Analytical Investigation of Cargo Motion Lengthwise the Wagon under the Action of Plane Force System

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Abstract- For the first time in the theory of solid cargo fastening there has been investigated a case when the cargo is in motion in relation to the wagon floor with acceleration For the first time in the theory of solid cargo fastening there has been investigated a case when the cargo is in motion in relation to the wagon floor with acceleration For the first time in the theory of solid cargo fastening there has been investigated a case when the cargo is in motion in relation to the wagon floor with acceleration $\overline{a}_r$, its speed being at this moment equal to $\overline{v}_r$. There have been set out the results of analytical investigation of cargo shift in dynamics and accordingly elongation and tension in flexible fastening elements under the action of plane force system. It has been established that the longitudinal force perceived by the flexible fastening elements in value is smaller than the force obtained when inertia in relative motion (at rest) is not taken into account. Hence, the cargo shift lengthwise the wagon in this case will be smaller. This, in its turn, will affect the decrease of elongation value and consequently the decrease of the effort of every flexible element, thus increasing their load-carrying capacity.

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ANALYTICALINVESTIGATIONOFCARGOMOTIONLENGTHWISETHEWAGONUNDERTHEACTIONOFPLANEFORCESYSTEM

Strictly as per the compliance and regulations of:

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Summary- For the first time in the theory of solid cargo fastening there has been investigated a case when the cargo is in motion in relation to the wagon floor with acceleration $\overline{a}_r$, its speed being at this moment equal to $\overline{v}_r$. There have been set out the results of analytical investigation of cargo shift in dynamics and accordingly elongation and tension in flexible fastening elements under the action of plane force system. It has been established that the longitudinal force perceived by the flexible fastening elements in value is smaller than the force obtained when inertia in relative motion (at rest) is not taken into account. Hence, the cargo shift lengthwise the wagon in this case will be smaller. This, in its turn, will affect the decrease of elongation value and consequently the decrease of the effort of every flexible element, thus increasing their load-carrying capacity.

Keywords: cargo, thrust bars, flexible fastening elements, cargo shift in dynamics, efforts in flexible fastening elements.

I. Formulation of a Problem

Formulas derived for determining efforts in flexible fastening elements of the cargo under the action of longitudinal and vertical forces presented in Appendix 8-Technical conditions [1, 2], (as has been pointed out in [3 – 22]) have been the result of incompletely solved problems when the longitudinal force value perceived by fastening means according to the gravity power of cargo G is understated (i. e. is always within the limits (0,97 ÷ 1,2)G) while during shunting collisions in a hump-yard or emergency braking this force may vary within – (1,2 ÷ 2)G . Moreover, they don't take into account the efforts of preliminary twisting of every fastening wire $R0$, without which the cargo is not liable to dispatching. Just because due to effort $R0$ the cargo is pressed against the wagon floor, friction force is increased. In [1, 2] there is no mention of the notion «shift of the cargo lengthwise the wagon" and hence, no mention of "elongation of each fastening element" to the value of which the efforts in each fastening elements are according to Hooke's law directly proportional. As a result, the efforts of each fastening element have one and the same value, which disagrees with reality. It should be noted that in [3 – 22]

a technical problem of cargo fastening under the action of space force system and, as a special case, under the action of plane force system, is solved within the fundamental law of dynamics during relative motion at rest. Unfortunately, there has not been yet considered the case when the cargo is moving lengthwise the wagon floor with acceleration $\overline{a}_r$ its speed at the moment being equal to $\overline{v}_r$ [23, 24].

On this basis it can be noted that determining of cargo shift lengthwise the wagon floor and correspondingly elongation and efforts in each fastening element during cargo motion with acceleration lengthwise the wagon floor at a given relative speed is an urgent technical problem for transport research.

a) *Problem Formulation In Dynamics (It is for the first time that the problem is set)*

To derive an analytical formula of cargo shift lengthwise the wagon, elongation and efforts in flexible fastening elements in case of the cargo moving in relation to the wagon floor with acceleration $\overline{a}_r$ at speed $\overline{v}_r$, as in case of motion of deformable thread on an imperfect curved surface [25].

b) *Problem Specification*

As in [7], let us consider the case, when cargo with gravity force $\overline{G}$, located on the wagon on down grade at angle Ψ_0 (rad. 0.006 ÷ 0.021 or 0.344 ÷ 1.2 degrees which agrees with grade within 6 ÷ 21°/∘∘) in the mode of both brake release and service braking is kept from lengthwise shifting by flexible fastening elements. The contours of the cargo when it is placed on the wagon the effective area makes it possible to use thrust and/or spacer wooden bars (Fig. 1a, b).

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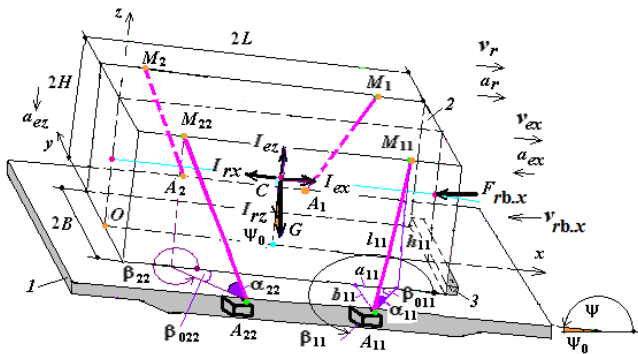*e-mail: khturanov@yandex.ru*

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Figure 1 a : Diagram of allocating cargo and thrust bar on the wagonmoving down grade on the tangent
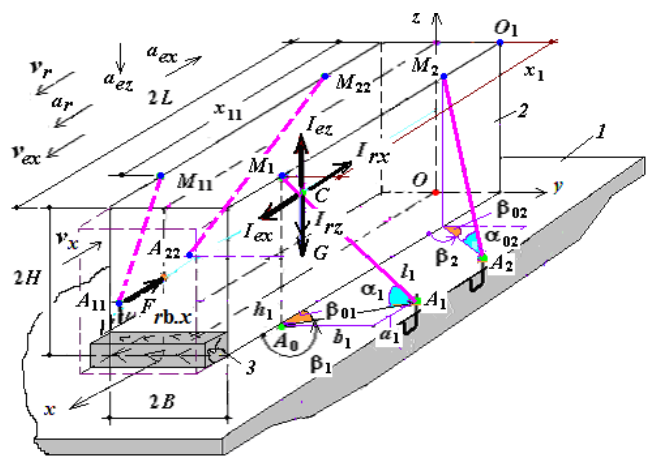1 – wagon , 2 – cargo, 3 – thrust bar

Figure 1 b : Diagram of allocating cargo and thrust bar on the wagonmoving on a tangent

In Fig. 1 a, b as an example the following symbols are accepted: A_1M_1 , $A_{11}M_{11}$ and A_2M_2 , $A_{22}M_{22}$ are flexible fastening elements of both directions; M_1, M_2, M_{11}, M_{22} are shipping loops (eyelets) [17]. It also has symbols: $I_{rx} = (-Ma_{rx})$ and $I_{rz} = (-Ma_{rz})$ – inertia forces in relative motion on lengthwise and vertical axes; $\overline{I}_{ex}$ – lengthwise and vertical transferring inertia forces; $\overline{F}_{rb.x}$ – aerodynamic resistance force [24].

c)   Man-Made Assumption

In working out a computable model as in [17] we assume wagon frame to be the major constrain for the cargo (object) and flexible elastic fastening elements and thrust bar to be additional constraints [10, 18, 23, 24].

We assume that effective longitudinal and vertical forces are perceived by flexible elastic fastening

elements $A_{11}M_{11}$ and $A_{22}M_{22}$ located oppositely the action of longitudinal forces while fastening elements of opposite direction A_1M_1 and A_2M_2 sag (Fig. 1a). Fastening elements A_1M_1 and A_2M_2 , as applied to Fig. 1b,on the contrary, perceive external forces while $A_{11}M_{11}$ and $A_{22}M_{22}$ sag (i.e. lose a constraint). An additional constraint (a thrust bar) is also a non-ideal and non-retentive (single-sided) one, it prevents the cargo from shifting from the contact plane to one side (to the right) and not keeping it from shifting to the other side (to the left).

Let us assume that flexible fastening elements are pre-tensioned by efforts $R0_i$ (for example, $R0_1 = R0_{11} = R0_2 = R0_{22} = 20$ kN), and they increase normal constituent $\overline{N}$ of constraint reaction (platform floor), therefore, cargo and floor cohesion force $\overline{F}_{coh}$ (and hence sliding friction $\overline{F}_{slid.x}$ meaning that $\overline{F}_{slid.x} < \overline{F}_{coh.}$).

As it is known [7, 10, 18], external constraint reaction (non-ideal) $\overline{R}$ is resolved into normal $\overline{N}$ and tangent $\overline{F}_{\tau}$ component, i.e. $\overline{R} = \overline{N} + \overline{F}_{\tau}$. Coordinates x_R , y_R (or x_N , y_N), points of application of external constraints reaction $\overline{R}$ are not known and are to be defined.

d)   Formation of Dynamic Model

We apply theoretically to the mass center of material system (cargo) C just as in Fig. 1a,b the active force – gravity force $\overline{G}$, inertia force at relative motion along the lengthwise $I_{rx} = (-Ma_{rx})$ and vertical axis $I_{rz} = (-Ma_{rz})$, longitudinal and vertical transferring inertia forces $\overline{I}_{ex}$ and $\overline{I}_{ez}$ and direct them from the object and also aerodynamic resistance force $\overline{F}_{rb.x}$ which we direct to the object. We will take into account the fact that these forces exert influence on the external constraints (platform and fastening means). As to the object (cargo), we formally apply external forces (reactive forces) – normal component $\overline{N}$ of wagon floor reaction and tangent component of this reaction as ultimate friction force (the force of cargo cohesion with wagon floor) $\overline{F}_{coh}$ or sliding friction force $\overline{F}_{slid.x}$; reactions $\overline{R0}_i$ of preliminary twistings of fastening wire and bar reaction $\overline{R}_{bar.x}$ (Fig.2 a, b) [17].
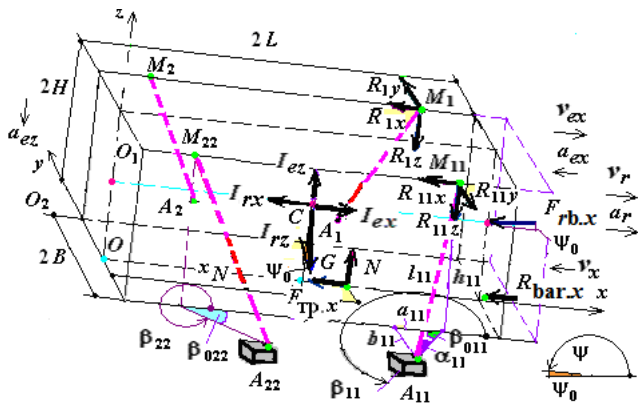
Figure 2 a : Dynamic model of cargo on the wagon moving along the tangent on down grade
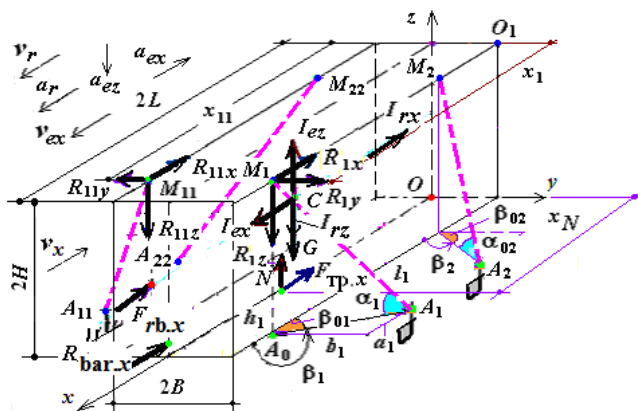

Figure 2 b : Dynamic model of cargo on the wagon moving on a tangent

Longitudinal and vertical transferring inertia forces $\bar{I}_{ex}$ and $\bar{I}_{ez}$ will experience external constraints (platform floor and thrust bar). It is only the thrust bar, not the pre-tensioned flexible elastic fastening elements that will experience longitudinal transferring inertia force $\bar{I}_{ex}$. Flexible fastening elements may experience the

action of external forces only when there takes place a shift of thrust bars with a bend of fastening items (nail).

II. METHODS OF SOLUTION

The formation of dynamic and constructing a mathematical model of cargo movement on a wagon is based on classical concepts and provisions of theoretical mechanics (for example, Constraint and their reactions, the Principle of ties release of the fundamental law of dynamics of the relative motions of records) [23, 24]).

a) Problem Analytical Solution

Unlike in [7, 10, 18], for deriving an engineering formula we will use the fundamental law of relative transferring cargo motion during rolling stock movement along tangent described by the equation in vector form

$$M\bar{a}_r = \bar{F} + \bar{R} + \bar{I}_e + \bar{I}_C , \qquad (1)$$

where $\bar{a}_r$ is cargo relative acceleration (or acceleration of cargo relative to the wagon floor).

As applied to the problem in question $\bar{F} = \bar{G}$ is active force, $\bar{R} = \bar{N} + \bar{F}_\tau$ is reactive force, $\bar{I}_e \in (\bar{I}_{ex}, \bar{I}_{ez})$ are longitudinal and vertical tran-sferring inertia forces, $\bar{I}_C$ is Carioles inertia force (for the problem in question it is assumed to be equal to zero).

Let us assume that as in [25], cargo is in motion lengthwise the wagon floor with acceleration $\bar{a}_r$ and let its speed be $\bar{v}_r$ at the moment. Then equation (1) in projections upon coordinate axes Ox and Oz is presented in the form

$$I_{ex} + (G_x - F_{rb.x}) - F_x - F_{\tau x} - R_{bar.x} = Ma_{rx} ; \qquad (2)$$

$$-(G_z - I_{ez}) - F_z - F_{rb.z} + N = Ma_{rz} , \qquad (3)$$

where $I_{rx} = (-Ma_{rx})$, $I_{rz} = (-Ma_{rz})$ and $G = \{G_x, G_z\}$ is projection of inertia force in relative motion onto coordinate axes Ox and Oz ; $\bar{I}_{ex}$, $\bar{I}_{ez}$ and $F_{rB} = \{F_{rB.x}, F_{rB.z}\}$ are active forces; $F^{(i)} = F_{(i)} = \{F_x, F_z\}$, $\bar{N}$ and $\bar{F}_{\tau x}$, $\bar{R}_{bar.x}$ are

reactive forces. At that, F_x and F_z are orthogonal projections of elastic forces (effort) of *i*-flexible elements with allowance made for wire preliminary twisting of fastening $R0_i$ of additional constraint (cargo fastening flexible element) onto coordinate axes Ox and Oz ; $\bar{N}$ and $\bar{F}_{\tau x} = \bar{F}_{slid.x}$ are so far unknown normal

Oz; $\overline{N}$ and $\overline{F_{\tau x}} = \overline{F}_{\text{slid}.x}$ are so far unknown normal and tangent components of wagon reaction (constraints) $\overline{R}$; $\overline{R}_{\text{bar}.x}$ is a design value of reactions of fastening thrust elements (thrust bars) calculated according to the chosen number of fastening elements (nail) in agreement with clearance outline. Designation to the power $i$of elastic force $\overline{F}^{(i)}$ means that the force is dependent on the number of fastening elements but it doesn't mean that it is to be summed according to i. Elastic force $\overline{F}^{(i)}$ has only one value.

III. RESULTS OF SOLUTION

a) *Mathematical Solution of The Problem*

In compliance with this and applied to (2) and (3) the following can be derived

$$G_x = G\sin(\psi_0); \quad F_x = \sum_{i=1}^{n_p} R_{ix} + \sum_{i=1}^{n_p} R0_{ix}; \quad F_{\text{rb}.x} = F_{\text{rb}}\cos(\psi_0); \tag{2a}$$

$$G_z = G\cos(\psi_0); \quad F_z = \sum_{i=1}^{n_p} R_{iz} + \sum_{i=1}^{n_p} R0_{iz}; \quad F_{\text{rb}.x} = F_{\text{rb}}\sin(\psi_0); \tag{3a}$$

According to the Coulomb law we'll write down

$$F_\tau \le fN, \tag{4}$$

where f is sliding friction coefficient $f = 0{,}7f_{\text{coh}}$ with allowance made for f_{coh} being the coefficient of cohesion friction between contact surfaces of cargo and wagon floor which are accepted according to reference data) [7, 10, 17, 18].

Substituting (4) in (3) we'll find

$$F_\tau = f\left[(G_z - I_{ez}) + F_z + F_{\text{rb}.z}\right],$$

or taking into account (3,a) we have

$$F_\tau = f\left[(G\cos(\psi_0) - I_{ez}) + \sum_{i=1}^{n_p} R_{iz} + \sum_{i=1}^{n_p} R0_{iz} + F_{\text{rb}}\sin(\psi_0)\right] \tag{5}$$

Just as in [10, 17, 18] let us present expression (5) in the form

$$F_\tau = F_\tau^{\text{elast.}} + F_\tau^e, \tag{6}$$

where $F_\tau^{\text{elast.}}$ and F_τ^e are friction forces of elastic and external forces

$$F_\tau^{\text{elast.}} = f\sum_{i=1}^{n_p} R_{iz} \tag{6a}$$

$$F_\tau^e = f\left[(G\cos(\psi_0) - I_{ez}) + \sum_{i=1}^{n_p} R_{iz} + \sum_{i=1}^{n_p} R0_{iz} + F_{\text{rb}}\sin(\psi_0)\right] \tag{6 b}$$

Substituting (2a) in (2) taking into account (6) we'll get

$$I_{ex} + G\sin(\psi_0) - \sum_{i=1}^{n_p} R_{ix} - \sum_{i=1}^{n_p} R0_{ix} - F_\tau^{\text{elast.}} - F_\tau^e -$$

$$- F_{rb}\cos(\psi_0) - R_{\text{bar}.x} = Ma_{rx}.$$

Let us rewrite the above expression taking into account (6a) and (6b)

$$I_{ex} + G\sin(\psi_0) - \sum_{i=1}^{n_p} R_{ix} - \sum_{i=1}^{n_p} R0_{ix} - f\sum_{i=1}^{n_p} R_{iz} -$$

$$- f\left[(G\cos(\psi_0) - I_{ez}) + \sum_{i=1}^{n_p} R0_{iz} + F_{rb}\sin(\psi_0)\right] - \tag{7}$$

$$- F_{rb}\cos(\psi_0) - R_{\text{bar}.x} = Ma_{rx}.$$

After elementary manipulations with the above expression we determine projections of elastic forces (effort or tension) of *i*-flexible fastening elements onto axis *Ox*

$$\sum_{i=1}^{n_p} R_{ix} + f\sum_{i=1}^{n_p} R_{iz} = I_{ex} + G(\sin(\psi_0) - f\cos(\psi_0)) + fI_{ez} -$$

$$- \sum_{i=1}^{n_p} (R0_{ix} + fR0_{iz}) - F_{rb}(\cos(\psi_0) + f\sin(\psi_0)) - R_{\text{bar}.x} - Ma_{rx}.$$

We rewrite the derived expression in the form

$$\sum_{i=1}^{n_p} R_{ix} + f\sum_{i=1}^{n_p} R_{iz} = \Delta F_x, \tag{8}$$

where $\Delta F_x = \Delta F_{\text{long.}}$ are longitudinal forces perceived by flexible and thrust cargo fastening elements

$$\Delta F_x = I_{ex} + G(\sin(\psi_0) - f\cos(\psi_0)) + fI_{ez} -$$

$$- \sum_{i=1}^{n_p} (R0_{ix} + fR0_{iz}) - F_{rb}(\cos(\psi_0) + f\sin(\psi_0)) -$$

$$- R_{\text{bar}.x} - Ma_{rx}.$$

Here we are to take into account the fact that $\overline{R}_{\text{bar}.x}$ is a design value of fastening thrust elements reactions (thrust bars) calculated either according to arbitrarily chosen or just as in [22] in accordance with scientifically grounded number of fastening elements (nail) in agreement with clearance outline. For example, it is in this way that reactions of thrust bars are determined (12 [21]):

[R_{nail}] = 1, 08 is an assumed value of force per one fastening item (nail), (Table 32 Appendix 14 to International Rail Freight Transportation Agreement);

$n_{\text{bar}.x}$ is accepted number of thrust bars according to cargo location and fastening scheme, item (Fig. 1);

k_1 is a strength coefficient of fastening thrust bars, taking into account the state of wagon floor, items (usually accepted to be 0,5 ÷ 0,6);

$n_{\text{nail}.x}$ is accepted value of needed number of nails per each thrust bar, item;

$R_{\text{bar}.x} = k_1 \cdot n_{\text{nail}.x} \cdot n_{\text{bar}.x} \cdot [R_{\text{nail}}]$ is allowable load per one fastening item (nail) (46 Appendix 14 to). For example, for a nail with ∅4and length 100 ÷ 120 mm

$[R_{nail.}] = 0,047$ кН, for a nail of $\varnothing 6$ and length $150 \div 200$ mm $[R_{nail.}] = 1,08$ кN. According to the standards of European countries for a fastening item of $\varnothing 5$mm and length $100 \div 150$ $[R_{nail.}] = 1,25$ кN, and for an item of $\varnothing 6$mm and length $150 \div 200$mm $[R_{nail.}] = 1,5$ кN [26].

By introducing notions of "shearing" and "retentive" forces [7, 10, 11, 18] we rewrite the above expression

$$\Delta F_x = F_{shear.} - F_{ret.} , \tag{9}$$

where

$$F_{shear.} = I_{ex} + G\sin(\psi_0) ; \tag{10a}$$

$$F_{reet.} = f\left(G\cos(\psi_0) - I_{ez}\right) + \sum_{i=1}^{n_p}\left(R0_{ix} + fR0_{iz}\right) + \tag{11}$$
$$+ F_{rb}\left(\cos(\psi_0) + f\sin(\psi_0)\right) + R_{bar.x} + Ma_{rx}.$$

From now on, for simplicity of problem solving we'll study a case when cargo is retained lengthwise the wagon by i-flexible fastening elements ($i = \overline{1, n_p}$) and thrust bar (Fig.1). Then according to the method of determining deformations at minor displacement we project a new point position first onto "original" or "old" direction of thrust element [17, 18 – 21]. As a thrust element is arbitrarily located in space for calculating the projection it is necessary to make use of the method of double projection the way it is done in theoretical mechanics for arbitrarily located force [10, 15, 18]. Based on this we'll derive a formula for finding elongation of fastening flexible elements depending on cargo shift along wagon Δx and fastening geometrical parameters

$$\Delta l_i = \Delta x \frac{a_i}{l_i}, \tag{12}$$

where l_i is the length for each flexible elastic fastening element, m: $l_i = \sqrt{a_i^2 + b_i^2 + h_i^2}$ taking into account the fact that a_i, b_i and h_i are projections of each flexible fastening element onto longitudinal Ox, crosswise Oy, and vertical axis Oz.

It is obvious that elongation in the flexible elastic fastening element will occur only when there is cargo shift lengthwise the wagon at value Δx .

According to [7, 10, and 15] the movement of cargo lengthwise the wagon as one-mass oscillatory system can be presented in the following way (Fig. 3).
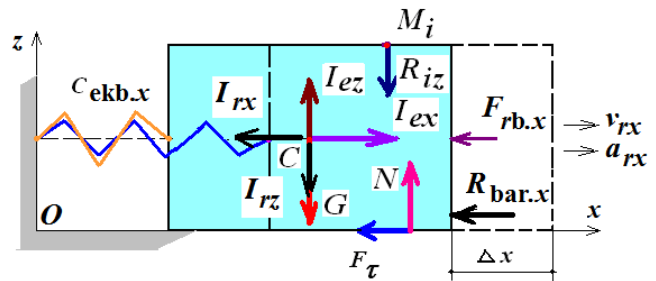

Figure 3 : Cargo movement lengthwise the wagon as one-mass oscillatory system

Here, just as in [7], the sum of projections of the efforts of flexible fastening elements $\overline{R}_{ix}$ onto the longitudinal axis which were according to Hooke's law the results of cargo shift lengthwise the wagon Δx is replaced by a spring with equivalent rigidity $c_{ekb.x}$ stretched out at the value of the shift. In its turn, spring with equivalent rigidity $c_{ekb.x}$ (Fig. 3) stretched out at
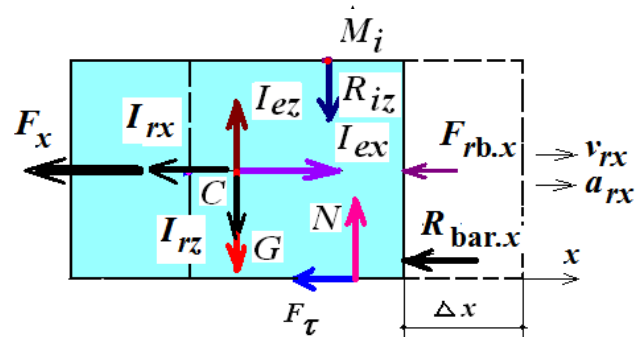

Figure 4 : Dynamic model of cargo loaded by longitudinal and vertical forces.

Designations in Fig. 3 and 4 are the same as in Fig. 2, except $c_{ekb.x}$ being equivalent (either reduced or generalized) rigidity of flexible fastening elements along longitudinal axis Ox.

In expressions (8) and (11) projections of elastic forces (effort and tension) of i-flexible elastic fastening elements onto longitudinal and vertical axes Ox and Oz are determined according to the formulas

$$\sum_{i=1}^{n_p} R_{xi} = \sum_{i=1}^{n_p} c_i \frac{a_i}{l_i} \cdot \Delta l_i ; \quad \sum_{i=1}^{n_p} R0_{xi} = \sum_{i=1}^{n_p} R0_i \frac{a_i}{l_i} ; \tag{13}$$

$$\sum_{i=1}^{n_p} R_{zi} = \sum_{i=1}^{n_p} c_i \frac{h_i}{l_i} \cdot \Delta l_i ; \quad \sum_{i=1}^{n_p} R0_{zi} = \sum_{i=1}^{n_p} R0_i \frac{h_i}{l_i}, \tag{14}$$

or taking into account (12),

$$\sum_{i=1}^{n_p} R_{xi} = \sum_{i=1}^{n_p} c_i \frac{a_i}{l_i} \cdot \Delta x \cdot \frac{a_i}{l_i} ; \quad \sum_{i=1}^{n_p} R0_{xi} = \sum_{i=1}^{n_p} R0_i \frac{a_i}{l_i} ; \tag{13a}$$

$$\sum_{i=1}^{n_p} R_{zi} = \sum_{i=1}^{n_p} c_i \frac{a_i}{l_i} \cdot \Delta x \cdot \frac{h_i}{l_i} ; \quad \sum_{i=1}^{n_p} R0_{zi} = \sum_{i=1}^{n_p} R0_i \frac{h_i}{l_i}, \tag{14a}$$

where c_i is rigidity of i-flexible fastening element (kN/m): are projections a_i and h_i are projections of i-flexible fastening elements onto longitudinal and vertical axes: l_i is length of i-flexible fastening element; Δx is cargo shift lengthwise the wagon; $R0_i$ is tension of preliminary twistings of i-flexible fastening element (kN) (we'll assume 20 kN); $i = \overline{1, n_p}$ is a number of flexible elastic fastening elements.

In formulas (13) and (14) rigidity of i-flexible fastening element (kN/m) with a number of threads n_i (item), diameter d_i(mm) and $l_i$of length (m) of fastening wire:

$$c_i = \frac{10^{-6}\pi E}{4} d_i^2 \frac{n_i}{l_i} = 7{,}854 \cdot d_i^2 \frac{n_i}{l_i}, \tag{15}$$

where$E$ is elasticity module of annealed fastening wire (usually assumed $E = 1 \cdot 10^7 \text{кN/m}^2$, and for steel cable $E = 2{,}1 \cdot 10^8 \text{kN/m}^2$); 10^{-6} is conversion factor of wire diameter mm into m.

Putting the first expressions (13a) and (14a) in (8) we obtain

$$\sum_{i=1}^{n_p} c_i \frac{a_i}{l_i} \cdot \Delta x \cdot \frac{a_i}{l_i} + f \sum_{i=1}^{n_p} c_i \frac{a_i}{l_i} \cdot \Delta x \cdot \frac{h_i}{l_i} = \Delta F_x,$$

or

$$\sum_{i=1}^{n_p} c_i \left(f \frac{h_i}{l_i} + \frac{a_i}{l_i} \right) \cdot \Delta x \cdot \frac{a_i}{l_i} = \Delta F_x.$$

The above expression with consideration for (15) has the form

$$7{,}854 d_i^2 \sum_{i=1}^{n_p} \frac{n_i}{l_i} \left(f \frac{h_i}{l_i} + \frac{a_i}{l_i} \right) \cdot \Delta x \cdot \frac{a_i}{l_i} = \Delta F_x.$$

Hence we can find cargo shift lengthwise the wagon

$$\Delta x = \frac{\Delta F_x}{7{,}854 d_i^2 \sum\limits_{i=1}^{n_p} \dfrac{n_i}{l_i}\left(f \dfrac{h_i}{l_i} + \dfrac{a_i}{l_i} \right) \cdot \dfrac{a_i}{l_i}}, \tag{16}$$

where $\Delta F_x = \Delta F_{long.}$ – is longitudinal force, determined by formula (9), (10) and (11) with consideration for second expressions (13), (14):

$$F_{shear.} = I_{ex} + G\sin(\psi_0); \tag{10}$$

$$F_{ret.} = f\big(G\cos(\psi_0) - I_{ez}\big) + \sum_{i=1}^{n_p} R0_i\left(f\frac{h_i}{l_i} + \frac{a_i}{l_i} \right) + \tag{11a}$$

$$+ F_{rb}\big(\cos(\psi_0) + f\sin(\psi_0)\big) + R_{bar.x} + Ma_{rx}.$$

Here, if aerodynamic resistance force $\overline{F}_{rb}$ acts from the cargo rear back, this force should be put in the formula with a negative sign with consideration for coordinates of its application.

Just as in [7, 10, 15) cargo shift lengthwise the wagon Δx is the distance from the cargo butt surface that is able to provide joint performance of flexible and thrust fastening means if a thrust bar is nailed to the wagon floor from the cargo butt at a distance less than Δx.

It can be observed from (16) that first, cargo shift lengthwise the wagon will occur only when $\Delta F_{long} > 0$ and second, breakup of flexible fastening elements will not take place only on condition that $\Delta x \leq [\Delta x]$ where $[\Delta x]$is an allowable value of cargo shift lengthwise the wagon (mm) determined according to value $[R_i]$ (Table 30 Appendix 14 to International Rail Freight Transportation Agreement).

Summarizing the results of mathematical modeling of fastening of cargo asymmetrically (or symmetrically) located on the wagon it can be noted that there has been derived an analytical formula for determination of cargo shift lengthwise the wagon Δx withconsideration for physical-geometrical character-ristics of flexible elements (i.e. E, n, d, l), values of external forces $(\overline{G}, \overline{I}_x, \overline{I}_z, \overline{F}_{rb}, \overline{R}_{bar.x}, M\overline{a}_{rx})$, perceived by flexible fastening elements, thrust bar and cargo and the state of cargo contact surfaces and wagon floor taken into account by friction coefficient (f).

In a special case when a wagon with cargo is moving on a tangent without braking and in the release schedule from (10) and 11a) there will be excluded descend angle ψ (i. e. $\psi_0 = 0$). In these cases (10) and (11a) will be

$$F_{shear.} = I_{ex}; \tag{10b}$$

$$F_{ret.} = f\big(G - I_{ez}\big) + \sum_{i=1}^{n_p} R0_i\left(f\frac{h_i}{l_i} + \frac{a_i}{l_i} \right) + F_{rb} + \tag{11b}$$

$$+ R_{bar.x} + Ma_{rx}.$$

14

Let us recall that in (11b) vertical transferring inertia force $\overline{I}_z$ occurs only during the movement of the wagon on a tangent without braking and in release schedule as in during schedule – $\overline{I}_z = 0$.

In case of shunting collision in a marshaling hump-yard $\overline{F}_{rb} = 0$ and $\overline{I}_z = 0$ (Fig.1b and 2b). That is why (11b) will have a simple form of:
(11в)

While solving practical problems by using formula (9) or (16) just as in [10, 15, 17, 18] let us assume that maximum regulatory values of longitudinal transferring accelerations $\overline{a}_{ex}^{\max} = \overline{a}_{ex}$ are equal to $a_{ex} = 0,3g$ – on a tangent, $a_{ex} = (0,7 \div 1,2)g$ during service brake application, $a_{ex} = (1,2 \div 2)g$ during wagon collisions in a hump-yard ,and vertical

transferring accelerations $\overline{a}_{ez}^{\max} = \overline{a}_{ez}$ occurring because of deviations in track maintenance standards , $– a_{ez} = (0,46 \div 0,66)g$. In accordance with this statement it is possible to accept $I_{ex} = 0,3G$ on a tangent, $I_{ex} = (0,7 \div 1,2)G$ in service braking, $I_{ex} = (1,2 \div 2)G$ during wagon collisions in a hump-yard and $I_{ez} = (0,4 \div 0,66)G$.

Using the derived value of cargo shift lengthwise the wagon Δx, just as in [10, 18] in compliance with Hooke's law (as the product of (15) multiplied by (12)) we determine effort (tension) R_i in i-flexible fastening element, kN:

$$R_{\text{elast}.i} = 7,854 d_i^2 \frac{n_i}{l_i} \Delta l_i \leq [R_i], \qquad (17)$$

or with consideration for (12)

$$R_{\text{elast}.i} = 7,854 d_i^2 \frac{n_i}{l_i} \Delta x \frac{a_i}{l_i} \leq [R_i], \qquad (17a)$$

where $[R_i]$ is allowable value of effort in fastenings, determined according to the Table 30 Appendix 14 to International Rail Freight Transportation Agreement depending on a number of threads n_i and wire diameter d_i.

Effort (tension) R_i in a flexible fastening element is according to axiom: to every action there is an equal reaction is equal to the reaction in this element.

IV. CONCLUSION

a) For the first time in the theory of fastening of solid-state goods there has been derived a formula for determining "retentive" force with allowance made for reactions of thrust fastening elements and inertia forces in relative motion on condition that the cargo moves in relation to the wagon floor with acceleration R_i its speed at the moment being $\overline{v}_r$.

b) When considering cargo motion in relation to the wagon floor with acceleration $\overline{a}_r$ at speed $\overline{v}_r$ it should be noted that longitudinal force perceived by flexible fastening is smaller in value than the force the value of which was obtained without taking into account the inertia force in relative motion (*at rest*). Therefore, cargo shift lengthwise the wagon in this

case will also be smaller. This, in its turn, will affect the decrease of elongation value and hence the decrease of the effort of each fastening element meanwhile increasing their loading capacity.

The results obtained in analytical investigation are an important contribution to the theory of cargo fastening.

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