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FINITEVOLUMESTUDYOFLAMINARBOUNDARYLAYERPROPERTIESFORFLOWOVERAFLATPLATEATZEROANGLEOFINCIDENCE

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Finite Volume Study of Laminar Boundary Layer Properties for Flow Over a Flat Plate at Zero Angle of Incidence

Toukir Islam [α], Md. Minal Nahin [σ] & Md. Abu Abrar [ρ]

Abstract - Boundary layer theory is considered to be the cornerstone of our knowledge about the fluid flow over a surface which not only describes some intriguing physical phenomena of fluid dynamics that were rather obscure before the year 1904 when Prandtl proposed the theory, but also pivotal in practical fields of engineering. The boundary layer which is known as the distance from the surface to a particular point perpendicular to the direction of flow where the flow velocity has retained 99% of the free stream velocity providing 'no-slip' condition at the surface i.e. zero velocity of flow at the surface; can be laminar or turbulent and there is a zone of 'transition' from laminar to turbulent depending on Reynolds number. In this paper the intriguing properties of laminar boundary layer such as development of velocity profile along the flow direction, boundary layer thickness, displacement thickness, momentum thickness, shape factor, wall shear stress, friction coefficient, drag coefficient etc. for flow over a smooth flat plate of 1 meter are studied by exact solution of Blasius's equationand 'Momentum Equation Method 'using Finite Volume solution of Navier-Stokes equations.

Keywords : laminar boundary layer, blasius's equation, momentum equation method, finite volume method, boundary layer thickness, displacement thickness, momentum thickness, shapes factor, shear stress.

I. Introduction

At the interface between a fluid and a surface in relative motion, a condition known as 'no slip' dictates an equivalence between fluid and surface velocities. Away from the surface, the fluid velocity rapidly increases; the zone in which this occurs is known as the boundary layer. The boundary layer is the thin region of flow adjacent to a surface, where the flow is retarded by the influence of friction between a solid surface and fluid. Although the boundary layer occupies geometrically only a small portion of flow field, its influence on different aerodynamic and heat transfer phenomena to the body is immense as Prandtl described it as 'marked results' (Anderson, 2010). Smooth thin flat plate has long been considered to be simplest form to describe boundary layer as there is no pressure gradient involved and it was probably the first example illustrating the application of Prandtl's boundary layer theory.

Shear stress acts as a pivotal parameter for the existence of boundary layer. The shear stress on the smooth surface is a direct function of the velocity gradient at the surface of the plate. This shear stress acting at the plate surface sets up a shear force which opposes the fluid motion and fluid close to the wall is decelerated. If the flow travels further along the surface, at zero pressure gradients, the shear force is effectively increased due to the increased plate surface wetted area. More and more of fluid retarded and the thickness of the fluid layer increases. Reynolds number (Re) can be considered as the measure-stick for behavior of the boundary layer. If the Re; calculated locally is low, the fluid flow close to the wall may be categorized as laminar. For smooth, polished plates the transition from laminar to turbulent may be delayed until Re 500000 i.e. below this Re the flow can be considered as laminar. However, for rough plates or for turbulent approach flows, transition may occur at much lower values.

There are number of intriguing properties of boundary layer which are decisive for analyzing different flow phenomena like drag or shear stress. These properties can be expressed through mathematical expressions which are direct function of local Re and distance of the point under consideration on the plate from the leading edge. Boundary layer thickness δ is the distance from the surface of the plate in perpendicular direction up to a point where the velocity of the flow is 99% of the free stream velocity. Displacement thickness δ^* can be considered as missing mass flow which is the difference between actual mass flow and hypothetical mass flow through the boundary layer if the boundary layers were not present. Another boundary layer property of importance is the momentum thickness θ, which is an index that is proportional to the decrement in momentum flow due to the presence of the boundary layer. It is the height of a hypothetical streamtube which is carrying the missing momentum flow at free stream conditions. Shape factor H of velocity profile is the ratio of the displacement thickness to the momentum thickness which increases in an adverse pressure gradient. For laminar flow with zero pressure gradient (such as a flat plate), it is 2.59 and it reaches to 3.5 at separation. Local friction coefficient $C_{f,x}$ is the

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dimensionless number defined as the ratio of wall shear stress to dynamic pressure.

Blasius (1908) was the first one to illustrate Prandtl's boundary layer theory through the application of flow over a flat plate. He provided the legendary equation known as 'Blasius's equation'. Bairstow (1925), Goldstein (1930) solved it through analytical procedure while Töpfer (1912) solved it using Runge-Kutta numerical method. Howarth (1938) solved the equation with greater accuracy using numerical procedure. Steinheuer (1968) published a systematic review of the solutions to Blasius's equation. Filobello-Nino et. al. (2012) provided with an approximate solution of Blasius's equation by using HPM (Homotopy Perturbation Method) and described the behavior of a two-dimensional viscous laminar flow over flat plate. Aminikhah (2012) persuaded analytical approximation to the solution of non-linear Blasius's viscous flow equation by LTNHPM (Laplace Transform and New Homotopy Perturbation Method). The exact solutions of boundary-layer equation have some mathematical difficulties associated with it. Thus exact solutions can be replaced by some sophisticated approximate methods like 'Momentum Equation Method'. Kármán (1921) and Pohlhausen (1921) linked shear stress with momentum thickness which provides alternative way of finding the wall shear stress rather than depending on velocity gradient at wall. In this paper the laminar boundary layer properties are illustrated using exact solution of Blasius's equation and 'Momentum Equation Method' and these properties are analyzed using flow over one side of a smooth flat plate with no pressure gradient by solving the Navier-Stokes equation set using the Finite Volume Method.

II. Mathematical Model

a) *Blasius Equation and Exact Solution*

Incompressible, two dimensional flows over a thin flat plate at 0° angle of incidence is simplest example used in the first place to describe Prandtl's boundary layer theory. For such flow the density and viscosity are constant and the pressure gradient is zero as inviscid flow over the smooth flat plate at 0° angle of attack yields constant pressure over the surface. Thus the Navier-Stokes equations reduce to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial p}{\partial y}=0 \quad (3)$$

Here ν is the kinematic viscosity defined as $\nu = \mu/\rho$. The exact solution is described by Blasius (1908); a student of Prandtl in his doctor's thesis at Goettingon. The independent variable (x, y) are then transformed into (ξ, η) as $\xi = x$ and $\eta = y\sqrt{\frac{V_\infty}{x\,\nu}}$ and the stream function is considered to be $\psi = f(\eta) \times \sqrt{x\,\nu V_\infty}$ (Huges and Brigton, 1967), (Resnick and Halliday, 1977), (Landau and Lifshitz, 1987) where f is strictly a function of η. Blasius concluded with a legendary equation known as Blasius's Equation' as form of

$$2f''' + ff'' = 0 \quad (4)$$

Where the function $f(\eta)$ has the property that is f' is described as $\frac{u}{U_\infty}$ where u is velocity at any point normal to the plate and U_∞ is the free stream velocity. This is a third order non-linear differential equation which requires three boundary conditions to solve which are: at $\eta = 0 :\ f = 0, f' = 0$ and $\eta = \infty : f' = 1$. The equation was solved by Blasius using a series of approach. The properties of boundary layer are defined as in Table.

Table 1 : Properties of laminar boundary layer over flat plate

Boundary layer property	Mathematical expression
Boundary layer thickness (δ)	$5x/\sqrt{Re_x}$
Displacement thickness (δ*)	$1.72x/\sqrt{Re_x}$
Momentum thickness (θ)	$0.664x/\sqrt{Re_x}$
Shape factor (H)	δ^*/θ
Friction coefficient ($C_{f,x}$)	$0.664/\sqrt{Re_x}$
Drag coefficient (C_D)	$1.328/\sqrt{Re_L}$

Here Re_x refers to be local Reynolds number and Re_L is the overall Reynolds number. To calculate the Re_x, the distance is measured from the leading edge of the flat plate. In case of Re_L, the distance is the total length of the plate.

b) *Momentum Equation Method*

Von Kármán first applied the momentum equation to a general section of a boundary layer. Regardless of the position of the section in either the laminar or turbulent boundary layer regions, it is possible to equate the skin friction drag force as a rate of change of mass and momentum of the fluid affected by the boundary layer. Consider a rectangle ABCD where the boundary AB parallel to the plate is placed at such a distance from the body that it lies in undisturbed region of velocity U_∞.There is no pressure gradient to affect the momentum. When we calculate the momentum flux across the control surface it should be considered that, owing to continuity, fluid must leave

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through AB is equal to the difference of fluid entering through AD to fluid leaving through BC.

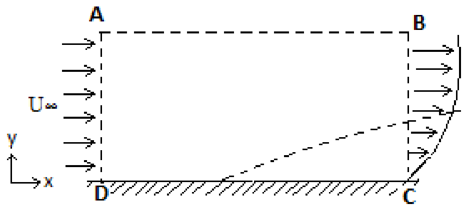

Figure 1 : Momentum equation method

The DC boundary does not contribute to the momentum in x direction as no-slip condition is considered. The mass entering the section is taken as positive and mass leaving the section is taken as negative. Now the net momentum flux is equal to the drag on a flat plate wetted on one side.

Table 2 : Momentum flow at different boundary of Fig. 1

Boundary	Rate of flow	Momentum in direction x
DC	0	0
AD	$b\int_0^y U_\infty\, dy$	$b\rho\int_0^y U_\infty{}^2\, dy$
BC	$-b\int_0^y u\, dy$	$-b\rho\int_0^y u^2\, dy$
AB	$-b\int_0^y (U_\infty - u)\, dy$	$-b\rho\int_0^y u(U_\infty - u)\, dy$
Total	0	$b\rho \times \int_0^y u(U_\infty - u)\, dy$

Thus we have: drag, $D = b\rho \times \int_{y=0}^{\infty} u(U_\infty - u)\, dy$.Now, from the definition of wall shear stress at the wallτ_0, where b is the breadth of the plate,

$$D = b \times \int_0^x \tau_0\, dx \qquad (5)$$

Comparing two equations we find the expression ofτ_0. Other parameters like boundary layer thickness, displacement thickness, momentum thickness, shape factors etc. can be found by following the procedure described by Schlichting (1979).Now different approximation of f(η)allows us to evaluate the coefficients which are different from exact solution. Point to be noted here that α_1, $\alpha_2$and β are defined as $\alpha_1 = \int_0^1 f(1-f)d\eta$, $\alpha_2 = \int_0^1 (1-f)d\eta$and $\beta = f'(0)$.Now several approximations are made for f(η) and depending on them α_1, $\alpha_2$and β are calculated. These values of α_1, $\alpha_2$and β are used to evaluate the coefficients A to G.

Table 3 : Properties of laminar boundary layer from momentum equation method (Schlichting, 1979)

Boundary layer property	Mathematical expression	Coefficients
Shear stress (τ)	$\mu\, U_\infty \times A \times \sqrt{U_\infty/\nu x}$	$A=\sqrt{\alpha_1\beta/2}$
Friction coefficient ($C_{f,x}$)	$B/\sqrt{Re_x}$	$B=\sqrt{2\alpha_1\beta}$
Boundary layer thickness (δ)	$(x \times C)/\sqrt{Re_x}$	$C = \sqrt{2\beta/\alpha_1}$
Momentum thickness (θ)	$(x \times D)/\sqrt{Re_x}$	$D=\sqrt{2\alpha_1\beta}$
Displacement thickness (δ*)	$(x \times E)/\sqrt{Re_x}$	$E = \alpha_2\sqrt{2\beta/\alpha_1}$
Shape factor (H)	δ^*/θ or F	$F = \alpha_2/\alpha_1$
Drag coefficient (C_D)	$G/\sqrt{Re_L}$	$G = 2 \times \sqrt{2\alpha_1\beta}$

Table 4 : Different approximations of f(η) and coefficients of boundary layer properties

Items	Approximation				
	Exact	1	2	3	4
$f(\eta)$	-	η	$1.5\eta - 0.5\eta^3$	$2\eta - 2\eta^3 + \eta^4$	$\text{Sin}(\frac{\pi}{2}\eta)$
α_1	-	$1/6$	$39/280$	$37/315$	$\frac{4-\pi}{2\pi}$
α_2	-	$1/2$	$3/8$	$3/10$	$\frac{\pi-2}{\pi}$
β	-	1	$3/2$	2	$\frac{\pi}{2}$
A		0.2886	0.3232	0.3427	0.328
B	0.664	0.5774	0.6464	0.6854	0.655
C	5.0	3.464	4.641	5.8355	4.795
D	0.664	0.5774	0.6464	0.6854	0.655
E	1.72	1.732	1.74	1.75	1.742
F	2.59	3.0	2.7	2.55	2.66
G	1.328	1.1548	1.2928	1.3708	1.310

III. Numerical Procedure

a) *Computational Design*

The computational design is comprised of a frame of 1m×0.1m×0.03m whose base is used as the smooth flat surface under consideration. Now the boundary conditions are assigned as; 'Surface A' is the 'Velocity Inlet' of 5 m/s uniform velocity, 'Surface B' is the 'Pressure Opening' at 101325 Pa. 'Surface C' above is considered as the 'Ideal Wall' while the 'Surface D' is the 'Real Wall' with no-slip condition which resembles the smooth flat plate with zero angle of incidence.

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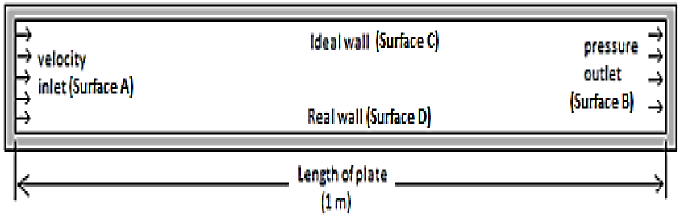

Figure 2 : Computational Design

b) Computational Meshing

The rectangular computational domain is constructed, so it encloses the solid body and has the boundary planes orthogonal to the specified axes of the Cartesian coordinate system. Then, the computational mesh is constructed in the following several stages.

First of all, a basic mesh is constructed. For that, the computational domain is divided into slices by the basic mesh planes, which are evidently orthogonal to the axes of the Cartesian coordinate system. The basic mesh is determined solely by the computational domain and does not depend on the solid/fluid interfaces.

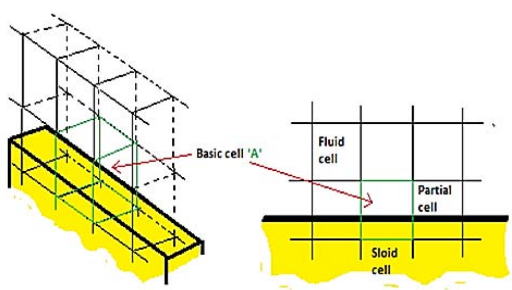

Figure 3 : Basic cell construction

Then, the basic mesh cells intersecting with the solid/fluid interface (like cell 'A' in Fig. 3) in are split uniformly into smaller cells in order to capture the solid/fluid interface with mesh cells of the specified size i.e. with respect to the basic mesh cells. The following procedure is employed: each of the basic mesh cells intersecting with the solid/fluid interface is split uniformly into 8 child cells (Child cell 'B') shown in Fig. 4.

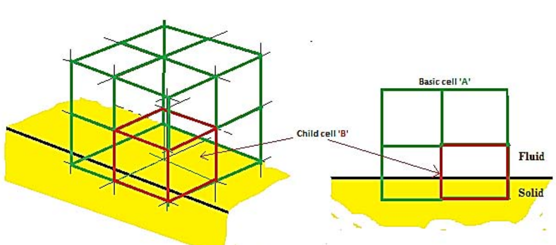

Figure 4 : Child cell 'B' formation

Each of the child cells (like Child cell 'B') intersecting with the interface is in turn split into 8 cells of next level, and so on, until the specified cell size (Child cell 'C') is attained shown in Fig. 5.

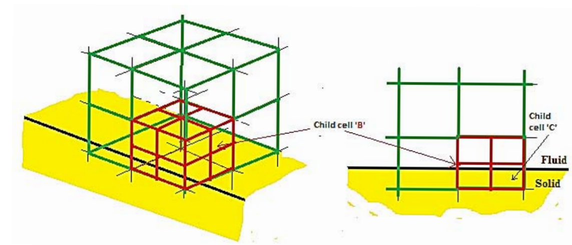

Figure 5 : Child cell 'C' formation

At the next stage of meshing, the mesh obtained at the solid/fluid interface with the previous procedure is refined (i.e. the cells are split further or probably merged) in accordance with the solid/fluid interface curvature. The criterion to be satisfied is established as follows: the maximum angle between the normals to the surface inside one cell should not exceed certain threshold; otherwise the cell is split into 8 cells. As a result of all these meshing procedures, a locally refined rectangular computational mesh is obtained and used then for solving the governing equations on it. As we are using a flat plate with no curvature or surface roughness, this step can be omitted.

c) Governing Equations and Finite Volume Scheme

Equation set consisting equation no (1), (2) and (3) are solved using 'Finite Volume' method. The cell-centered finite volume (FV) method is used to obtain conservative approximations of the governing equations on the locally refined rectangular mesh. The governing equations are integrated over a control volume which is a grid cell, and then approximated with the cell-centered values of the basic variables. The integral conservation laws may be represented in the form of the cell volume and surface integral equation:

$$\frac{\partial}{\partial t}\int \mathbf{U}\,\mathrm{dv} + \oint F.ds = \int Qdv \qquad (6)$$

Which is replaced by : $\frac{\partial}{\partial t}(Uv) + \sum_{cell\ faces} F.s = Qv$

The second-order upwind approximations of fluxes Fare based on the implicitly treated modified Leonard's QUICK approximations (Roache, 1998) and the Total Variation Diminishing (TVD) method (Hirsch, 1988).

IV. Results and Discussion

For air flow over a smooth flat plate of 1 meter length without any pressure gradient and heat transfer, the local Re never crossed the critical Re (500000) that could cause transition of laminar flow to turbulent flow. So the boundary layer generated at the vicinity of the lower wall of the computational design can be considered as laminar boundary layer. The difference between Reobtained from finite volume solution of Navier-Stokes equations and theoretical Reynolds number is considerably low which indicates the acceptability of the computational process used for this particular analysis.

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Table 5 : Re at different distance from leading edge

Distance from Leading edge, x (m)	Re (theoretical) $= \frac{\rho v d}{\mu}$	Re (numerical)	Deviation (%)
0.1	33425.41	33328.98	0.28
0.2	66850.82	66871.84	0.03
0.3	100276.24	100593.78	0.31
0.4	133701.65	134449.18	0.56
0.5	167127.07	168409.15	0.77
0.6	200552.48	202480.73	0.96
0.7	233977.90	236622.62	1.13
0.8	267403.31	270818.32	1.28
0.9	300828.73	305028.96	1.40

a) Boundary layer Thickness

At the area closer to the leading edge of the plate, the boundary layer thickens rapidly (Fig. 6 (a), (b), (c)). As the flow travels further downstream, the rate of thickening of the boundary layer decreases and at some points around 70-90% of the plate length, the thickening effect becomes more obscure (Fig. 6(e)). With increasing distance from leading edge, the point at which the local velocity of the flow becomes almost equal to the free stream velocity; travels more to the perpendicular direction of the plate i.e. y direction. Both from Fig. 7 and Fig. 8 it is evident that the thickness of the boundary layer δ increases as the flow travels more downstream because more and more fluid particles pile up due to increase of wall shear stress at that direction.

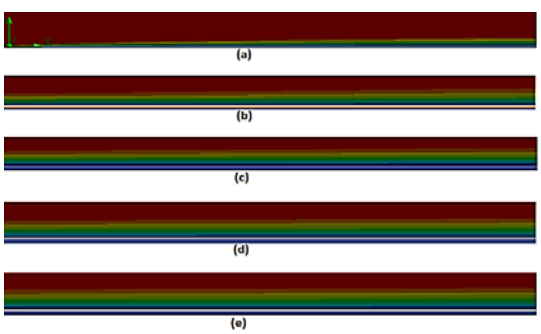

Figure 6 : Development of boundary layer at (a) x=0.0 to 0.1, (b) x=0.1 to 0.2, (c) x=0.2 to 0.3, (d) x=0.4 to 0.5 and (e) x= 0.7 to 0.8

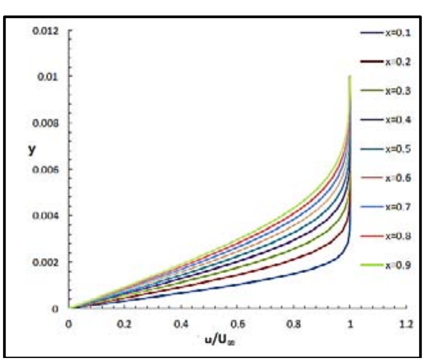

Figure 7 : Velocity profile at different position of the plate

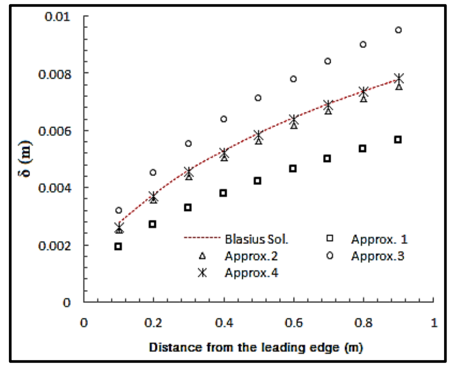

Figure 8 : Boundary layer thickness (δ) at different position of the plate

In case of momentum equation method, the approximation no. 2 and 4 provides nearly similar thickness of boundary layer at any position of the plate which are close to the exact solution while approximation no. 1 is proved to be under-estimated and 3 is over-estimated approximation.

b) Displacement and Momentum Thickness

Boundary layer thickness is so far referred to only in physical terms. It is however, possible to define boundary layer thickness in terms of the effect on the flow. Displacement thickness is defined as the 'distance' the surface would have to move in the y direction to reduce the flow passing by a volume equivalent to the real effect of the boundary layer. Displacement thickness δ* for the boundary layer increases with increasing distance from the leading edge of the plate (Fig.9). With increasing distance from the leading edge of the plate, δ* increases due to the same reason as δ increases. That means the plate would have to move further in y direction in case there is no boundary layer to compensate the flow reduction due to boundary layer. Similar outcome is found for momentum thickness, θ as decrement in momentum flow due to the boundary layer increases as the flow travels further downstream (Fig.10).

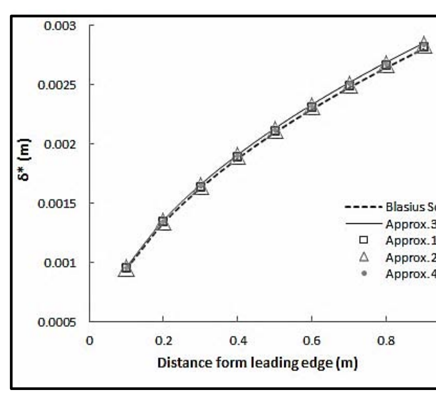

Figure 9 : Displacement thickness (δ*) at different position of the plate

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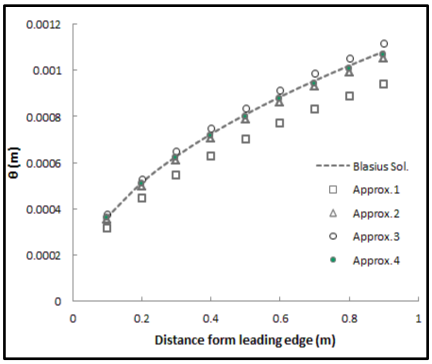

Figure 10 : Momentum thickness (θ) at different position of the plate

In case of displacement thickness, all four approximation seem cogent while for momentum thickness, approximation no. 4 seems convenient than approximation no. 2 and 3. Approximation no. 1 deviates by huge percentage from the exact solution.

c) Shape Factor

The shape factor from Blasius's calculation (Blasius, 1908) is 2.59 for the flat plate while from present calculation, it is the approximation 3 and 4 for momentum equation method that are closer to the exact solution and as this value would be around 3.5 at separation (Fox, McDonald and Pritchard, 2009), it can be concluded that separation of flow from the plate surface did not occur.

d) Shear Stress, Local Friction Coefficient and Skin Friction Drag Coefficient

The shear stress on a smooth plate is a direct function of the velocity gradient at the surface of the plate and this velocity gradient exists in a direction perpendicular to the surface. In immediate neighborhood of the body in which the velocity gradient normal to the wall is very large and the very small viscosity of the fluid exerts an essential influence that results in larger shear stress (Fig. 11). As we travel further upward from the plate, the influence of viscosity becomes trivial and flow at this region can be considered frictionless. As flow travels further downstream from the leading edge, Re increases and the velocity gradient decreases (in laminar boundary layer region) and thus the shear stress decreases. As the friction coefficient is directly proportional to the shear stress, it also decreases as the flow travels towards the downstream (Fig. 12).

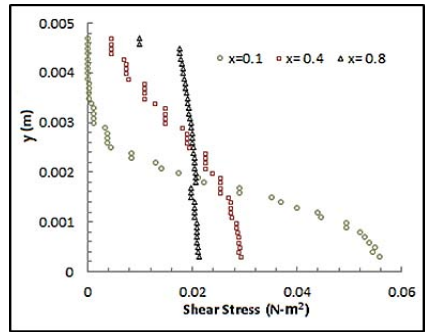

Figure 11 : Shear stress distribution in perpendicular direction of the plate

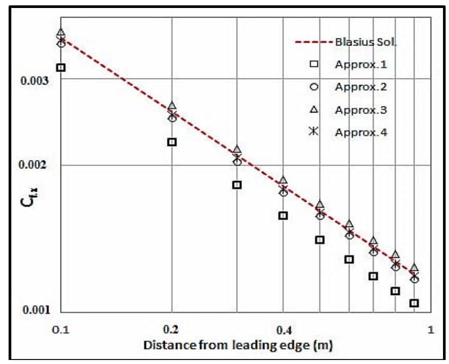

Figure 12 : Friction coefficient ($C_{f,x}$) at different Re_x

In calculating the skin friction drag, the approximation no.4 for the momentum equation method converged very closely to the exact solution of Blasius equation. Similar case happens when we calculate the drag coefficient for flow over the flat plate at zero angle of attack. Both approximation no. 2 and 4 deviate from exact solution by small fraction while the approximation no. 1 strays well way from the exact drag coefficient.

Table 6 : Coefficient of drag for laminar flow over flat plate at zero angle of incidence

Process	C_D	Deviation from exact solution (%)
Exact Sol.	0.002296	-
Approx.1	0.001997415	13.0
Approx.2	0.0022361	2.61
Approx.3	0.00237102	2.6
Approx.4	0.002266205	1.3

V. Conclusion

Properties of laminar boundary layer are analyzed numerically using Finite Volume Method solution of the Navier-Stokes equations and these intriguing and rather decisive properties of boundary layer are evaluated through exact solution of Blasius equation and different approximation of Momentum Equation Method. Authors have reached to several concluding remarks through this study:

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- As the theoretical Reynolds number and the Reynolds number calculated numerically differ by very small percentages at different position of the plate, it can be concluded that Finite Volume Method of solving Navier-Stokes equations serves well the purpose of analyzing the laminar boundary layer.
- Different boundary layer thickness increase as the flow travels further downstream from the leading edge of the flat plate.
- Shear stress and the local friction coefficient decreases as the flow travels downstream from the leading edge.
- Among the four approximations used in Momentum Equation Method, the fourth approximation conversed convincingly towards the exact solution.

Future research works could be conducted by applying different other approximations to the momentum equation method for laminar flow over a flat plate at different free stream velocities and compare the relative outcomes. Mesh shapes other than rectangular mesh at the fluid solid interface can be implied to find out whether the characteristics of boundary layer are responsive to mesh size and shape or not.

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