Global Journal of Researches in Engineering
Mechanical and Mechanics Engineering
Volume 13 Issue 2 Version 1.0 Year 2013
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 Print ISSN:0975-5861

Investigation of Stresses in a Beam with Fixed Connection using Finite Difference Technique

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GJRE-A Classification : FOR Code: 249999p, 290501p

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Muhammad Ziaur Rahman [α], Tousif Ahmed [σ] & Raihan Md. Imtiaz [ρ]

Abstract - In this study a simple rectangular beam has been analyzed. For simplicity two dimensional plane strain problem is considered. The beam is fixed at the left and right edges of the bottom surface. Uniformly distributed load is applied along the length of the beam. A new formulation, known as displacement potential function formulation has been used here. Stresses and displacements at different sections along the length and depth of the beam have been plotted and discussed. The curves are found to be in complete conformity with the available literature and physical condition of the beam. The present literature reveals that the fixed connections of the beam are extremely critical regions and have the highest probability to fail.

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I. Notations

Symbol	Meaning
x,y	rectangular co-ordinates
u,v	displacement component along x and y directions respectively
σ_x	normal stress component along x direction
σ_y	normal stress component along y direction
σ_{xy}	shear stress
ψ	displacement potential function
E	modulus of elasticity
μ	poisson's ratio
a	depth of the beam
b	length of the beam
h	mesh length in the x-direction
k	mesh length in the y-direction
R	ratio of the mesh length k/h
m	number of mesh points in x-direction
n	number of mesh points in y-direction

II. Introduction

Fixed connections are very common. Steel structures of many sizes are composed of elements which are welded together. A cast-in-place concrete structure is automatically monolithic and it becomes a series of rigid connections with the proper placement of the reinforcing steel. Fixed connections demand greater attention during construction and are often the source of building failures.

This paper is an attempt to find out what actually happens in a beam with fixed connection at the bottom when it is loaded with uniformly distributed force. In this regard two dimensional elastic problem has been used. It enabled to manage the mixed mode of the boundary conditions as well as the zone of their transition.

In modern age, stress related problems are classical topics. But still there are some shortcomings in analyzing the problems. Specially at the boundary, where stress and deformation co-exist, the problem turns into a complex situation.

The formulation of two dimensional elastic problems used here was first introduced by Uddin[1], later Idris *et. al.* [5-7] used it for obtaining analytical solutions of a number of mixed boundary value elastic problems and Ahmed [2,8-9] extended its use where he obtained finite difference solution of a number of mixed boundary value problems of simple rectangular bodies. Later, Akanda developed a numerical scheme [10-12] by which he solved irregular shaped elastic bodies under mixed mode of loading. This study focuses on the solution of the problem of rectangular beam with fixed connection.

In this paper stresses and deformations at different longitudinal and transverse sections of the beam have been plotted and discussed elaborately. For the convenience of analysis the body is divided into 30 meshes in y direction and 10 meshes in x direction.

III. Formulation of the Problem

Analysis of stresses in an elastic body is generally a three-dimensional problem. But in the cases of plane stress or plane strain, the stress analysis of three-dimensional body can easily be resolved into two-dimensional problem. The problem studied here has been considered to be a plane strain problem. In the case of the absence of any body forces, the equations governing the three stress components σ_x, σ_y, σ_{xy} under the state of plane stress or plain strain are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} = 0 \quad (2)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\sigma_x + \sigma_y\right) = 0 \quad (3)$$

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Replacement of the stress components in Eqs. (1-2) by their relations with the displacement components u and v makes Eq.(3) surplus and Eq.(1) and (2) becomes :

$$\frac{\partial^2 u}{\partial x^2} + \left(\frac{1-\mu}{2}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{1+\mu}{2}\right)\left(\frac{\partial^2 v}{\partial x \partial y}\right) = 0 \quad (4)$$

$$\frac{\partial^2 v}{\partial y^2} + \left(\frac{1-\mu}{2}\right)\frac{\partial^2 v}{\partial x^2} + \left(\frac{1+\mu}{2}\right)\left(\frac{\partial^2 u}{\partial x \partial y}\right) = 0 \quad (5)$$

This two equations can be used for the solution. But it is still difficult to solve for two functions simultaneously. To overcome the difficulty, the two equations are transformed into a single equation with a single function. So a new function called displacement potential function (ψ) is defined as a function of displacement components to reduce the no. of governing differential equations into a single equation like following :

$$u = \frac{\partial^2 \psi}{\partial x \partial y}$$

$$v = -\frac{1}{1+\mu}\left[(1-\mu)\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial^2 \psi}{\partial x^2}\right] \quad (6)$$

When the displacement components in Eqs.(4) and (5) are replaced by ψ (x,y) Eq.(4) is automatically satisfied and the only condition that ψ (x,y) has to satisfy becomes [1]:

$$\frac{\partial^4 \psi}{\partial x^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} = 0 \quad (7)$$

So the problem is now reduced in such a way that a single function ψ (x,y) has to be evaluated from the bi-harmonic Eq. (7), satisfying the boundary conditions that are specified at the boundary.

IV. Boundary Conditions

Fig. 1 shows the simple rectangular beam subjected to uniformly distributed load. Surface AB is under compression and points C, D are fixed. On the other hand AC and BD surfaces are free from any external load.

The normal and tangential stress components for the top surface AB are given by

$$\sigma_{xy}(x,y) = 0,$$

$$\sigma_x/E\,(x,y) = -1.10\text{E-}04 \text{ for } 0 \le y/b \le 1,\ x/a=0$$

The normal and tangential stress components for the surfaces AC and BD are given by

$$\left.\begin{array}{l}\sigma_{xy}(x,y) = 0 \\ \sigma_y(x,y) = 0\end{array}\right\} \begin{array}{l}\text{for } 0 \le x/a \le 1 \text{ and} \\ y/b=0, y/b=1\end{array}$$

The normal and tangential stress components of the surface CD except the points C, D are given by

$$\sigma_{xy}(x,y) = 0,$$

$$\sigma_x(x,y) = 0 \quad \text{for } x/a=1,\ 1/15 \le y/b \le 14/15$$

The normal and tangential displacement components for the points C and D are given by

$$u(x,y)=0$$

$$v(x,y)=0 \ \text{ for } x/a=1 \text{ and } y/b= 0,\ 1/30,\ 29/30,\ 1$$

In order to solve the problem using the bi-harmonic equation (7) it is necessary to express the known boundary functions interms of the single potential function ψ like following

$$u = \frac{\partial^2 \psi}{\partial x \partial y} \quad (8)$$

$$v = -\frac{1}{1+\mu}\left[(1-\mu)\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial^2 \psi}{\partial x^2}\right] \quad (9)$$

$$\sigma_x(x,y) = \frac{E}{(1+\mu)^2}\left[\frac{\partial^3 \psi}{\partial x^2 \partial y} - \mu\frac{\partial^3 \psi}{\partial y^3}\right] \quad (10)$$

$$\sigma_y(x,y) = -\frac{E}{(1+\mu)^2}\left[\frac{\partial^3 \psi}{\partial y^3} + (2+\mu)\frac{\partial^3 \psi}{\partial x^2 \partial y}\right] \quad (11)$$

$$\sigma_{xy}(x,y) = \frac{E}{(1+\mu)^2}\left[\mu\frac{\partial^3 \psi}{\partial x \partial y^2} - \frac{\partial^3 \psi}{\partial x^3}\right] \quad (12)$$

So it is clear that all the boundary conditions can be discretized in terms of displacement potential function ψ with the help of finite difference technique.

V. Solution Procedure

a) *Method of Solution*

The finite-difference technique, frequently used to transform differential equations into corresponding algebraic equations has been used here to generate the algebraic equations of the bi harmonic Eq.(7) and partial differential Eq. (8-12) which are associated with boundary conditions. The discrete values of the potential function ψ (x,y) at the mesh points of the domain concerned (Fig. 2) are obtained from a system of linear algebraic equations resulting from the discretization of the governing equation and the prescribed boundary conditions.

The problem is discretized in a desired number of mesh points and the value of ψ (x,y) is evaluated only at these points. A false boundary, exterior to the physical boundary is introduced to keep the order of error of the difference equations to a minimum. The discretization of the domain concerned is shown in Fig. 2. The division into mesh points can be done in any regular and irregular shape. But due to the shape of the problem rectangular meshing is performed. The governing bi-harmonic equation which is used to evaluate the function only at the internal mesh points is expressed in its corresponding difference equation using central difference operators. Replacing the derivatives of Eq. (7) with central difference formulae, the finite difference equation stands like following

$$R^4\{\psi(i-2,j)+\psi(i+2,j)\}-4R^2(1+R^2)\{\psi(i-1,j)+\psi(i+1,j)\}-4(1+R^2)\{\psi(i,j+1)+\psi(i,j-1)\}+(6R^4+8R^2+6)\,\psi(i,j)+2R^2\{\psi(i-1,j-1)+\psi(i-1,j+1)+\psi(i+1,j-1)+\psi(i+1,j+1)\}+\psi(i,j-2)+\psi(i,j+2)=0 \tag{13}$$

Where $R=k/h$

Let $O\ (i,j)$ is an internal mesh point as shown in Fig.(2). From Eq.(13) it is evident that O has 13 mesh points including itself.

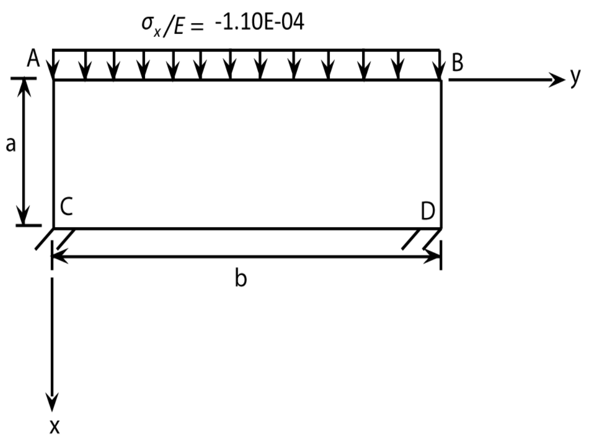

Figure 1 : Rectangular beam with fixed connection and subjected to uniformly distributed load

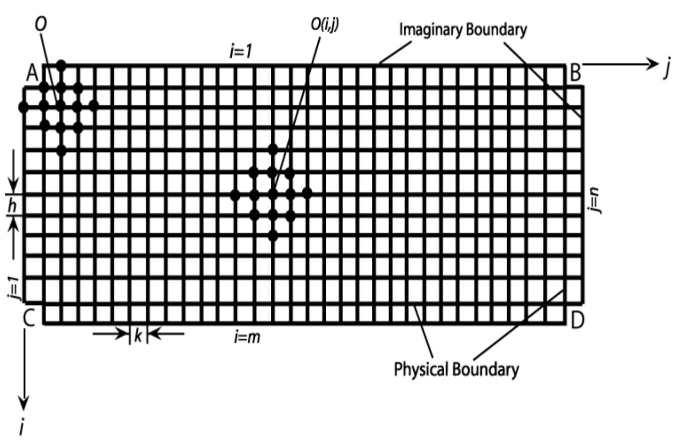

Figure 2 : Rectangular mesh generation of the domain in relation to the coordinate system and the finite-difference discretization of the bi-harmonic equation at an arbitrary internal mesh point

When O becomes an immediate neighbor of the physical boundary, this equation will contain mesh points both interior and exterior to the physical boundary. But it will not create any difficulty provided that a false boundary is introduced external to the physical boundary.

b) Management of Boundary Conditions

Boundary conditions are specified either in deformation parameter or stress parameter of combination of two. There are four possible combination of boundary condition. They are (1)normal stress and shear stress (2) normal stress and tangential displacement (3) shear stress and normal displacement (4) normal displacement and tangential displacement. Each mesh point on the physical boundary entertains two conditions at a time out of four. The computer program is organized in such a fashion that out of these two conditions one is used for evaluation of $\psi\ (x,y)$ at the concerned boundary point and the other point for the corresponding point on the exterior false boundary. Thus when the boundary conditions are expressed by their appropriate difference equations, every mesh point of the domain will have a single linear algebraic equation. Table 1 is the demonstration of boundary conditions for all the surfaces namely AB, CD, AC and BD.

Table 1 : Specification of the boundary conditions in relation to corresponding mesh points on boundary

Boundary	Given boundary conditions	Correspondence between mesh points and given boundary conditions	
		condition/mesh point	condition/mesh point
Top AB	σ_x, σ_{xy}	$\sigma_x/(2,j)$	$\sigma_{xy}/(1,j)$
Bottom CD (except point C and D)	σ_x, σ_{xy}	$\sigma x/(m,j)$	$\sigma_{xy}/(m-1,j)$
Left AC	σ_y, σ_{xy}	$\sigma_y/(i,1)$	$\sigma_{xy}/(i,2)$
Right BD	σ_y, σ_{xy}	$\sigma_y/(i,n)$	$\sigma_{xy}/(i,n-1)$
Point C,D	u,v	u/(m-1,2), (m-1,3), (m-1,n-1), (m-1,n-2)	v/(m,2), (m,3),(m,n-1), (m,n-2)

Three point backward or forward difference formulae has been used here for the replacement of the derivatives of the boundary expressions. Because, the differential equations contain second and third order derivatives of the function ψ for which the application of the central difference formulae will lead to the inclusion of points exterior to the false boundary. As an example, the finite-difference expressions for the normal and tangential components of stress on top boundary AB closer to B are given by

$$\sigma_x(2,j) = \frac{E\mu}{(1+\mu)^2 R^3 h^3}\Big[\Big(\frac{3R^2}{\mu} - 5\Big)\psi(2,j) + 1.5\psi(2,j+1) + \Big(6 - \frac{4R^2}{\mu}\Big)\psi(2,j-1) + \Big(\frac{R^2}{\mu} - 3\Big)\psi(2,j-2) + 0.5\psi(2,j-3) - \frac{3R^2}{2\mu}\{\psi(1,j) + \psi(3,j)\} + \frac{2R^2}{\mu}\{\psi(1,j-1) + \psi(3,j-1)\} - \frac{R^2}{2\mu}\{\psi(1,j-2) + \psi(3,j-2)\}\Big] \quad (14)$$

$$\sigma_{xy}(1,j) = \frac{E\mu}{(1+\mu)^2 R^2 h^3}\Big[\frac{3R^2}{2\mu}\psi(1,j) + \Big(3 - \frac{5R^2}{\mu}\Big)\psi(2,j) + \Big(\frac{6R^2}{\mu} - 4\Big)\psi(3,j) + \Big(1 - \frac{3R^2}{\mu}\Big)\psi(4,j) + \frac{R^2}{2\mu}\psi(5,j) - 1.5\{\psi(2,j-1) + \psi(2,j+1)\} + 2\{\psi(3,j-1) + \psi(3,j+1) - 0.5\{\psi(4,j-1) + \psi(4,j+1)\}\Big] \quad (15)$$

In Fig. 3(a) and (b) the discretization process used for the two equations above has been demonstrated.

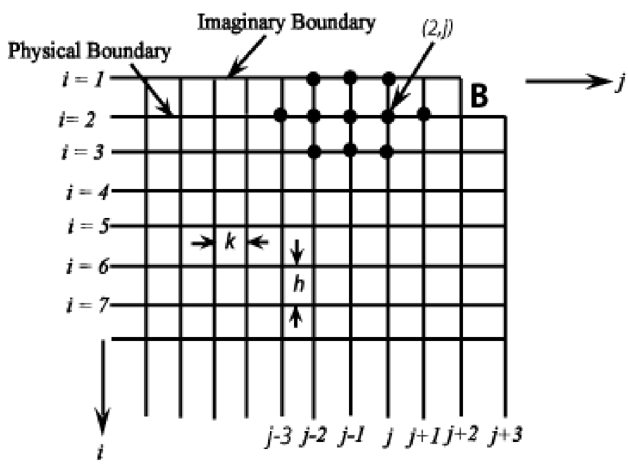

Fig. 3(a)

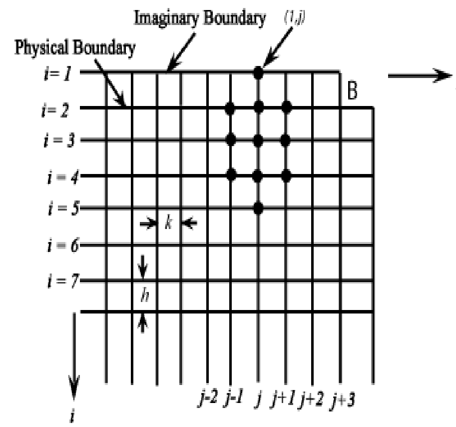

Fig. 3(b)

Figure 3 : Grid-points for expressing the boundary conditions on the top edge at points closer to B (a) for normal stress component σ_x (b) for tangential stress component σ_{xy}

Referring to Fig.4, assuming B as the corner mesh point, it is evident that B is common for both the edges AB and BD. So point B has four boundary conditions, two from each edges. In the present solution, three of the four conditions have been used and one is considered to be redundant. Thus the values of ψ at the point B and 1 have been evaluated from the boundary conditions coming from the edge AB and point 2 has been evaluated from the boundary BD. Management of the boundary conditions at the corner mesh points has been demonstrated in table 2 for the problem shown in Fig. 1.

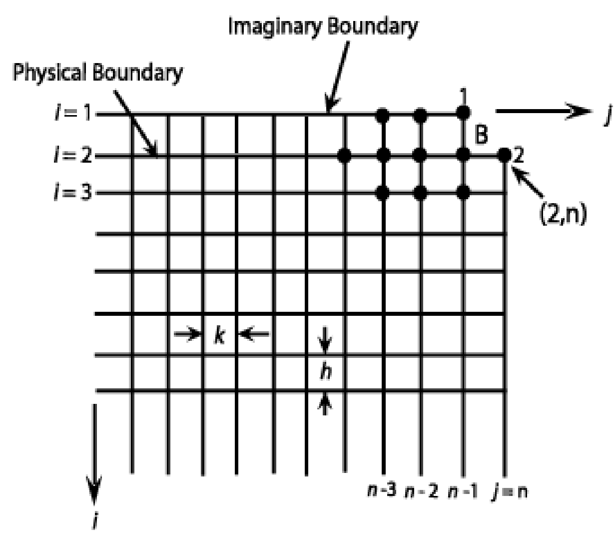

Figure 4 : Treating the transaction Point B

Table 2 : Management of the boundary conditions at corner mesh points

Corner point	Possible boundary conditions	Conditions used	Corresponding mesh points for evaluation of ψ
Point A	$[\sigma_x, \sigma_{xy}]$ on AB $[\sigma_y, \sigma_{xy}]$ on AC	$[\sigma_x, \sigma_{xy}, \sigma_y]$	(2,2),(1,2),(2,1)
Point B	$[\sigma_x, \sigma_{xy}]$ on AB $[\sigma_y, \sigma_{xy}]$ on AC	$[\sigma_x, \sigma_{xy}, \sigma_y]$	(2,n-1),(1,n-1),(2,n)
Point C	$[\sigma_y, \sigma_{xy}]$ on AC $[u,v]$ on CD	$[u, v, \sigma_y]$	(m-1,2),(m,2),(m-1,1)
Point D	$[\sigma_y, \sigma_{xy}]$ on AC $[u,v]$ on CD	$[u, v, \sigma_y]$	(m-1,n-1),(m,n-1),(m¯1,n)

The finite difference equation corresponding to corner mesh point B of Fig. 4 is like following

c) Solution of the system of algebraic equations

A system of equations having a numerous number of unknowns needs to be solved in the present problem. An iterative method could be chosen in solving the system. But it has some short comings like it has very slow convergence rate and it fails to produce any solution for other complex boundary conditions. Considering all these, a triangular decomposition method has been used here ensuring better reliability and better accuracy of solution in a shorter period of time. The matrix decomposition method used here solves the present system of equations directly.

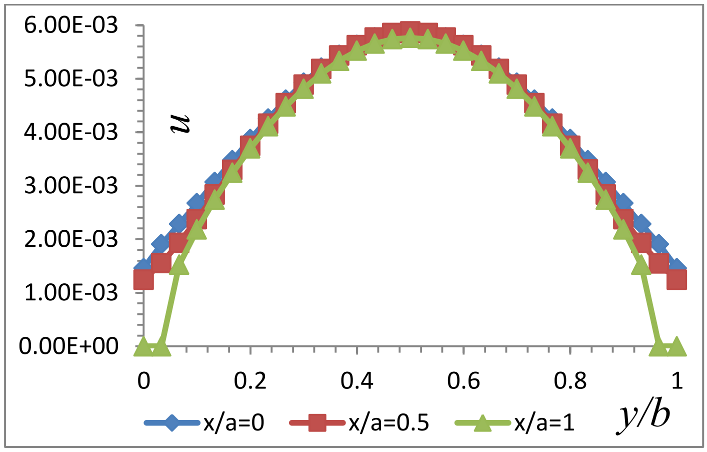

Figure 5 : Distribution of the displacement component u along the neutral axis of beam with fixed connection

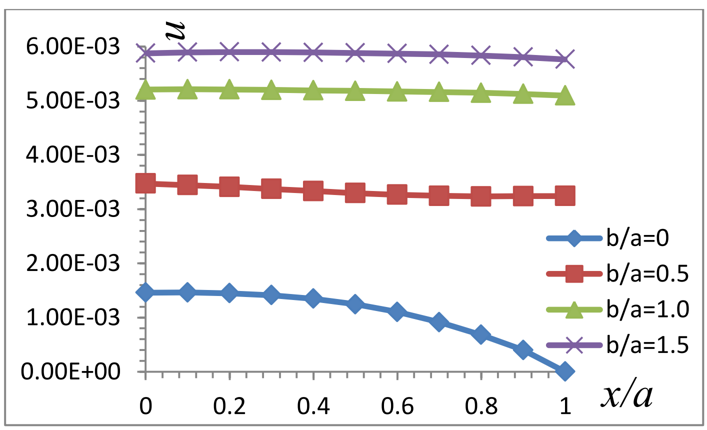

Figure 6 : Distribution of the displacement component u along the depth of beam with fixed connection

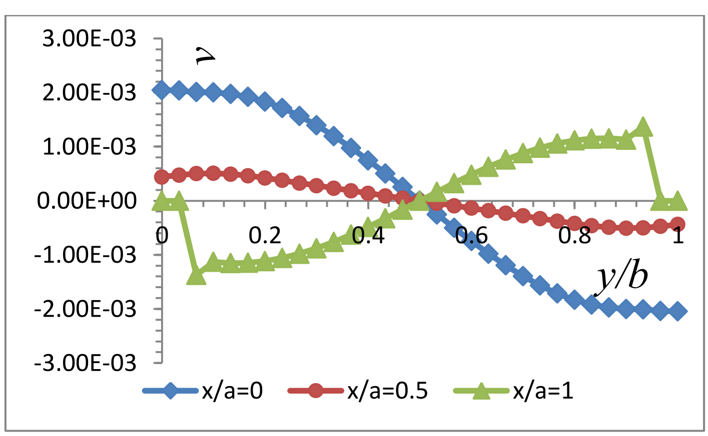

Figure 7 : Distribution of the displacement component v along the neutral axis of beam with fixed connection

VI. Results and Discussion

Distribution of the deformation parameters and stress parameters has been plotted along the length and depth of the beam. All the solutions of interest obtained through the ψ formulation conform to the symmetric and anti-symmetric characteristics of the problem. Again the famous Saint-Venant's principle that the effects of sharp variation of parameter on the boundary die down and become uniform with the increase of distance of points in the body from the boundary is also supported by all the solutions obtained.

The elastic problem that has been studied here is considered to be made of ordinary steel having poison ratio, μ=0.3 and modulus of elasticity E=200 GPa. Uniformly distributed load is applied at the top surface of the beam. The stresses and deformation are symmetrically distributed about the transverse mid-section of the beam.

a) Distribution of u

Fig.5 shows the distribution of u, displacement along x axis, along the length of the beam. The distribution is symmetric about the mid-section of the beam. At the mid-section, value of u is maximum. Again u is zero at points C and D. All are in complete conformity with the physical condition of the beam. Change of the distribution of u with the increase of distance from the top fiber is also evident in the figure. With the increase of distance, change in the distribution of u is minor.

Fig. 6 is a demonstration of the distribution of u along the depth of the beam. u remains almost constant along the depth. One exception is at the left most transverse section. Here at the point C, u is zero. It is in conformity with the physical condition because point C is fixed. The effect of b/a ratio along the neutral axis is also shown in the figure. With the increase of the ratio b/a, u increases. That means at the transverse sections nearer to mid transverse section, u is higher and it is highest at the mid section.

b) Distribution of v

Distribution of displacement along y axis, v is shown in the Fig.7. The distribution is counter symmetric about the transverse mid section of the beam. At the mid-section, v is zero. All these is in complete conformity with the physical condition and loading of the problem.

Fig. 8 shows the distribution of v along the depth of the beam. It is zero at the left most transverse section where value of x/a is 1. The effect of b/a ratio is also evident in the Fig. 8. With increasing b/a ratio, v decreases. That means at the inner sections the displacement along y direction dies down and at the mid-section there is no v value.

c) Distribution of σ_x

Fig. 9 shows the distribution of normal stress component σ_x with respect to x at various longitudinal sections of the beam.

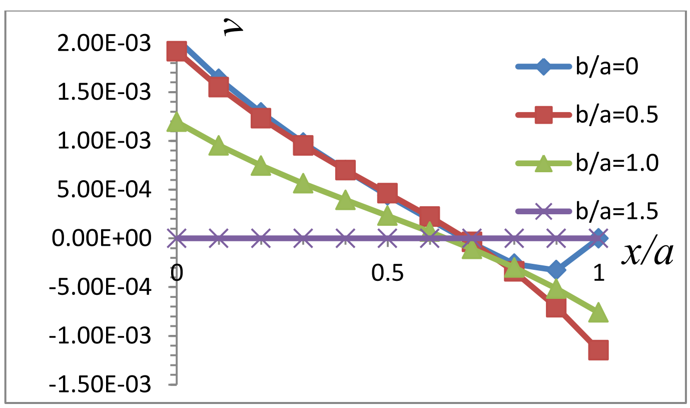

Figure 8 : Distribution of the displacement component v along the depth of beam with fixed connection

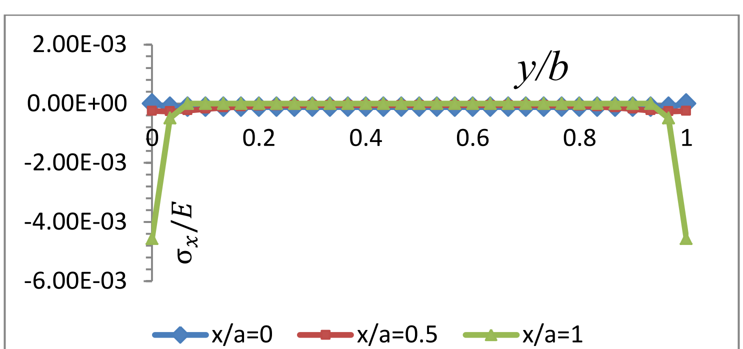

Figure 9 : Distribution of normal stress component σ_x at various longitudinal sections of a beam with fixed connection

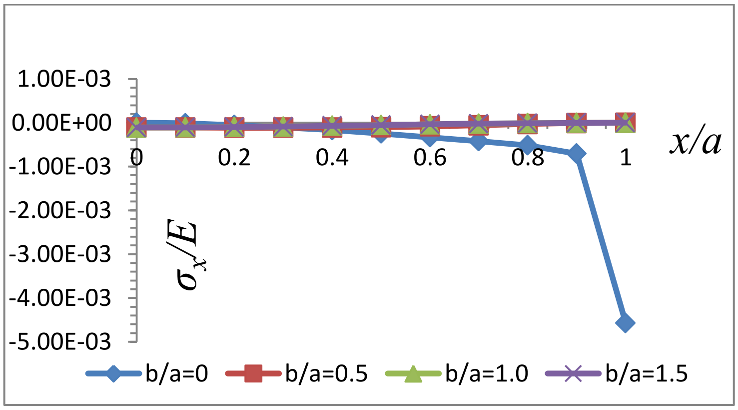

Figure 10 : Distribution of normal stress component σ_x at various transverse sections of a beam with fixed connection

From the figure it is observed that the distribution is symmetric about the transverse mid-section of the beam. It is also evident that the fixed region is the most critical region as far as the normal stress is concerned. It is supported by the available literature[9].

The effect of *b/a* ratio on the distribution of σ_x is shown in Fig.10. The value of σ_x decreases with an increasing *b/a* ratio. The most critical points are found at the region of fixed connection which is conforming to the available literature[9]. So it can concluded from the distributions shown in Fig.10 and Fig.11 that the most critical points with respect to σ_x are around $x/a = 1$, $y/b = 0$ and $x/a = 1$, $y/b = 1$.

d) Distribution of σ_y

Fig. 11 is a demonstration of the distribution of normal stress component σ_y along neutral axis of the beam. The distribution is completely symmetric about the transverse mid-section of the beam. The distribution is negative at the top fiber and positive at the bottom fiber. At the mid longitudinal section, the value of σ_y is negligible. All these are in complete conformity with the physical condition of the beam. The most critical points as far as σ_y is concerned, are found at the fixed region and they are at around $y/b =$ 0.03 and 0.97 respectively. Again it conforms to the available literature[9]

From Fig.12 the effect of b/a ratio on σ_y can be studied. It is zero at almost all transverse sections except at $b/a=0.1$. At the section $b/a=0.1$, there are at least two critical points between

$$0.8 \le x/a \le 1.0$$

e) Distribution of σ_{xy}

Distribution of shear stress σ_{xy} at various longitudinal sections(Fig.13) of a beam with fixed connection reveals that shearing stress is almost zero at all the longitudinal sections. It conforms to the physical and loading condition of the beam under consideration. But again there are atleast two critical points found at the fixed connection region which again reveals that fixed connection is an extremely critical region where the probability of failure is maximum.

Finally Fig.14 is a demonstration of shear stress at different transverse sections of the beam. Shear stress is almost zero at different sections except at the fixed region, which again conforms to the physical condition of the beam.

VII. Conclusions

In the elementary formulas of strength of materials, the boundary conditions are satisfied in an approximate way. As a result it often fails to give the exact distribution of stresses at the restrained boundaries. The present displacement potential function approach is free from this type of limitations. So it is capable of representing the actual distribution of normal and shear stresses at critical regions. For example, from the elementary solution it is observed that the magnitude of the shearing stress is maximum at the mid-section of the beam. It is not supported by the numerical solution presented here,

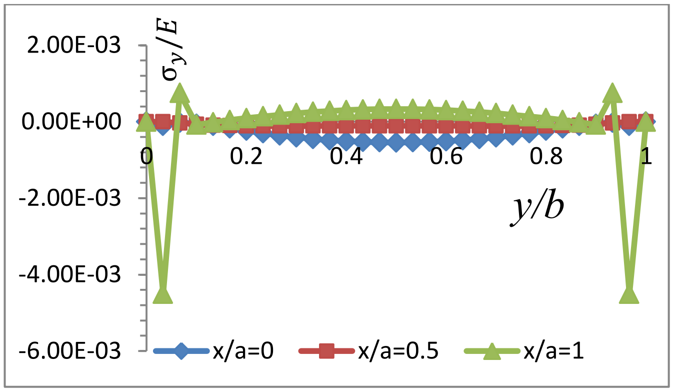

Figure 11 : Distribution of normal stress component σ_y at various longitudinal sections of a beam with fixed connection

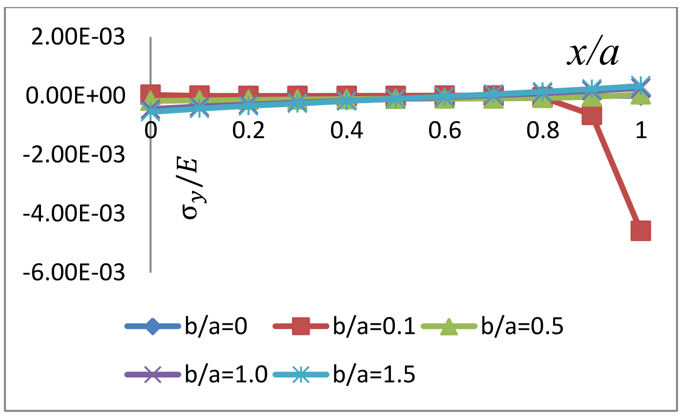

Figure 12 : Distribution of normal stress component σ_y at various transverse sections of a beam with fixed connection

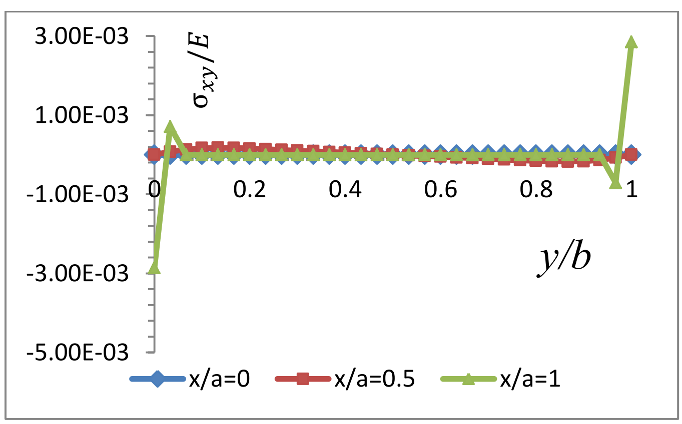

Figure 13 : Distribution of shear stress σ_{xy} at various longitudinal sections of a beam with fixed connection

because the approach used here reveals that shearing stress as well as normal stresses are maximum at the restrained boundaries. So, a more reliable and accurate study on the distribution of normal and shear stresses has been provided in the present literature. Besides, all the results obtained here are compared with the available literature and are in complete conformity with it and with the boundary and loading conditions. Such both the qualitative and quantitative results obtained through ψ-formulation of a beam with fixed connection at the bottom surface and subjected under uniformly distributed load establish the soundness and acceptance of the approach adopted here.

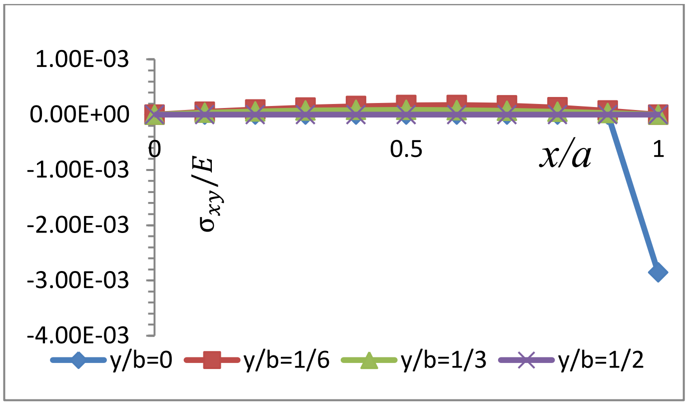

Figure 14 : Distribution of shear stress σ_{xy} at various transverse sections of a beam with fixed connection

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Global Journal of Researches in Engineering (A) Volume XIII Issue II Version I Year 2013

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