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By Sohag Sarker, Tanzila Lutfor & Md. Zahangir Alam
*Pabna University of Science and Technology, Bangladesh*

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*Strictly as per the compliance and regulations of :*

# Performance Analysis of Maximal-Ratio Combining and Equal Gain Combining in Fading Channels

Sohag Sarker [α], Tanzila Lutfor [σ] & Md. Zahangir Alam [ρ]

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## I. Introduction

In wireless communications, radio propagation refers to the behavior of radio waves when they are propagated from transmitter to receiver through medium [1]. The transmission path between the transmitter and receiver can vary from simple line-of-sight to one that is severely obstructed by buildings, mountains, and foliage which cause reflection, diffraction, and scattering[2]. The wireless channel environment mainly governs the performance of wireless communication systems. The wireless channel is dynamic and unpredictable rather than the typically static and predictable characteristics of a wired channel which make an exact analysis of the wireless communication system often difficult. A unique characteristic in a wireless channel is a phenomenon called 'fading', the rapid fluctuations of the signal amplitude over time and frequency. In addition to AWGN, fading is another source of signal degradation that is characterized as a non-additive signal disturbance in the wireless channel. Fading may either bemulti-path (induced) fading, which is due to multipath propagation, orshadow fading which is due to shadowing from obstacles that affect the propagation of a radio wave. The fading phenomenon in the wireless communication channel is modeled for 800MHz to 2.5 GHz by extensive channel measurements in the field. Includes the ITU-R standard channel models specialized for SISO. The MIMO (Multiple Input Multiple Output), MISO (Multiple Input Single Output), and SIMO (Single Input Multiple Output) systems have been recently developed by the various research and standardization activities, aiming at high-speed wireless transmission and diversity gain [3] which improves the performance of the communication links over radio fading channels[1].The basic concept of diversity is that if one signal path is week at a particular point of time, another independent path may be just fine. Here receiver is provided with multiple copies of the same information signal which are transmitted over two or more real or virtual communication channels. Thus it can be considered as the diversity is the repetition or redundancy of information [4].

Diversity-combining techniques are often used to combat the deleterious effect of channel fading [5,6]. Maximal ratio combining (MRC) or equal-gain combining (EGC) are widely applied to reduce the system bit error rate (BER) [6].

The *relative* advantage of diversity is greater for Rayleigh fading than Rician fading, because as the Rice factor *K* increases there is less difference between the instantaneous received signal-to-noise ratios on the various diversity branches [7].However, the performance will always be better with Rician fading than with Rayleigh fading, for a given average received signal-to-noise ratio and diversity order.

## II. System Model

Let us consider the transmission ofthe band-pass signal,

$$s(t) = \mathrm{Re}[\tilde{s}(t)e^{j2\pi f_c t}] \tag{1}$$

where, $\tilde{s}(t)$ = complex envelope of transmitted signal, $f_c$= carrier frequency, Re[z]= real part of z.If the channel is composed of N propagation paths, then the noiseless received band pass waveform is

$$r(t)=\mathrm{Re}[\sum_{n=1}^{N} C_n e^{j2\pi[(f_c+f_{D,n})(t-\tau_n)]}\tilde{s}(t-\tau_n)] \tag{2}$$

where, $C_n$ is the amplitude of $n^{th}$ propagation path, $\tau_n$is time delay associated with $n^{th}$ propagation path.

*Author α: Department of Information and Communication Engineering, Pabna University of Science and Technology, Pabna-6600, Bangladesh. e-mail: sohagsarker5614@gmail.com*
*Authors σ ρ: School of Science and Engineering, University of Information Technology and Sciences (UITS), Dhaka, Bangladesh. e-mail: ripon83.ru@gmail.com*

Now, the received band pass signal,

$$r(t)=\mathrm{Re}[\tilde{r}(t)e^{j2\pi f_c t}] \tag{3}$$

where, the received complex envelope,

$$\tilde{r}(t)=\sum_{n=1}^{N} C_n e^{-j\varphi_n(t)}\tilde{s}(t-\tau_n) \tag{4}$$

$$\varphi_n(t)=2\pi\{(f_c+f_{D,n})\tau_n-f_{D,n}t\} \tag{5}$$

Now, the channel impulse response,

$$g(t,\tau)=\sum_{n=1}^{N} C_N e^{-j\varphi_n(t)}\delta(\tau-\tau_n) \tag{6}$$

where $g(t,\tau)$ is the channel response at time t due to an impulse applied at time t-τ, δ(.) is the dirac delta function[4].

Here, the channel assumed to be wide sense stationary (WSS), so the received complex envelope $g(t)=g_I(t)+jg_Q(t)$; where, $g_I(t)$ and $g_Q(t)$ are independent identically distributed zero mean Gaussian random variables at time $t_1$ with variance $b_0$.Under these conditions, the received complex envelope's magnitudeα(t) = |g(t)| has a Rayleigh distribution at any time $t_1$, that is $P_\alpha(x)=\frac{x}{b_0}exp\left\{-\frac{x^2}{2b_0}\right\}$.The average envelope power is$E[\alpha^2]=\Omega_p=2b_0$.

So, for Rayleigh distribution,

$$P_\alpha(x)=\frac{2x}{\Omega_p}\exp\{\frac{-x^2}{\Omega_p}\}\ (x\geq 0) \tag{7}$$

For Rician fading, line of sight (LoS) component will be present; so equation (7) can be re-written for Rician channel as:

$$P_\alpha(x)=\frac{2x}{\Omega_p}\exp\{\frac{-x^2+s^2}{2b_o}\}I_0(\frac{xs}{b_0})\ (x\geq 0) \tag{8}$$

where, $s^2=m_I^2(t)+m_Q^2(t)$

Where, $m_I(t)$ and $m_Q(t)$ corresponds to in-phase and quadrature component of LoS signal as:

$$m_I(t)=s\cdot\cos(2\pi f_m\cos\theta_0 t+\phi_0)$$

$$m_Q(t)=s\cdot\sin(2\pi f_m\sin\theta_0 t+\phi_0)$$

where, $f_m\cos\theta_0$ and $\varphi_0$are the Doppler shift and random phase off set associated with the LoS or specular component, respectively.The Rice factor (K)is the ratio of the specular power $s^2$to scattered power $2b_0$, that is, $K= s^2/2b_0$. When K = 0 the channel exhibits Rayleigh fading, and when K =∞ the channel does not exhibit any fading at all [7].

*a) Maximal Ratio Combiner (MRC)*

In MRC, each signal branch is multiplied by a weight factor that is proportional to the signal amplitude. Signals from all the MR branches are weighted according to their individual SNR and then summed.

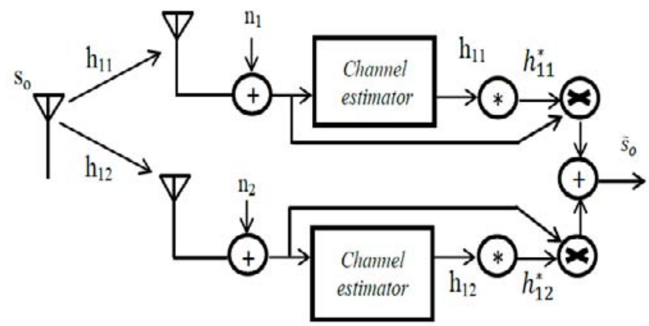


*Figure 1 :* MRC with 1 Tx and 2 RX

Our system, as indicates in fig-1, modeled with 1 transmitter and 2 receivers. The symbol to be transmitted is weighted with transmit weighting vector to form the transmitted signal vector. Thus the resulting receive signal can be represented as [1]:

$$\tilde{r}_1=h_{11}\tilde{s}_0+n_1 \tag{9}$$

$$\tilde{r}_2=h_{12}\tilde{s}_0+n_2 \tag{10}$$

Where, $h_{11}$ and $h_{12}$ represents the channel coefficients for receive antenna 1 and 2 respectively at time t and, $n_1$and $n_2$ represents additive white Gaussian noise with zero mean and unit variance [8].

The combined signal can be written as:

$$\hat{s}_o=h_{11}*\tilde{r}_1+h_{12}*\tilde{r}_2 \tag{11}$$

This system, particularly designed for 1 Transmitter antenna and L Receiver Antennas. Thus the channel consists of 1*L statistically independent coefficients which form the channel matrix **h** = $[h_{11}\ h_{12}\ \ldots\ldots\ldots h_{1L}]^T$. Then the received signal vector can be written as the product of the Transmitted signal and the channel coefficient plus noise.

$$\tilde{\boldsymbol{r}}=\boldsymbol{h}*\boldsymbol{x}+\boldsymbol{n} \tag{12}$$

where, $\tilde{\boldsymbol{r}}\triangleq(\tilde{r}_1,\ \tilde{r}_2,\cdots,\tilde{r}_L)^T$, the noise vector **n** = $[n_1\ldots\ldots\ldots\ldots.n_L]^T$. Here, the noise is assumed to be uncorrelated and white Gaussian with the signal [5].

As it is inevitable that MRC realizes Maximum Likelihood (ML) detector, the receiver chooses the message vector $\tilde{s}_m$ that maximizes the metric,

$$\mu(\tilde{\mathbf{s}}_m)=-\sum_{K=1}^{L}\left\|\tilde{\mathbf{r}}_K-h_{1K}\tilde{\mathbf{s}}_m\right\|^2$$

$$=-\sum_{K=1}^{L}\{\left\|\tilde{\mathbf{r}}_K\right\|^2-2\mathrm{Re}(h^*_{1K}\tilde{\mathbf{r}}_K,\tilde{\mathbf{s}}_m)+\left|h_{1K}\right|^2\left\|\tilde{\mathbf{s}}_m\right\|^2\} \tag{13}$$

The diversity combiner generates the sum

$$\tilde{\boldsymbol{r}} = \sum_{k=1}^{L} h_{1k}^{*} r_k \qquad (14)$$

After weighting, co-phasing and combining, the envelope of the composite signal component can be written as:

$$\alpha_M = \sum_{K=1}^{L} \alpha_K^2 \qquad (15)$$

The weighted sum of the branch noise power is

$$\sigma_{\acute{n},tot}^2 = N_0 \sum_{k=1}^{L} \alpha_k^2 \qquad (16)$$

Hence the symbol energy to noise ratio is

$$\gamma_s^{mr} = \frac{\alpha_M^2 E_{av}}{\sigma_{\acute{n},tot}^2} = \sum_{K=1}^{L} \frac{\alpha_k^2 E_{av}}{N_0} = \sum_{k=1}^{L} \gamma_K \qquad (17)$$

where $E_{av}$ is the average symbol energy in the signal constellation.

Here, it is assumed that all antennas are balanced and uncorrelated and γ has a chi-squared distribution with 2L degrees of freedom i.e.,

$$p_{\gamma_s}^{mr}(x) = \frac{1}{(L-1)!(\bar{\gamma}_c)L} x^{L-1} e^{-x/\bar{\gamma}_c} \qquad (18)$$

Where, $\bar{\gamma}_c = E[\gamma_k]$; k=1……..L (19)

For BPSK the bit error rate can be represted as:

$$P_b = \int_0^{\infty} P_b(x) P_\gamma(x) dx$$

$$= \int_0^{\infty} Q(\sqrt{2x}) \frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c} dx \qquad (20)$$

Similarly, bit error rate for M-QAM as follows [ 9] :

$$P_{QAM} = 1 - (1 - P_{QAM,Rician,\sqrt{M}})^2 \qquad (21)$$

where,

$$P_{QAM,Rician,\sqrt{M}}$$

$$= (1 - \frac{1}{\sqrt{M}}) \sum_{n=0}^{\infty} \frac{(LK)^n e^{-LK}}{\Gamma(n+1)} * \left[1 - \sum_{i=0}^{L+n-1} \mu (\frac{1-\mu^2}{4})^i \binom{2i}{i}\right]$$

and $\mu = \sqrt{\dfrac{3\bar{\gamma}_s}{2M - 2 + 3\bar{\gamma}_s}}$

*b) Equal Gain Combiner(EGC)*

The similarity between EGC and MRC is that in both cases diversity branches are co-phased, but for EGC diversity branches are not weighted. The complete channel vector is required anyway and MRC might as well be used. In EGC, the receiver maximizes the metric

$$\mu(\tilde{\boldsymbol{s}}_m) = \sum_{k=1}^{L} Re(\tilde{\boldsymbol{r}}_k e^{-j\Phi_k}, \tilde{\boldsymbol{s}}_m) = \sum_{k=1}^{L} Re\left(e^{-j\Phi_k} \int_0^T \tilde{r}_k(t) \tilde{\boldsymbol{s}}_m^*(t)\, dt\right) \qquad (22)$$

The combiner generates the sum

$$\tilde{\boldsymbol{r}} = \sum_{k=1}^{L} e^{-\Phi_k} \tilde{\boldsymbol{r}}_k \qquad (23)$$

The envelope of the composite signal, after co-phasing and combining, is

$$\alpha_E = \sum_{k=1}^{L} \alpha_k \qquad (24)$$

And $LN_0$ is the sum of branch powers. The resulting symbol energy-to-noise ratio is

$$\gamma_s^{eg} = \frac{\alpha_E^2 E_{av}}{LN_0} \qquad (25)$$

With EGC, the average symbol energy-to-noise ratio is

$$\tilde{\gamma}_s^{eg} = \frac{E_{av}}{LN_0} E\left[\left(\sum_{k=1}^{L} \alpha_k\right)^2\right]$$

$$= \frac{E_{av}}{LN_0} \sum_{i=1}^{L} \sum_{l=1}^{L} E[\alpha_k \alpha_l] \qquad (26)$$

The bit error probability, with coherent BPSK signaling, is

$$P_b = \int_0^{\infty} P_b(x) P_{\gamma_s^{eg}}(x)\, dx$$

$$= \frac{1}{2}\left(1 - \sqrt{1-\mu^2}\right) \qquad (27)$$

where $\mu = \dfrac{1}{1 + \tilde{\gamma}_c}$

The Probability of error can be expressed as the following equation as well:

$$P_S = \frac{1}{\pi} \int_0^{\infty} \mathrm{Re}\, al\{G(\omega)\phi_x^*(\omega)\} d\omega \qquad (28)$$

The formula can be calculated using Gauss-Chebychev quadrature (GCQ)

$$G(\omega) = \frac{1}{\sqrt{\pi}\omega} \times$$

$$\left\{F(\frac{\omega}{2\sqrt{\kappa_2}}) - \exp(\frac{-\omega^2}{4\kappa_2}\sin^2(\eta)) \times F(\frac{\omega\cos(\eta)}{2\sqrt{\kappa_2}})\right\}$$

$$+ j\omega \frac{1}{2\pi} \int_0^{\eta} \frac{\sin^2(\theta)}{\kappa_2} \times \phi(1, \frac{3}{2}; \frac{-\omega^2 \sin^2(\theta)}{4\kappa_2})\, d\theta \qquad (29)$$

Where, $\eta = \pi - \pi/4$ and $\kappa_2 = \sin^2(\pi/4)$ and F (.) denotes Dawson's integral. [10]

Global Journal of Researches in Engineering ( F ) Volume XIII Issue XVII Version I Year 2013

## III. Simulation Results

In this section, we discussed the simulation results of the BER Vs SNR performance of BPSK, QPSK and 16-QAM modulation schemes with MRC and EGC diversity combining techniques over Rayleigh and Rician fading channels using one or more receiving antennas with the help of MATLAB.

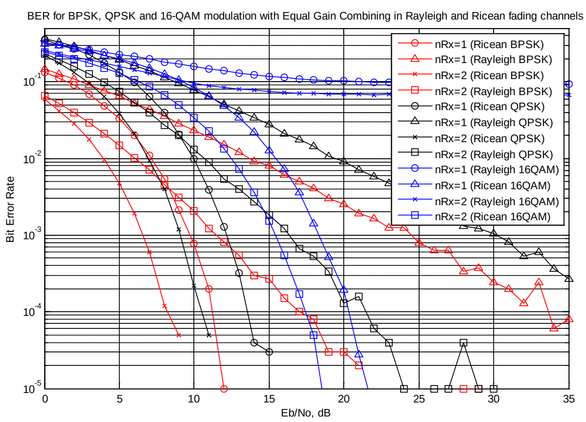


*Figure 1 :* BER for BPSK,QPSK and 16-QAM modulation schemes with Equal Gain Combining technique in Rayleigh and Rician fading channels

In Figure 1, it is observable that at very low SNR value area, the system performance is comparatively better under deployment of BPSK and Rician fading channel with Equal Gain Combining technique. It is also observable that with increasing number of receiving antennas BER is decreasing significantly.

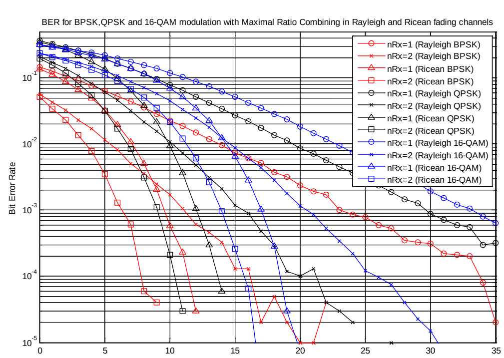


*Figure 2 :* BER for BPSK,QPSK and 16-QAM modulation schemes with Maximal Ratio Combining Technique in Rayleigh and Rician fading channels

In Figure 2, it is observed that with Maximal Ratio Combining technique, BPSK performs comparatively better than QPSK with Rician fading channel. It is also observable that BER decreases significantly with increasing number of receiving antennas.

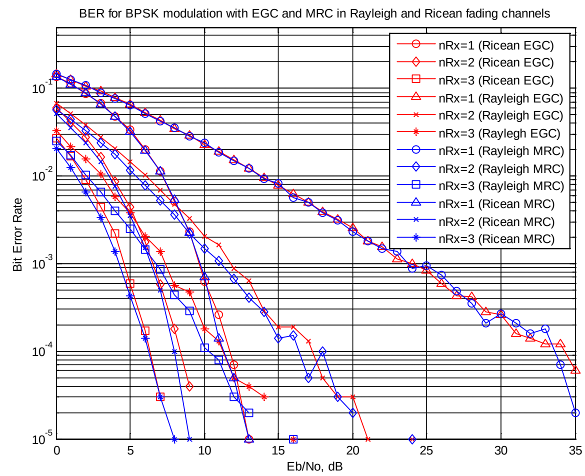


*Figure 3 :* BER for BPSK modulation schemes with EGC and MRC in Rayleigh and Rician fading channels

Figure 3 shows the BER for BPSK modulation schemes with EGC and MRC in Rayleigh and Rician fading channels. We observed that with single receiving antenna BER for BPSK is same in both Maximal Ratio Combining technique and Equal Gain Combining technique. With two or more receiving antennas Maximal Ratio Combining technique performs better than Equal Gain Combining technique with BPSK.

## IV. Conclusion

In this paper, we have investigated simulation results of Maximal ratio combining and Equal Gain Combining under Rayleigh and Rician fading channels using various modulation techniques. In the context of system performance, it can be concluded that BPSK Maximal Ratio Combining technique gives satisfactory results than Equal Gain Combining. The results presented in this paper are expected to provide useful information and guidelines to radio systems design engineers to exploit the use of diversity combining under realistic imperfect channel estimation scenarios.

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