



GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING  
ELECTRICAL AND ELECTRONICS ENGINEERING

Volume 13 Issue 1 Version 1.0 Year 2013

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 2249-4596 & Print ISSN: 0975-5861

## Performance Comparison of MIMO Systems over AWGN and Rayleigh Channels with Zero Forcing Receivers

By Navjot Kaur & Lavish Kansal

*Lovely Professional University, Phagwara*

**Abstract** - Multiple-Input Multiple-Output (MIMO) wireless antenna systems have been recognized as a key technology for future wireless communications. The performance of MIMO system can be improved by using multiple antennas at transmitting and receiving side to provide spatial diversity. In this paper, the effects of the number of transmit and receive antennas on the performance of MIMO system over AWGN and Rayleigh fading channels with ZF receiver is analyzed. The bit error rate performance characteristics of Zero-Forcing (ZF) receiver is investigated for M-PSK modulation technique. The AWGN and Rayleigh channels have been used for the analysis purpose and their effect on BER have been presented.

**Keywords** : MIMO (multiple input multiple output), AWGN (additive white gaussian noise), rayleigh, ZF (zero forcing), BER (bit error rate), M-PSK (M-ary phase shift keying), spatial diversity.

**GJRE-F Classification** : FOR Code: 100510



PERFORMANCE COMPARISON OF MIMO SYSTEMS OVER AWGN AND RAYLEIGH CHANNELS WITH ZERO FORCING RECEIVERS

*Strictly as per the compliance and regulations of :*



# Performance Comparison of MIMO Systems over AWGN and Rayleigh Channels with Zero Forcing Receivers

Navjot Kaur <sup>α</sup> & Lavish Kansal <sup>σ</sup>

**Abstract** - Multiple-Input Multiple-Output (MIMO) wireless antenna systems have been recognized as a key technology for future wireless communications. The performance of MIMO system can be improved by using multiple antennas at transmitting and receiving side to provide spatial diversity. In this paper, the effects of the number of transmit and receive antennas on the performance of MIMO system over AWGN and Rayleigh fading channels with ZF receiver is analyzed. The bit error rate performance characteristics of Zero-Forcing (ZF) receiver is investigated for M-PSK modulation technique. The AWGN and Rayleigh channels have been used for the analysis purpose and their effect on BER have been presented.

**Keywords** : MIMO (multiple input multiple output), AWGN (additive white gaussian noise), rayleigh, ZF (zero forcing), BER (bit error rate), M-PSK (M-ary phase shift keying), spatial diversity.

## 1. INTRODUCTION

In wireless communication technology, the high data rates can be achieved by using the multiple antennas at both transmitter and receiver through multiplexing or performance can be improved through diversity compared to single antenna systems [1]. The system that makes use of multiple antennas at the transmitter and receiver is referred as MIMO systems.

This method offers higher capacity to wireless systems and the capacity increases linearly with the number of antennas. There are two fundamental aspects of wireless communication as compared to wire line communication:

- First is the phenomenon of fading i.e. the time variation of the channel strengths due to the small-scale effects as well as largescale effects of multipath fading such as path loss via distance attenuation and shadowing by obstacles [2].
- Second, wireless users communicate over the air whereas in the wired communication each transmitter receiver pair is considered as an isolated point-to-point link.

A MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment. MIMO systems may be implemented in a number of

different ways to obtain either a diversity gain to combat signal fading or to obtain a capacity gain.

There are various categories of MIMO techniques. The first one aims to improve the power efficiency by maximizing spatial diversity. Such techniques include delay diversity, STBC [3], [4] and STTC [5]. The second one uses a layered approach to increase capacity. One popular example of such a system is V-BLAST where full spatial diversity is usually not achieved [6].

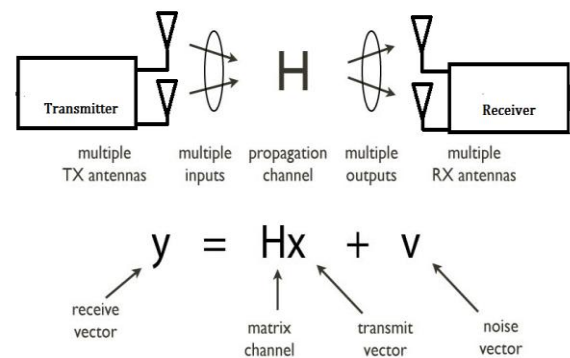


Figure 1.1 : Basic Representation of MIMO System (2X2 MIMO Channel)

In the Fig. 1.1, the basic representation of MIMO system is presented. The main idea of MIMO is to improve quality (BER) and/or data rate (bits/sec) by using multiple TX/RX antennas [7]. The core scheme of MIMO is space-time coding (STC). The two main functions of STC: diversity & multiplexing. The maximum performance needs tradeoffs between diversity and multiplexing.

In this paper, we consider the effects of fading channels like AWGN and Rayleigh channel on the performance of MIMO systems using ZF receivers. AWGN channel is a universal channel model for analyzing modulation schemes. In this, channel adds a white Gaussian noise to the signal passing through it. This implies that the channel's amplitude frequency response is flat and phase frequency response is linear for all frequencies so that modulated signals pass through it without any amplitude loss and phase distortion of frequency components. Constructive and destructive nature of multipath components in flat fading

channels can be approximated by Rayleigh distribution if there is no line of sight which means when there is no direct path between transmitter and receiver.

## II. BENEFITS OF MIMO SYSTEMS

### a) *Interference reduction and avoidance*

Interference in wireless networks results from multiple users sharing time and frequency resources. Interference may be mitigated in MIMO systems by exploiting the spatial dimension to increase the separation between users. Interference reduction and avoidance improve the coverage and range of a wireless network.

### b) *Spatial multiplexing*

Spatial multiplexing offers a linear (in the number of transmit-receive antenna pairs or min (MR, MT) increase in the transmission rate (or capacity) for the same bandwidth and with no additional power expenditure.

### c) *Diversity gain*

Multipath fading is a significant problem in communications. In a fading channel, signal experiences fade (i.e they fluctuate in their strength). When the signal power drops significantly, the channel is said to be in deep fade. This gives rise to high BER. The diversity is used to combat fading. This involves providing replicas of the transmitted signal over time, frequency, or space.

### d) *Array gain*

Array gain is the average increase in the SNR at the receiver that arises from the coherent combining effect of multiple antennas at the receiver or transmitter or both. Basically, multiple antenna systems require perfect channel knowledge either at the transmitter or receiver or both to achieve good array gain.

## III. LITERATURE REVIEW

Digital communication using MIMO is one of the most significant technical breakthroughs in wireless communications. A. I. Sulyman [8] describes the impact of antenna selection on the performance of multiple input-multiple output (MIMO) systems over nonlinear communication channels. The author has derived exact analytical expressions for evaluating the PWEF performance of space-time trellis codes over nonlinear MIMO channel, case of Rayleigh fading, when antenna selection is employed at the receiver side. Performance degradation due to nonlinearity in the channel reduces as less numbers of antennas are selected at the receiver, representing some savings in SNR penalty due to nonlinearity for the reduced complexity system.

C. Wang [9] explains the approach to exploit the capacity of MIMO systems is to employ spatial multiplexing where independent information streams are transmitted from the antennas. These information

streams are then separated at the receiver by means of appropriate signal processing techniques such as maximum likelihood (ML) which achieves optimal performance or linear receivers like Zero-Forcing (ZF) which provide sub-optimal performance but it also offers significant computational complexity reduction with tolerable performance degradation.

The comparison of MIMO with conventional Single-Input Single-Output (SISO) technology was discussed by S. G. Kim et. al [10]. MIMO can not only improve spectral efficiency, but also enhance link throughput or capacity. The authors presented a tight closed form BER approximation of MPSK for MIMO ZF receiver over continuous flat fading channels in the presence of practical channel estimation errors. The authors concluded from the numerical results that the BER performances depend not only on Doppler spread but also on the channel estimation error, and the difference between the number of transmit antennas and the number of receive antennas. The larger the difference is, the better performance is.

A simple two-branch transmit diversity scheme was presented by S. Alamouti [11]. The scheme uses two transmit antennas and one receive antenna. It provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit antenna, and two receive antennas. The scheme may easily be generalized to two transmit antennas and M receive antennas to provide a diversity order of 2M. The new scheme does not require any bandwidth expansion or any feedback from the receiver to the transmitter and its computation complexity is similar to MRRC.

A. Lozano et. al [12] explained the fades are very much localized in space and frequency: a change in the transmitter or receiver location (on the order of a carrier wavelength) or in the frequency (on the order of the inverse of the propagation delay spread) leads to a roughly independent realization of the fading process. Antenna diversity is a preferred weapon used by mobile wireless systems against the deleterious effect of fading. While narrowband channelization and non adaptive links were the norm, antenna diversity was highly effective. In modern systems, however, this is no longer the case. The prevalence of MIMO has opened the door for a much more effective use of antennas: spatial multiplexing. Indeed, the spatial degrees of freedom created by MIMO should be regarded as additional 'bandwidth' and, for the same reason that schemes based on time/frequency repetition waste bandwidth, rate-sacrificing transmit diversity techniques (e.g., OSTBC) waste 'bandwidth'.

An efficient implementation of space-time coding for the broadband wireless communications is presented by R. S. Blum et. al [13]. The authors predicted the improved performance and diversity gains of a space time (ST) coding system through a number of parameters including type of trellis codes and

channel fading. The results demonstrate the versatility of the developed simulator for predicting the performance of a ST coding system under different coding and channel conditions.

N. S. Kumar et. al [14], investigated about the three types of equalizer for MIMO wireless receivers. The authors made analysis by varying the receiver antenna keeping transmitter antenna constant for a particular type of equalizer based receiver at a particular  $E_b/N_0$  value using BPSK modulation method. The authors discussed about a fixed antenna MIMO antenna configuration and compare the performance with all the three types of equalizer based receiver namely ZF, ML, and MMSE. BER performance of ML Equalizer is superior than zero forcing Equalizer and Minimum Mean Square Equalizers. Based on the mathematical modeling and the simulation result it is inferred that the ML equalizer is the best of the three equalizers.

#### IV. MODULATION TECHNIQUE

Modulation and channel coding are fundamental components of a digital communication system. Modulation is the process of mapping the digital information to analog form so it can be transmitted over the channel. Consequently every digital communication system has a modulator that performs this task. Closely related to modulation is the inverse process, called demodulation, done by the receiver to recover the transmitted digital information. The design of optimal demodulators is called detection theory.

Phase-shift keying (M-PSK) for which the signal set is:

$$X_i(t) = \frac{\sqrt{2E_s}}{T_s} \cos\left(2\pi * f_{ct} + \frac{2(i-1)}{M}\right) \quad i = 1, 2, \dots, M \quad \& \quad 0 < t < T_s \quad 1.1$$

where  $E_s$  the signal energy per symbol  $T_s$  is the symbol duration and  $f_{ct}$  is the carrier frequency.

This phase of the carrier takes on one of the  $M$  possible values.

$$\theta = 2(i-1)\pi/M \quad i = 1, 2, \dots, M \quad 1.2$$

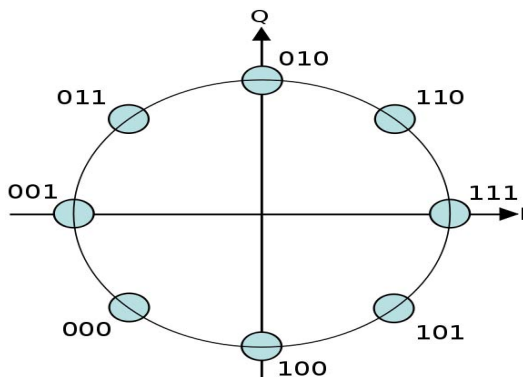


Figure 1.2 : Signal Space Diagram for 8-PSK

In M-ary PSK modulation, the amplitude of the transmitted signals was constrained to remain constant, thereby yielding a circular constellation as shown in Fig. 1.2.

#### V. MIMO SYSTEM MODEL

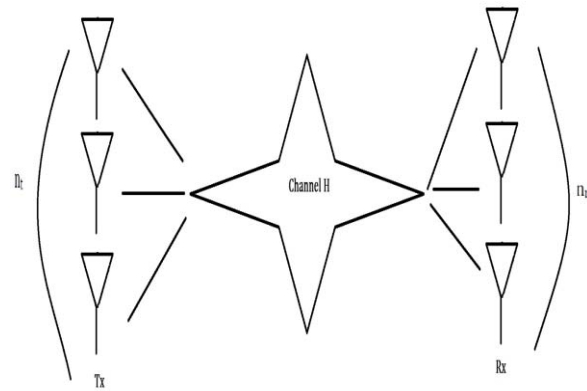


Figure 1.3 : The MIMO Channel

The MIMO channel is represented in Fig. 1.3 with an antenna array with  $n_t$  elements at the transmitter and an antenna array with  $n_r$  elements at the receiver is considered. The impulse response of the channel between the  $j^{\text{th}}$  transmitter element and the  $i^{\text{th}}$  receiver element is denoted as  $h_{ij}(\tau, t)$ . The MIMO channel can then be described by the  $n_r \times n_t$   $H(\tau, t)$  matrix:

$$H(\tau, t) = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \dots & h_{1,n_t}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \dots & h_{2,n_t}(\tau, t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_r,1}(\tau, t) & h_{n_r,2}(\tau, t) & \dots & h_{n_r,n_t}(\tau, t) \end{bmatrix} \quad 1.3$$

The matrix elements are complex numbers that correspond to the attenuation and phase shift that the wireless channel introduces to the signal reaching the receiver with delay  $\tau$ .

The input-output notation of the MIMO system can now be expressed by the equation:

$$y(t) = H(\tau, t) \otimes s(t) + u(t) \quad 1.4$$

where  $\otimes$  denotes convolution,  $s(t)$  is a  $n_t \times 1$  vector corresponding to the  $n_t$  transmitted signals,  $y(t)$  is a  $n_r \times 1$  vector corresponding to the  $n_r$  and  $u(t)$  is the additive white noise.

#### VI. ZERO FORCING EQUALIZER

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems, which inverts the frequency response of the channel. This equalizer was first proposed by Robert Lucky. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero Forcing corresponds to bringing down

the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise [5].

For a channel with frequency response  $F(f)$  the zero forcing equalizer  $C(f)$  is constructed such that  $C(f) = 1/F(f)$ . Thus the combination of channel and equalizer gives a flat frequency response and linear phase  $F(f)C(f) = 1$ . If the channel response for a particular channel is  $H(s)$  then the input signal is multiplied by the reciprocal of this. This is intended to remove the effect of channel from the received signal, in particular the Inter symbol Interference (ISI).

For simplicity let us consider a 2x2 MIMO channel, the channel is modeled as,

The received signal on the first receive antenna is,

$$y_1 = h_{1,1} x_1 + h_{1,2} x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad 1.5$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1} x_1 + h_{2,2} x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad 1.6$$

where

$y_1, y_2$  are the received symbol on the first and second antenna respectively,

$h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to the 1<sup>st</sup> receive antenna,

$h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to the 1<sup>st</sup> receive antenna,

$h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to the 2<sup>nd</sup> receive antenna,

$x_1, x_2$  are the transmitted symbols and  $n_1, n_2$  are the noise on 1<sup>st</sup> and 2<sup>nd</sup> receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad 1.7$$

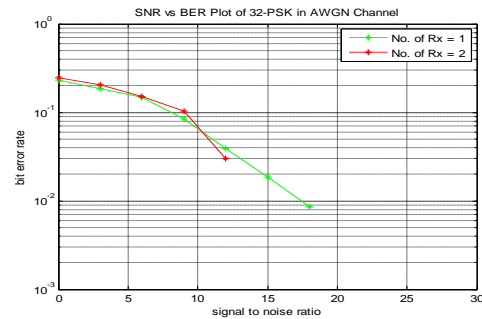
Therefore,

$$y = Hx + n \quad 1.8$$

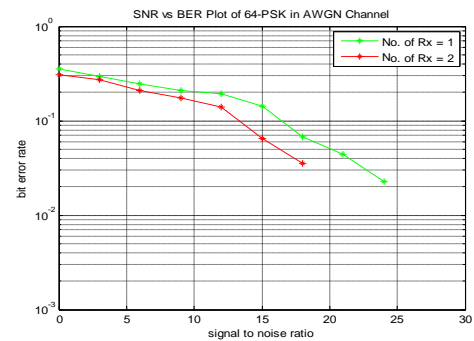
## VII. SIMULATED RESULTS

In this section, BER analysis of MIMO system using STBC code structure is done for M-PSK Modulation techniques over AWGN and Rayleigh fading channels. The BER analysis of MIMO system is done for M-PSK over AWGN and Rayleigh fading channels where M can be 32, 64, 128, 256, 512 and 1024. In this section, the BER performance of MIMO system is analyzed using M-PSK over both the fading channels.

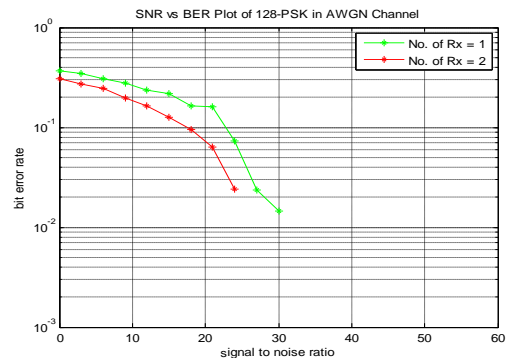
### a) M-PSK over PWGN channel



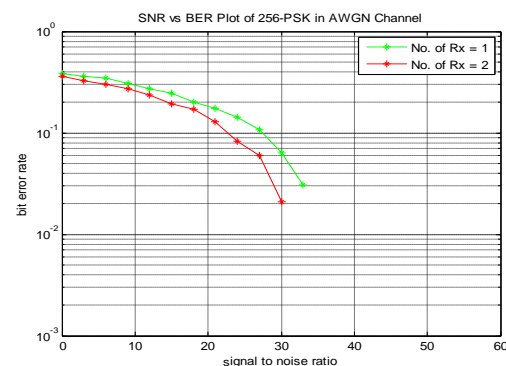
(a) 32-PSK



(b) 64-PSK

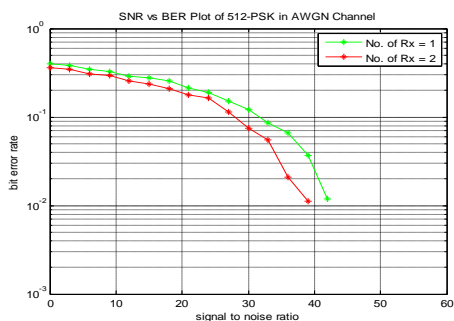


(c) 128-PSK

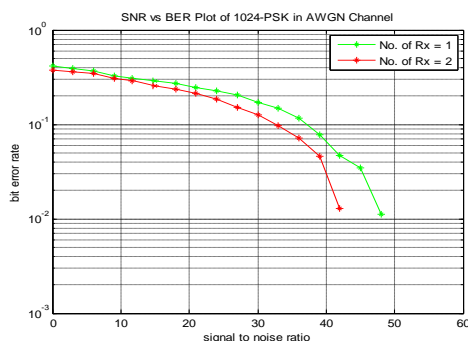


(d) 256-PSK





(e) 512-PSK

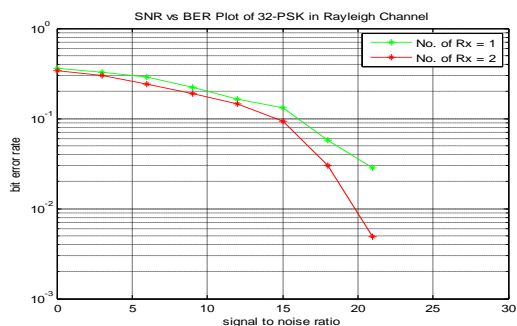


(f) 1024-PSK

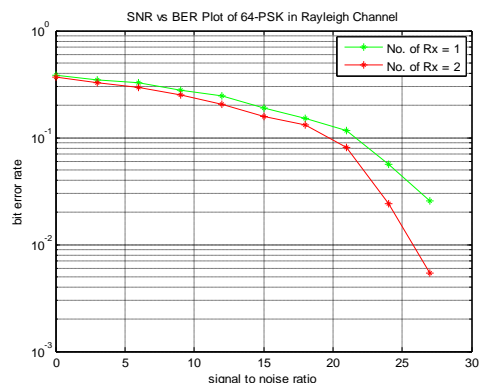
Figure 1.4 : SNR vs BER plots for M-PSK over AWGN channel

SNR vs BER plots for M-PSK over AWGN channel for MIMO system employing different antenna configurations have been presented in Fig. 1.4 (a) – (f). Here the graph depicts that in MIMO system as we goes on increasing the number of receiving antennas, the BER keeps on decreasing due to space diversity. This system provides better BER performance as compared to the other antenna configurations.

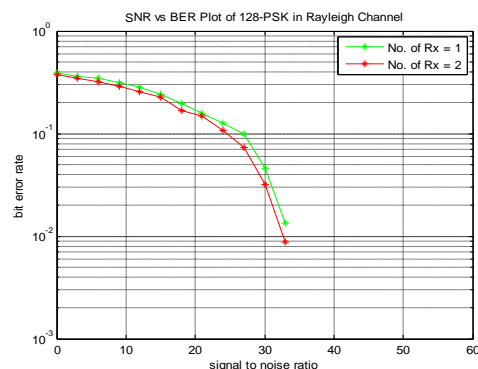
#### b) M-PSK over Rayleigh channel



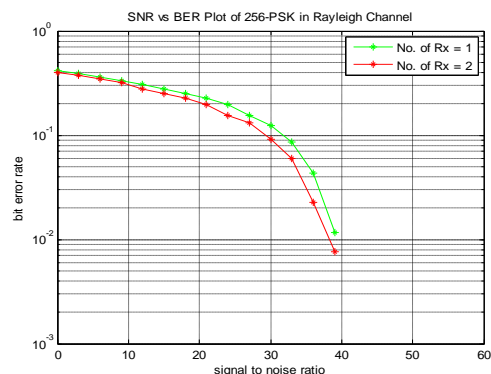
(a) 32-PSK



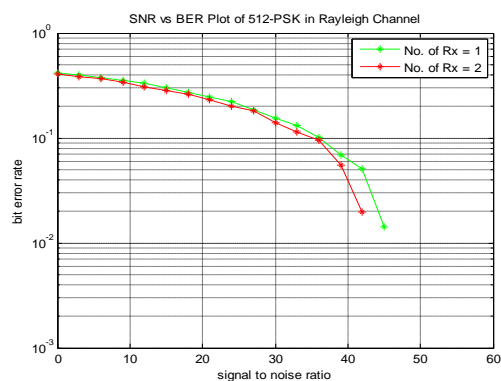
(b) 64-PSK



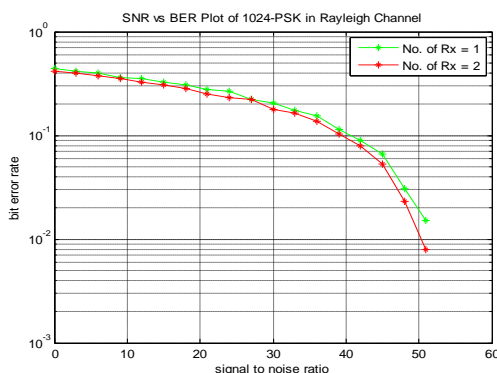
(c) 128-PSK



(d) 256-PSK



(e) 512-PSK



(f) 1024-PSK

Figure 1.5 : SNR vs BER plots for M-PSK over Rayleigh channel

In Fig. 1.5 (a) – (f) SNR vs BER plots for M-PSK over Rayleigh channel for MIMO system using different antenna configurations are presented. Here the graph depicts that in MIMO system as we goes on increasing the number of receiving antennas, the BER keeps on decreasing due to space diversity. But the BER is greater than the AWGN channel.

## VIII. CONCLUSION

In this paper, SNR vs. BER plots for M-PSK over AWGN and Rayleigh fading channels for MIMO system employing different antenna configurations are presented. It can be concluded that in MIMO system, the BER keeps on decreasing due to space diversity as we goes on increasing the number of receiving antennas and the proposed system provide better BER performance. But BER is greater in Rayleigh channel as compared to that of AWGN channel.

## REFERENCES RÉFÉRENCES REFERENCIAS

1. R. W. Heath, "Multimode antenna selection for spatial multiplexing systems with linear receivers", IEEE Transactions on Signal Processing, Vol. 53, Issue 8, pp. 3042-3056, 2005.
2. E. Casas & C. Leung, "Performance of OFDM/FM scheme for data transmission over fading mobile radio channels", 36<sup>th</sup> IEEE Vehicular Technology Conference, Vol. 36, Issue 5, pp 103-108, 1986, Dallas, Texas.
3. S. Alamouti, "A simple transmit diversity technique for wireless communications", IEEE Journal on Selected Areas of Communication, Vol. 16, Issue 8, pp. 1451–1458, 1998.
4. Tarokh, H. Jafarkhani & A. R. Calderbank, "Space-time block codes from orthogonal designs", IEEE Transactions on Information Theory, Vol. 45, Issue 5, pp. 1456–1467, 1999.
5. G. Ganesan & P. Stoica. 2001. "Spacetime block codes: a maximum SNR approach", IEEE Transactions on Information Theory, Vol. 47, Issue 4, pp. 1650–1656, 2001.
6. P. W. Wolniansky, G. J. Foschini, G. D. Golden & R. A. Valenzuela, "V-Blast: An architecture for realizing very high data rates over the rich-scattering channel", International Symposium on Signals, Systems and Electronics, pp. 295–300, 1998.
7. X. Zhang, Z. Lv & W. Wang, "Performance Analysis of Multiuser Diversity in MIMO Systems with Antenna Selection", IEEE Transactions on Wireless Communications, Vol. 7, Issue 1, pp. 15-21, 2008.
8. A. I. Sulyman, "Performance of MIMO Systems With Antenna Selection Over Nonlinear Fading Channels", IEEE Journal of Selected Topics in Signal Processing, Vol. 2, Issue 2, pp. 159-170, 2008.
9. A. Wang, "On the Performance of the MIMO Zero-Forcing Receiver in the Presence of Channel Estimation Error", IEEE Transactions on Wireless Communications, Vol. 6, Issue 3, pp. 805 – 810, 2007.
10. S. G. Kim, D. Yoon, Z. Xu & S. K. Park, "Performance Analysis of the MIMO Zero-Forcing Receiver over Continuous Flat Fading Channels", IEEE Journal of Selected Areas in Communications, Vol. 20, Issue 7, pp. 324 – 327, 2009.
11. S. Alamouti, "A simple transmit diversity technique for wireless communications", IEEE Journal on Selected Areas of Communication, Vol. 16, Issue 8, pp. 1451–1458, 1998.
12. A. Lozano & N. Jindal, "Transmit Diversity vs. Spatial Multiplexing in Modern MIMO Systems", IEEE Transactions on Wireless Communications, Vol. 9, Issue 1, pp. 186-197, 2010.
13. R. S. Blum, Y. Li, J. H. Winters & Q. Yan, "Improved Space-Time Coding for MIMO-OFDM Wireless Communications", IEEE Transaction on Communications, Vol. 49, Issue 11, pp. 1873-1878, 2001.
14. N. S. Kumar, G. J. Foschini, G. D. Golden & R. A. Valenzuela, "Bit Error Rate Performance Analysis of ZF, ML and MMSE Equalizers for MIMO Wireless Communication Receiver", European Journal of Scientific Research, Vol. 59, Issue 4, pp. 522–532, 2011.