, X_1, ..., X_{L-1}]^T \). Then precoding is applied to this complex vector which transforms this complex vector into new vector of length \( L \) that can be written as \( K=PX=[K_0, K_1, ..., K] \), where \( P \) is a precoder matrix of size \( N=L \times L \) and \( Y \) can be written as follows:

\[
P_{m,i} = \sum_{k=0}^{L-1} P_{m,k} i, \quad m=0, 1, ..., L-1 \quad (13)
\]

\( P_{m,i} \) means \( m \)th row and \( i \)th column of precoder matrix. The complex baseband OFDM signal with \( N \) subcarriers can be written as:

\[
x_n = \sum_{m=0}^{N-1} X_n e^{j2\pi m^2 N}^L, \quad n=0, 1, 2, ..., N-1 \quad (14)
\]

After applying the DCT-II on OFDM signal the signal get compressed decreasing the PAPR value. Clipping is performed on the I and Q outputs of the IFFT after precoding. As the word length at the IFFT output is decreased, the power consumption and complexity of the DAC/ADC decreases. Also clipping at the IFFT output increases the resolution giving a better average signal.
VI. Simulation Results

In this section, computer simulations are used to evaluate the peak-to-average ratio reduction capability with proposed scheme. In simulation, an OFDM system is considered where data is randomly generated with subcarriers N=2400, then the signal is modulated by M-PSK and M-QAM respectively. The channel modeled is an additive white Gaussian noise (AWGN). The PAPR reduction capability is measured by the complementary cumulative distribution (CCDF = \( \text{Prob}(\text{PAPR}>\text{PAPR}_0) \)), which indicates the probability that PAPR is above a certain threshold. We compared the simulation results of proposed system with DCT-II precoded OFDM signal and precoded clipped signal. The results are presented in following figures and then the reduction in PAPR value was observed for precoded signal and for clipped precoded signal.

\( a) \) M-PSK Modulation

In this section clipping technique is applied over DCT-II precoded OFDM signal with phase shift keying modulation. We assume CCDF clip rate of \( 10^{-0.5} \) using M-ary PSK technique for N=2400 and M=16, 32, 64, 128,256, 512 and 1024.

Figure 3: Signal Block scheme of DCT-II Precoding technique with clipping in OFDM system

Figure 4: CCDF of clipping with proposed DCT-II technique for 16 PSK

At CCDF of for \( 10^{-0.5} \) OFDM signal Fig. 4 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. The PAPR value decreases by 0.6 dB for PSK modulation at M=16 for precoded clipped signal over DCT-II precoded signal.
At CCDF of for $10^{-0.5}$ OFDM signal Fig. 5 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. The PAPR value decreases by 0.6 dB for PSK modulation at $M=32$ for precoded clipped signal over DCT-II precoded signal.

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 6 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. The PAPR value decreases by 0.6 dB for PSK modulation at $M=64$ for precoded clipped signal over DCT-II precoded signal.

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 7 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. The PAPR value decreases by 0.7 dB for PSK modulation at $M=128$ for precoded clipped signal over DCT-II precoded signal.

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 8 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. The PAPR value decreases by 0.6 dB for PSK modulation at $M=256$ for precoded clipped signal over DCT-II precoded signal.
Figure 9: CCDF of clipping with proposed DCT-II technique for 512 PSK

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 9 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 0.6 dB for PSK modulation at $M=512$ for precoded clipped signal over DCT-II precoded signal.

Figure 10: CCDF of clipping with proposed DCT-II technique for 1024 PSK

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 10 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 0.6 dB for PSK modulation at $M=1024$ for precoded clipped signal over DCT-II precoded signal.

Figure 11: CCDF of clipping with proposed DCT-II technique for 16 QAM

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 11 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 0.5 dB for QAM modulation at $M=16$ for precoded clipped signal over DCT-II precoded signal.

Figure 12: CCDF of clipping with proposed DCT-II technique for 64 QAM

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 12 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 0.8 dB for QAM modulation at $M=16$ for precoded clipped signal over DCT-II precoded signal.

b) QAM Modulation

For $M=16$, 64, 256 and 1024 M-ary quadrature amplitude modulation (QAM) technique is used here.
Figure 13: CCDF of clipping with proposed DCT-II technique for 256 QAM

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 13 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 1.5 dB for QAM modulation at $M=256$ for precoded clipped signal over DCT-II precoded signal.

Figure 14: CCDF of clipping with proposed DCT-II technique for 1024 QAM

At CCDF of for $10^{-0.5}$ OFDM signal Fig. 14 shows the CCDF performance of the proposed clipped scheme compared with that of the DCT-II precoded technique. A The PAPR value decreases by 1.52 dB for QAM modulation at $M=1024$ for precoded clipped signal over DCT-II precoded signal.

**VII. Conclusion**

In this paper, we evaluated clipping with DCT-II precoding techniques that can be used for PAPR reduction in OFDM systems. The proposed combined technique is simple to implement and has no limitations on the system parameters such as number of subcarriers modulation order, and constellation type. This system produce the lowest PAPR and is efficient, signal independent, distortion less and do not require any complex optimizations representing better PAPR reduction methods than others existing techniques because it does not require any power increment, complex optimization and side information to be sent to the receiver. From simulation results, it can be observed that the proposed combined clipped method displays a better PAPR reduction performance than the DCT-II precoded OFDM signal. Thus, it is concluded that the proposed clipped scheme is more favorable than the precoded DCT-II transform.

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**References**

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