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MATHEMATICAL MODELING OF TEMPERATURE LAPSE RATE IN SOLAR CHIMNEY POWER PLANT

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Mathematical Modeling of Temperature Lapse Rate in Solar Chimney Power Plant

Shashank.B. Thakre [α], L.B. Bhuyar [σ], Sachin.V. Dahake [ρ] & Pankaj Wankhade [ω]

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I. Introduction

The widespread use of solar energy, as an alternate and non depletable resource for agriculture and industry as well as other applications, is dependent on the development of solar systems which possess the reliability, performance and economic characteristics that compare favorably with the conventional systems. The solar chimney power plant system, which is composed of the solar collector, the chimney and the turbine, has been investigated all over the world since the German researcher Jorg Schliaich first made the brainchild in the 1970s. The main objective of the collector is collecting solar radiation to heat up the air inside. As the air density inside the system is less than that of the environment at the same height, natural convection affected by buoyancy which acts as driving force comes into existence. Due to the existence of the chimney, the cumulative buoyancy results in a large pressure difference between the system and the environment, then the heated air rises up into the chimney with great speed. If an axis-based turbine is placed inside the chimney where there is a large pressure drop, the potential and heat energy of the air can be converted into kinetic energy and ultimately into electric energy.

II. Description of the System

A typical solar chimney power plant consists of a solar hot air collector, a solar chimney and a turbine with generator. All three essential elements have been familiar from time immemorial. A solar chimney power plant simply combines them in a new way [1,2,3], as is shown in Fig. 1. Air is heated by solar radiation under a low circular transparent cover open at the periphery; this and the natural ground below it form a hot air collector. In the middle of the cover is a vertical chimney with large air inlets at its base. The joint between the cover and the chimney base is airtight. As hot air is lighter than cold air it rises up the chimney. Suction from the chimney then draws in more hot air from the collector, and cold air enters from the outer perimeter. Thus solar radiation causes a constant up-draught in the chimney. The energy that the hot air contains is converted into mechanical energy by pressure staged wind turbines at the base of the chimney, and into electrical energy conventional generators.

The characteristics of this solar chimney power plant are listed below.

- Efficient solar radiation use. The hot air collector used in the system, can absorb both direct and diffused radiation. Thus the solar chimney can operate on both clear and overcast days. The other major large-scale solar thermal power plants, which are often driven by high temperature steam generated from solar concentrators, can only use direct radiation.

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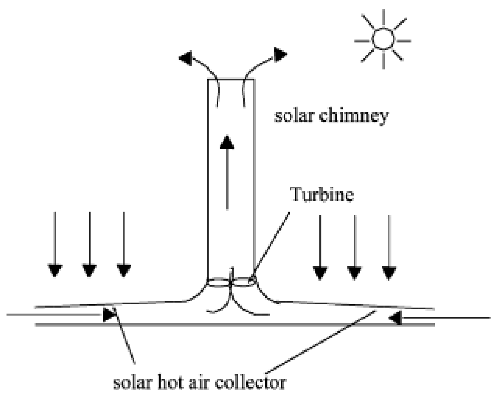

Fig. 1 : Schematic diagram of a solar chimney power plant

Free dual functions, natural energy storage and greenhouse effect. The collector provides storage for natural energy, as the ground under the transparent cover can absorb some of the radiated energy during the day and releases it into the collector at night. Thus solar chimneys also produce a significant amount of electricity at night. The collector itself can also be used as a greenhouse, which will benefit agriculture production accordingly.

- Low operation cost. Unlike conventional power stations, and also other solar–thermal type power stations, solar chimneys do not need cooling water. This is a key advantage in northwestern China where there have already been problems with drinking water.
- Low construction cost. The building materials needed for solar chimneys, mainly concrete and transparent materials, are available everywhere in sufficient quantities.
- Particularly important is that no investment in a high-tech manufacturing plant is needed, as both wind turbine and solar collectors are well- developed industrial products.

III. Mathematical Models Developed

Xinping Zhou et.al., (2009) developed a theoretical model for evaluating maximum height and optimum height of chimney for maximum power output. The models proposed are as follows.

The electric power generated by the turbine generators, Pout, can be expressed as

$$P_{out} = \eta_{tg} \Delta p_t . V . A_c \qquad (1)$$

Where η_{tg} is the efficiency of turbine generators. Total energy conversion efficiency can be expressed as the ratio of Pout to solar radiation input on the collector.

$$\eta = \frac{P_{out}}{\pi R_{coll}^2 G} \qquad (2)$$

The maximum chimney height can be found out by using expression

$$H_{Max} = \frac{c_p \dot{m}}{U \pi D} . \ln \left(\frac{\pi^2 UDG \eta_{coll} R_{coll}^2}{c_p \dot{m}^2 \left(g - \gamma_\infty c_p \right)} + 1 \right) \qquad (3)$$

Pasumarthi and Sherif (1998) conducted extensive research work on experimental and theoretical performance of a demonstration solar chimney model as part I of their research work. The purpose of the investigation on which this paper partly reports was to demonstrate that the solar chimney technology is a viable alternate energy technology suitable and adaptable to hot-climate areas such as those of Florida. Other objectives included developing a mathematical model that could predict the performance characteristics of solar chimneys and validate model results against experimental data. The theoretical models proposed are as follows. The collector was divided into 25 sections, starting from air entering the collector till the air entering the base of the chimney.
Collector: A simple energy balance across a section of the collector yields the fluid temperature at the outlet of that section. For example, the exit temperature at the first section can be evaluated using

$$t_{f.avg} = t_a + \frac{q_u}{\dot{m} c_p} \qquad (4)$$

The temperature $t_{f.avg}$ so obtained becomes the inlet temperature for the second section and so on. The temperature attained by the air at the end of the 25th section is the temperature of the air entering the chimney section.

Chimney: The velocity at the exit of each chimney section can be found using following equation.

$$\rho_{ch} a_{ch.\sec} v_i^2 + 1 = \rho_{ch} a_{ch.\sec} v_i^2 + \left(\rho_a - \rho_{ch} \right) g a_{ch.\sec} \Delta z - \tau_w Pe \Delta z \qquad (5)$$

Wind turbine generator: The wind turbine placed inside the chimney converts translational energy of the flow field into rotational energy.

$$P_{Max} = \frac{16}{27} \left(\frac{1}{2} \rho a v^3 \right) \qquad (6)$$

The electric power output can be related to the kinetic power by the following equation:

$$P_{elec} = \eta_{gen} P_{\max} \tag{7}$$

Typical values of the generator efficiency η_{gen} vary from 75% to 85%. A value of 80% for η_{gen} is used throughout this analysis. An overall efficiency can be defined for the solar chimney as the ratio of the useful electric power output and the insolation over the area of the collector:

$$\eta_{overall} = \frac{P_{elec}}{Sa_{coll}} \tag{8}$$

Having developed the models for the analysis, the authors finally concluded that mathematical model capable of predicting the performance of solar chimney systems has been developed. The model is capable of estimating the temperature and power output of solar chimneys as well as examining the effect of various ambient conditions and structural dimensions on the power output.

The potential of solar chimney for application in rural areas of developing countries is discussed by Onyango, F.N and Ochieng, R.M., (2006). The authors have considered the appropriateness of a solar chimney to rural villages and highlight some features of such a power generating plant. A mathematical correlation was developed for temperature ratio τ, and the instantaneous electric power produced by a single turbine P_i, for square collector of side L, a circular chimney of radius R and velocity V, at which the air impinges on the rotor blades. The important expressions are as follows.

$$\tau = \left(\frac{T_s - T_H}{T_m - T_s} \right) \tag{10}$$

Where, τ is the difference between the collector surface temperature and the temperature at the turbine (T_s–T_H) to the difference between the air mass temperature under the roof and the collector surface temperature (T_m–T_s), Where T_s and T_H are the temperatures at the surface covered by a selective material and at any position H in the covered area A and h is the convective transfer coefficient.

If it is assumed that the area swept by the rotor is the same as the cross-sectional area of the chimney, the instantaneous electric power Pi produced by a single turbine is readily derived as

$$P_i = \frac{16}{27}\left(\frac{1}{2} \rho_m \pi R^2 V^3 \right) \tag{11}$$

Where, ρ_m is the density of the air at temperature Tm and the factor of 16/27 is the ideal limit for the extraction of power.
Further can be written as,

$$P_i = 3.0x10^{-31} \beta \left(\frac{L^{15}}{H^{12} R^4} \right) \tau^{15} \tag{12}$$

Where at 300 K and one atmosphere

$$\beta = \left(\frac{\rho_m k^5}{\alpha^5 C_p^{15} \nu^7 \rho_a^{15}} \right) = 1.148X10^{-12} \tag{13}$$

Substituting the known the constants in equation (13) for dry air at one atmospheric pressure and ambient temperature (T = 300 K), the instantaneous power reduces to

$$P_i = 4.48X10^{-43} \left(\frac{L^{15}}{H^{12} R^4} \right) \tau^{15} \tag{14}$$

Thermal and technical analyses of solar chimneys were carried out by Bernardes, M.A. and Weinrebe, A. (2003). In the research work, an analysis for the solar chimneys was developed, aimed particularly at a comprehensive analytical and numerical model, which describes the performance of solar chimneys. This model was developed to estimate power output of solar chimneys as well as to examine the effect of various ambient conditions and structural dimensions on the power output. Results from the mathematical model were compared with experimental results and the model was further used to predict the performance characteristics of large-scale commercial solar chimneys. The results show that the height of chimney, the factor of pressure drop at the turbine, the diameter and the optical properties of the collector are important parameters for the design of solar chimneys.

Mullet (1987) presented an analysis to derive the overall efficiency of the solar chimney. The author found that the overall efficiency is directly related to the height of the chimney and he demonstrated it to be about 1% for the height of 1000m and finally concluded that the solar chimney is essentially a power generator of large scale.

The equations developed by the researcher are as follows.
Driving force inside the chimney is given by,

$$\Delta p = 11.67 \left(1 - \frac{K_0}{K_1} \right) h \tag{15}$$

Where, K_0 = Ambient temperature (293 k)
K_1 = Air temperature in the chimney.
h = Height of the chimney.

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Velocity of air flowing inside the chimney is given as,

$$V = \left[\frac{2\Delta Kgh}{K_0}\right]^{1/2} \quad (16)$$

Where, ΔK = Temperature rise in K

K_0 = Ambient temperature (293 k)

Power rating: The mechanical power rating can be obtained as the rate of delivery of kinetic energy at the top of the chimney.

$$W = \rho_1 \sqrt{2}\left[\frac{\Delta Kgh}{K_0}\right]^{3/2} \quad (17)$$

Where, ρ_1 = Density of air at temperature K_1.

W = Mechanical power rating.

Overall efficiency;

η_1 is assumed to be 80% with a reasonable rise in temperature.

$\eta_2 = 0.033 \times 10^{-3}$ h

η_3 is assumed to be in the range of 40 to 80 %.

$\eta = \eta_1 \times \eta_2 \times \eta_3$

= 1.07%

Zhou et.al., (2007) conducted experiments on a model having 8 PVC pipes measuring 0.35m radius and 1m height. Eight peaces of pipes were taken to get variation in height of the chimney. In the similar fashion diameter of chimney was made to vary from 1m to 5m. A multiple-blade designed on the operating principle of turbine blade was installed at the base of the chimney. The generator, commercially available, was a permanent magnetism motor with direct current. The updraft drove the turbine, which drove the generator to generate electricity. In the course of experimental measuring, the turbine-generator is under no load conditions. Platimum resistance thermometer sensors (Pt 100) were used to measure hot air temperatures; a mercury thermometer with an accuracy of ±0.5 1C was fixed outside the equipment to measure ambient temperature; a thermal anemometer with an accuracy of ±0.01 m/s was used to measure the velocity of airflow. Measurements were sampled every 10 min. Further a mathematical model based on energy balance has been developed to predict the performance of the solar chimney thermal power generating equipment for different conditions. The models are as follows.

$$V = \sqrt{2gH_{ch}\left(T_0 - T_\infty\right)/T_\infty} \quad (18)$$

Where, V denotes hot air velocity at the solar collector outlet.

Power output of the total system can be found as

$$P_{out} = \frac{1}{3}\eta_w \rho A_{ch} V^3 \quad (19)$$

Where η_w is the turbine efficiency, usually between 50% to 90%.

On the basis of results obtained the researchers finally concluded that the simulated power outputs are basically in agreement with the results calculated with the measurements, which validates the mathematical model of the solar chimney thermal power generating system. Furthermore, based on the simulation and the specific construction costs at a specific site, the optimum combination of chimney and collector dimensions can be selected for a required electric power output.

Analytical and numerical investigation of the solar chimney power plant systems was carried out by Tengzhen et.al., (2006). Different models were developed by the authors for determining static pressure, driving force, power output and efficiency. The authors felt that there is a surge in the use of the solar chimney power plant in the recent years which accomplishes the task of converting solar energy into kinetic energy and also the existing models are insufficient to accurately describe the mechanism. Therefore a more comprehensive model is presented by the authors to evaluate the performance of a solar chimney power plant system, in which the effects of various parameters on the relative static pressure, driving force, power output and efficiency have been further investigated.

The density difference may be expressed in terms of the volume coefficient of expansion as

$$\Delta p = \frac{\pi g \rho_0 \beta_0 q}{C_p \dot{m}} H R_{coll}^2 \quad (20)$$

The maximum power output can be expressed as the product of driving force and the volumetric flow rate and shows that the air properties, the chimney height, the collector radius and the solar radiation have significant effect on the maximum power output.

$$P_{max} = \frac{\rho_0}{\rho}\frac{\pi g}{C_p T_0} H R_{coll}^2 q \quad (21)$$

The maximum efficiency of the system can be expressed by,

$$\eta_{max} = \frac{\rho_0}{\rho}\frac{gH}{C_p T_0} \quad (22)$$

Which, demonstrates the functional dependence of the system maximum efficiency on the air density, the inlet temperature and the chimney height.

It can be anal sized from above literature that more emphasis is given in development of models for obtaining velocity of flow through the chimney, power generated by the plant and heat flow through the collector, but significant efforts for determination of adiabatic temperature lapse rate is not done, hence in this paper an attempt is made for obtaining the adiabatic temperature lapse rate with the help of simple mathematical model.

IV. Mathematical Correlation for Adiabatic Lapse Rate

Adiabatic lapse rate is given by

$$-\frac{dT}{dz} = \frac{g}{Cp} \tag{23}$$

Above equation can be written as

$$-dT = \frac{g}{Cp}dz$$

Integrating the above equation with limits T_1 to T_h, we get,

$$\int_{T1}^{Th} -dT = \frac{g}{Cp}\int_{Z_1}^{Z_h} dz$$

$$-\int_{T1}^{Th} T = \frac{g}{Cp}\int_{z1}^{zh} Z + A$$

$$-(T_h - T_1) = \frac{g}{Cp}(Z_h - Z_1) + A$$

$$\text{or, } (T_1 - T_h) = \frac{g}{Cp}(Z_h - Z_1) + A \tag{24}$$

at Z = 0, $T_h = T_1$

where,

Z – is the height at which turbine is installed.

T_1 – Temperature of air at Z=0.

T_h – Temperature of air at exit of the chimney.

Substituting in (2)

$$0 = \frac{g}{Cp}(Z_h) + A$$

Therefore,

$$A = -\frac{g}{Cp}(Z_h) \tag{25}$$

Substituting the value of (3) in (2), we get,

$$(T_1 - T_h) = \frac{g}{Cp}(Z_h - Z_1) - \frac{g}{Cp}Z_h$$

Therefore,

$$T_1 = T_h - \frac{g}{Cp}Z_1 \tag{26}$$

The temperature of air at any height (Z+h) can similarly be expressed as

$$T_{(Z+h)} = T_h - \frac{g}{Cp}(Z_1 + h) \tag{27}$$

$$T_{(Z+h)} = T_h - \frac{g}{Cp}Z_1 - \frac{g}{Cp}h \tag{28}$$

But from equation (26)

$$T_h - \frac{g}{Cp}Z_1 = T_1$$

Therefore, equation (28) reduces to

$$T_{(Z+h)} = T_1 - \frac{g}{Cp}h$$

or

$$T_{(Z+h)} = -\frac{g}{Cp}h + T_1 \tag{29}$$

This equation is comparable with an equation of straight line of the form

$$y = mx + c$$

It indicates that the plot between $T_{(Z+h)}$ and T_1 should yield a straight line. The slope of this line 'm' represents the value $-\frac{g}{Cp}$ from which the ratio $-\frac{g}{Cp}$ can be calculated and T_1 represents the intercept on Y-axis.

V. Conclusion

It is concluded that the mathematical models developed by the different researcher are quite useful for predicting the performance of solar chimney power plant. The theoretical mathematical model developed in the present study is quite useful for obtaining the adiabatic temperature lapse rate for a long chimney where the heat flow from the walls of the chimney cannot be measured accurately.

References Références Referencias

1. Schlaich J. The solar chimney: 1995. Electricity from the sun. In: Maurer C, editor. Germany: Geislingen.
2. Haaf W, Friedrich K, Mayr G, Schlaich J. Solar chimneys: 1983 Part I: Principle and construction of

the pilot plant in Manzanares. International Journal of Solar Energy;2(1):3–20.
3. Gannon AJ, Backstrom TWV.2000. Solar chimney cycle analysis with system loss and solar collector performance. ASME, Journal of Solar Energy Engineering; 122:133–7.
4. Xinping, Zhou., Jiakuan, Yang.,Bo Xiao, Guoxiang, Hou., Fang, Xing., (2009) "Analysis of chimney height for solar chimney power plant", Applied Thermal Engineering 29, pp. 178–185.
5. Pasumarthi N, and Sherif, S.A. (1998) Experimental and theoretical performance of a demonstration solar chimney model-Part I: mathematical model development, Int. J. Energy Res. 22, 277–288.
6. F.N. Onyango, R.M. Ochieng, (2006) The potential of solar chimney for application in rural areas of developing countries, Fuel 85, 2561–2566.
7. Mullet, L. B. (1987) "The solar chimney-overall efficiency, design and performance", Int. J. Ambient Energy, 8(1), 35-40.
8. X.P. Zhou, J.K. Yang, B. Xiao, G.X. Hou, (2007) Experimental study of temperature field in a solar chimney power setup, Appl. Therm. Eng. 27, 2044–2050.