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Optimal Pricing, Shipments and Ordering Policies for Single-Supplier Single-Buyer Inventory System with Price Sensitive Stock-Dependent Demand and Order-Linked Trade Credit

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I. Introduction

The conventional economic order quantity (EOQ) model is formulated under the assumption that the demand of the of the commodity is deterministic and constant; and the buyer must follow cash-on-delivery as a mode of payment. Both these assumptions are unrealistic in business world. The supplier aims at lowering investment tie-up for warehousing inventory, to estimate demand by offering sales promotional incentives and attracting more buyer. Goyal (1985) modeled the concept of permissible delay period to compute economic order quantity. He assumed demand to be constant and calculated interest earned by the buyer on the purchase cost of the unit. Thereafter, several researchers studied different variants of inventory models with allowable trade credits to discuss advantages of this promotional tool. One can refer review article on trade credit by shah *et al*. (2010).

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In the cited articles, it is observed that offer of credit from the supplier to the buyer is independent of the size of the order. Once again this is also somewhat impractical.

Khouja and Mehraj (1996) analyzed the vendor's credit polices to determine the optimal order quantity when credit terms are linked to order quantity. Shinn and Hwang (2003), Chang (2004), Chang *et al.* (2005) Chang and Liao (2004, 2006), Chang *et al.* (2009), Shah and Shukla (2010, 2011), Shah (2010), Shah *et al.*(2012) considered order-dependent trade credit and quadratic demand structure in their analysis. These models were derived either from vendor's or from buyer's point of view. Here, only one player takes decision and that is to be followed by the other willingly or un-willingly.

Globalization of the business forced the business players to device win-win strategy to survive in this competitive world. Goyal (1976) formulated a single-vendor single-buyer integrated inventory model. Banerjee (1986) extended above model when vendor follows lot-for-lot production policy. Goyal (1988) relaxed the assumption of lot-for-lot production policy and proved that the inventory cost lowers significantly if vendor's economic production quantity is an integral multiple of buyer's order quantity. Lu (1995), Goyal (1995), Vishwanathan (1998), Hill (1997,1999), Kelle *et al.* (2003), Yang and Wee (2003) advocated that more batching and frequent transfer polices are beneficial for the players of the supply chain.

Levin *et al.* (1992) quoted that "large piles of goods attract more customers". This demand is treated as stock-dependent demand. Urban (2005) analyzed effect of displayed goods on order quantity. Roy and Chaudhari (2006) assumed finite planning horizon by allowing shortages to study the effect of stock-dependent parameter on decision variable and objective function.

In this study, the goal is to analyze an integrated inventory system comprising of single-vendor single-buyer when demand rate is price sensitive stock-dependent. The production rate is proportion to demand rate. The buyer qualifies for delay payments only if the order is greater than that of pre-specified as announced by the vendor. The joint total profit per unit time of the supply chain is maximized with respect to retail price of the unit, order quantity for the buyer and number of shipments from the vendor to the buyer. An algorithm is outlined to decide the best optimal solution. The numerical example is given to elaborate the mathematical development of the proposed problem. Sensitivity analysis is carried out to discuss managerial insights.

II. Notation and Assumptions

a) Notation

$R(I(t),s)$ Price-sensitive Stock-dependent demand rate; $(\alpha+\beta I(t))s^{-\eta}$ where $\alpha>0$ denotes constant scale demand, $0<\beta<1$ denotes stock-dependent parameter, $\eta>1$ denotes price-elasticity and s denotes retail price of the product per unit by the buyer. (s is a decision variable)

A_v Vendor's set-up cost per set-up

A_b Buyer's ordering cost per order

C_p Production cost per unit

C_b Buyer's purchase cost per unit

Note: $s>C_b>C_p$

I_v Vendor's inventory holding cost rate per unit per year, excluding interest charges

I_b Buyer's inventory holding cost rate per unit per year, excluding interest charges

I_{vp} Vendor's opportunity cost /\$/ unit time

I_{bp} Buyer's opportunity cost /\$/ unit time

I_{be} Buyer's interest earned /\$/ unit time

ρ Capacity utilization which is ratio of demand rate to the production rate; $\rho<1$ is known constant

M Permissible credit period for the buyer given by the vendor

Q Buyer's order quantity per order (a decision variable)

Q_d Pre-specified order quantity to qualify for delayed payment

T Cycle time (a decision variable)

T_d The duration of time when Q_d - units are sold off

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n Number of transfers from the vendor to the buyer (a decision variable)

TVP Vendor's total profit per unit time

TBP Buyer's total profit per unit time

π $(=TBP+TVP)$ Joint total profit per unit time

b) Assumptions

The following assumptions are made in deriving the proposed model.

1. Single - vendor and single - buyer supply chain is studied for a single-item.
2. Lead - time is zero. Shortages are not allowed.
3. The demand rate is price - sensitive stock-dependent.
4. The buyer qualifies for settlement of account at a later date if the order is equal or larger than the pre-specified quantity Q_d by the vendor. Otherwise, the buyer must pay for the purchases immediately.
5. During the credit period, the buyer earns interest at the rate I_{be} per unit on the generated revenue. At the end of the credit period, the buyer pays incurs an opportunity cost at a rate of I_{bp} on the unsold items in the warehouse.

III. Mathematical Model

In this section, we develop an integrated inventory model when demand is price-sensitive stock-dependent demand and trade credit is offered only if buyer's order quantity is equal or greater than a pre-specified quantity.

a) Vendor's total profit per unit time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:

(1) Sales revenue: The total sales revenue per unit time is $(C_b-C_p)\frac{Q}{T}$. (See Appendix A for computation of Q)

(2) Set-up cost: nQ-units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is $\frac{A_v}{nT}$.

(3) Holding cost: Using Joglekar (1988), the vendor's average inventory per unit time is

$$\frac{C_p(I_v+I_{vp})\left[(n-1)(1-\rho)+\rho\right]}{T\beta^2}\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right).$$

(4) Opportunity cost: If Q_d or more units are ordered by the buyer, the credit period of $M-$ units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when $T \geq T_d$, the delay in payments is permissible and corresponding opportunity cost per unit time is $\frac{C_b I_{vp} QM}{T}$. On the other hand, when $T < T_d$ the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

$$TVP(n) = \begin{cases} TVP_1(n), T < T_d \\ TVP_2(n), T \geq T_d \end{cases} \tag{1}$$

Where

$$TVP_1(n) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp})\left[(n-1)(1-\rho)+\rho\right]}{T\beta^2}\alpha\left(e^{\beta s^{-\eta}T}s^{\eta} - s^{\eta} - \beta T\right) \tag{2}$$

$$TVP_2(n) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp})\left[(n-1)(1-\rho)+\rho\right]}{T\beta^2}\alpha\left(e^{\beta s^{-\eta}T}s^{\eta} - s^{\eta} - \beta T\right) - \frac{C_b I_{vp} QM}{T} \tag{3}$$

b) *Buyer's total profit per unit time*

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

(1) Sales revenue: The total sales revenue per unit time is $(s - C_b)\frac{Q}{T}$. (See Appendix A for computation of Q)

(2) Ordering cost: The ordering cost per unit time is $\frac{A_b}{T}$.

(3) Holding cost: The buyer's holding cost (excluding interest charges) per unit time is

$$\frac{C_b I_b \alpha\left(e^{\beta s^{-\eta}T}s^{\eta} - s^{\eta} - \beta T\right)}{T\beta^2}$$

(4) Opportunity cost: Based on the lengths of T, M and T_d, the following four cases arises (i) $0 < T < T_d$ (ii) $T_d \leq T \leq M$ (iii) $T_d \leq M \leq T$ (Fig. 1) and (iv) $M \leq T_d \leq T$

The cases (iii) and (iv) are similar. We discuss opportunity cost and interest earned in each case.

Opportunity cost per unit time

$$= \begin{cases} \frac{C_b I_{bp} Q}{T} & , 0 < T < T_d \\ 0 & , T_d \leq T \leq M \\ \frac{C_b I_{bp}}{\ } \alpha\left(s^{\eta}e^{-M\beta s^{-\eta}+\beta s^{-\eta}T} + \beta M - s^{\eta} - \beta T\right) & , T_d \leq M \leq T \text{ o } M \leq T_d \leq T. \end{cases}$$

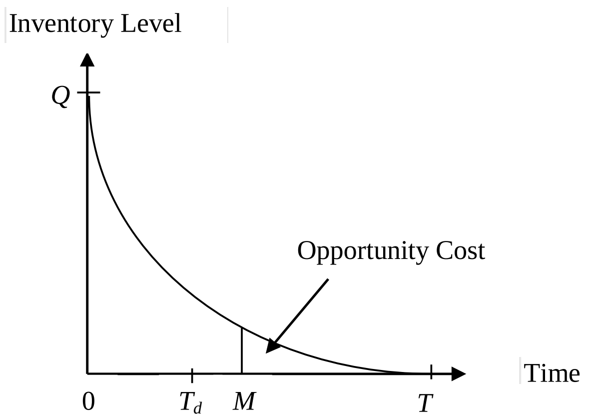

Figure 1 : Opportunity cost for $T_d \le M \le T$ or $M \le T_d \le T$

Interest earned: As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time

$$
=\begin{cases} 0 & ,\ 0 < T < T_d \ (\text{because payment is to be made on delivery}) \\ \dfrac{sI_{be}}{T}\left(\int_0^T R(I(t),s)\,t\,dt + Q(M-T)\right) & ,\ T_d \le T \le M \ (\text{figure } 2) \\ \dfrac{sI_{be}}{T}\left(\int_0^M R(I(t),s)\,t\,dt\right) & ,\ T_d \le M \le T \ or\ M \le T_d \le T \ (\text{figure } 3) \end{cases}
$$

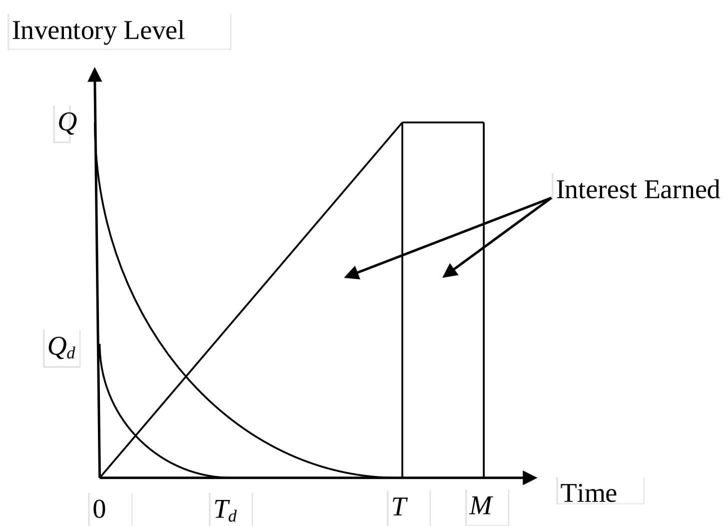

Figure 2 : Interest earned by buyer when $T_d \le T \le M$

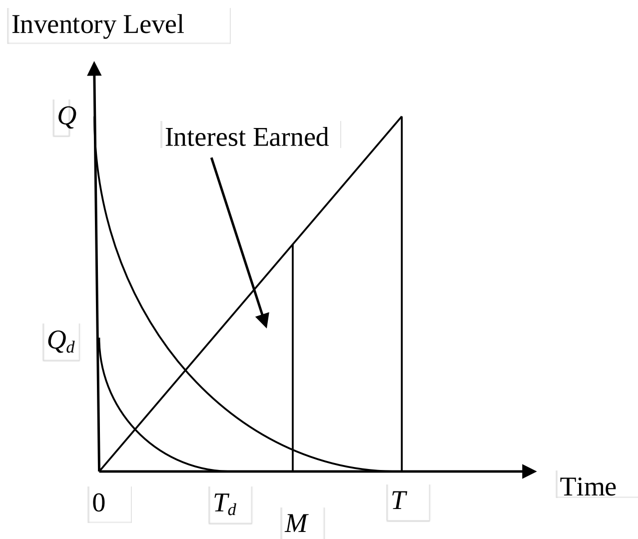

Figure 3 : Interest earned by buyer when $T_d \le M \le T$

Hence, the buyer's total profit per unit time is

$$TBP(T)=\begin{cases} TBP_1(T), \; 0<T<T_d \\ TBP_2(T), T_d \le T \le M \\ TBP_3(T), T_d \le M \le T \\ TBP_4(T), M \le T_d \le T \end{cases} \quad (4)$$

Where

$$TBP_1(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_b I_b \alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right)}{T\beta^2}-\frac{C_b I_{bp} Q}{T} \quad (5)$$

$$TBP_2(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_p I_b}{T\beta^2}\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right)$$
$$+\frac{sI_{be}}{T}\left(\int_0^T R\left(I(t),s\right)tdt+Q(M-T)\right) \quad (6)$$

$$TBP_3(T)=TBP_4(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_b I_b}{T\beta^2}\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right)$$
$$-\frac{C_b I_{bp}}{T\beta^2}\alpha\left(s^{\eta}e^{-M\beta s^{-\eta}+\beta s^{-\eta}T}+\beta M-s^{\eta}-\beta T\right)$$
$$-\frac{sI_{be}\alpha}{T\beta^2}\left(-e^{\beta s^{-\eta}T}s^{\eta}+s^{\eta}e^{-M\beta s^{-\eta}+\beta s^{-\eta}T}+e^{-M\beta s^{-\eta}+\beta s^{-\eta}T}M\beta\right) \quad (7)$$

c) Joint total profit per unit time

In integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain. The joint total profit per unit time for the integrated system is

$$\pi(n,T)=\begin{cases}\pi_1(n,T)=TVP_1(n)+TBP_1(T), & 0<T<T_d\\ \pi_2(n,T)=TVP_2(n)+TBP_2(T), & T_d\le T\le M\\ \pi_3(n,T)=TVP_2(n)+TBP_3(T), & T_d\le M\le T\\ \pi_4(n,T)=TVP_2(n)+TBP_3(T), & M\le T_d\le T.\end{cases} \tag{8}$$

Where

$$\pi_1(n,T)=\left(s-C_p-C_bI_{bp}\right)\frac{Q}{T}-\frac{\bar{A}}{T}-\frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right) \tag{9}$$

$$\begin{aligned}\pi_2(n,T)=&\left(s-C_p-\left(C_bI_{vp}-sI_{be}\right)M\right)\frac{Q}{T}-sI_{be}\,Q-\frac{\bar{A}}{T}\\ &-\frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right)+\frac{sI_{be}}{T}\int_0^T R(I(t),s)t\,dt\end{aligned} \tag{10}$$

$$\begin{aligned}\pi_3(n,T)=&\left(s-C_p-C_bI_{vp}M\right)\frac{Q}{T}-\frac{\bar{A}}{T}-\frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta s^{-\eta}T}s^{\eta}-s^{\eta}-\beta T\right)\\ &-\frac{C_bI_{bp}\alpha}{\beta^2T}\left(e^{\beta s^{-\eta}(T-M)}s^{\eta}-s^{\eta}-\beta(T-M)\right)\\ &+\frac{sI_{be}\alpha}{\beta^2T}\left(e^{\beta s^{-\eta}T}s^{\eta}-\left(s^{\eta}+\beta M\right)e^{\beta s^{-\eta}(T-M)}\right)\end{aligned} \tag{11}$$

$$\bar{A}=A_b+\frac{A_v}{n}$$

$$\phi=\frac{C_p\left(I_v+I_{vp}\right)\left[(n-1)(1-\rho)+\rho\right]}{\beta^2}$$

$$\psi=\frac{C_bI_b}{\beta^2}$$

IV. Computational Procedure

For fixed T, we note that $\pi(n,T)$ is a concave function of n because $\frac{\partial^2\pi(n,T)}{\partial n^2}=-\frac{2A_v}{n^3T}<0$. Therefore to find optimum number of shipments n^* we will have a local optimal solution. The optimum value of cycle time and retail price can be obtained by setting $\frac{\partial\pi}{\partial T}=0$ and $\frac{\partial\pi}{\partial s}=0$ simultaneously for fixed n.

Algorithm:

Step 1: Set parametric values.

Step 2: Compute T_d using $\frac{1}{\beta s^{-\eta}}ln\left(1+\frac{\beta Q_d}{\alpha}\right)$ for given value of Q_d.

Step 3: Set $n=1$.

Step 4: Knowing T_d and M, compute T and s by solving $\frac{\partial\pi_j}{\partial T}=0$ and $\frac{\partial\pi_j}{\partial s}=0$ simultaneously for $j=1,2,3$.

Step 5: Find corresponding profit π_j for $j=1,2,3$.

Step 6: Increment n by 1.

Step 7: Repeat step 4 and 6 until

$$\pi\left(n-1,T(n-1),s(n-1)\right)\le\pi\left(n,T(n),s(n)\right)\ge\pi\left(n+1,T(n+1),s(n+1)\right).$$

Once the optimal solution $\left(n^*, T^*, s^*\right)$ is obtained, the optimal order quantity can be obtained.

V. Numerical Examples and Interpretations

Example 1: Consider, $\alpha = 10000$ units, $\beta =$ 10%=0.1, $\eta = 1.25$, $\rho =$ 0.7, $C_b =$ \$10/unit, $C_p =$ \$5/unit, $A_v =$ \$400/setup, $A_b =$ \$50/order, $Iv =$ 10%/unit/annum, $I_b =$ 10%/unit/annum, $I_{bp} =$ 8%/unit/annum, $I_{be} =$ 5%/\$/annum, $I_{vp} =$ 2%/unit/annum, $s =$ \$25/unit and $M =$ 30 days.

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of Q_d are exhibited in Table 1.

Table 1 : Optimal solutions for different Q_d

Q_d	Q^*	n^*	T^* (days)	Profit(\$) Buyer	Vendor	Joint
100	74	11	184	2661	576	3237
200	74	11	184	2661	576	3237
300	300	9	219	2690	566	3256
400	400	9	219	2690	566	3256
500	74	11	184	2661	576	3237
600	74	11	184	2661	576	3237

From Table 1, it is seen that the vendor's total profit and joint total profit of the system increase with increase in Q_d and then further increase in pre-specified units lower their profits whereas for the buyer, it is opposite trend. It is seen that the buyer's optimal order quantity Q^* is equal to Q_d when and less than Q_d when $Q_d \geq 500$. Thus, vendor is advised to set threshold which is effective. If the threshold set by the vendor is too high, the buyer will be reluctant to order a quantity greater than the threshold to take advantage of delayed payments. The concavity of joint total profit for (n,s) for obtained T is exhibited in fig. 4, for (n,T) for $s = 29.63$ in fig. 5 and for (s,T) for 9 - shipments in fig. 6.

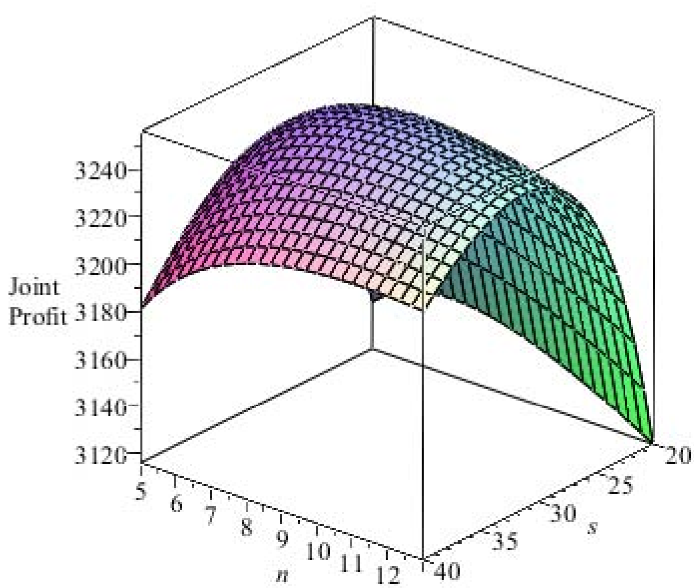

Figure 4 : Concavity of joint total profit for (n,s) for obtained T

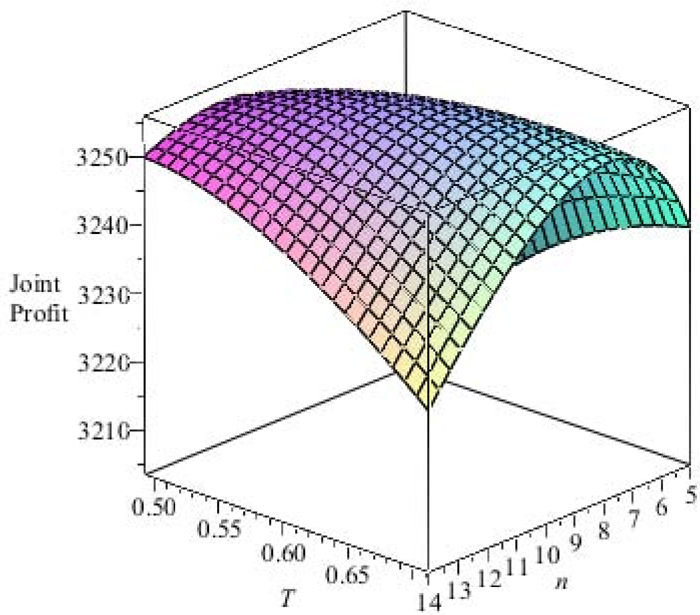

Figure 5 : Concavity of joint total profit for (n,T) for s

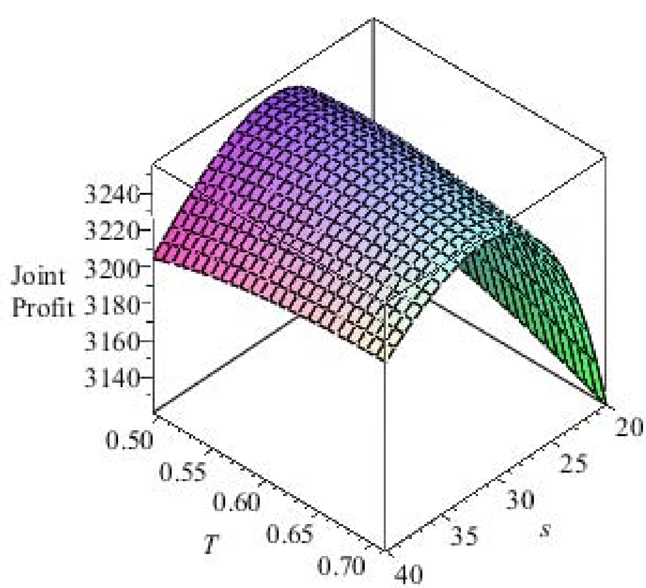

Figure 6 : Concavity of joint total profit for (s,T) for n

Example 2 Consider the data given in Example 1. We study the effect of delayed payments for $Q_d = 300$ units.

Table 2 : Optimal solutions for different *M* $(Q_d = 300)$

M (days)	Q^*	n^*	T^* (days)	Profit($)		
				Buyer	Vendor	Joint
20	300	9	219	2684	567	3251
30	300	9	219	2687	567	3253
40	300	9	219	2690	566	3256
50	300	9	218	2693	566	3259
60	300	9	218	2696	566	3261

From Table 2, it is observed that longer credit period increases buyer's total profit and joint profit of the supply chain. The longer credit period reduces vendor's total profit because payment will be received late for the purchases made. This suggests that late payment increases risk of cash shortage for the vendor.

Example 3. In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the optimal cycle time, purchase quantity and joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and 20%. the results are exhibited in Figure 7.

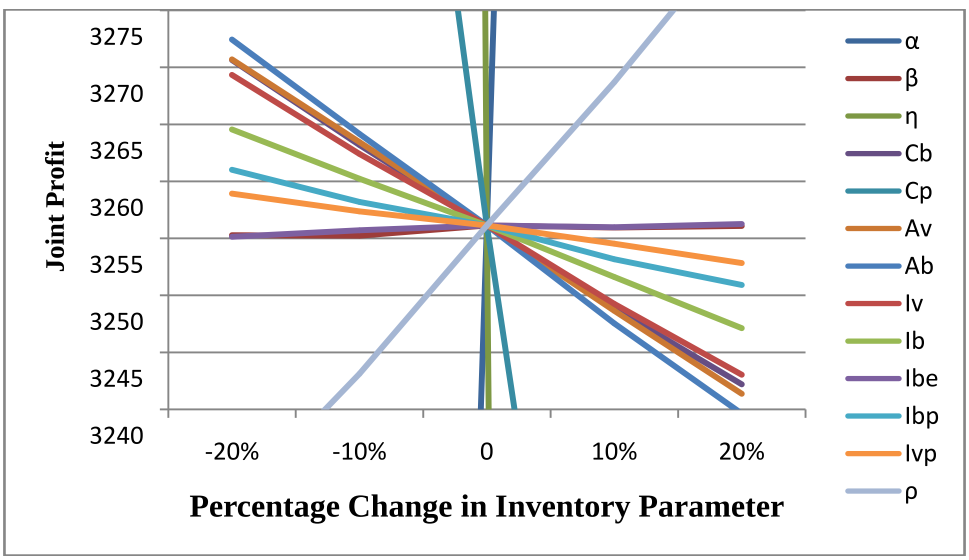

Fig. 7 : Variation in joint total profit

It is observed from fig. 7 that joint profit increases positively with increase in scale demand. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small perturbations in profit of the supply chain.

Example 4. In table 3, we compare independent v*s.* joint decision, for pre-specified quantity $Q_d = 300$ units at which buyer qualifies for getting delay period facility.

Global Journal of Researches in Engineering (G) Volume XIII Issue I Version I Year 2013

Table 3 : Optimal Solution of independent and integrated scenario

Scenario			Buyer	Vendor	Joint
Independent	Total Shipments	9			
	Ordering Quantity	62			
	Cycle Time	329			
	Total Annual Profit		2901	236	3137
Integrated	Total Shipments	9			
	Ordering Quantity	87			
	Cycle Time	219			
	Total Annual Profit		2690	567	3256
	Readjusted Total Annual Profit		3011	245	3256

Where

$$\text{Buyer's profit } = \pi(n,T)\times\frac{TBP(P,T)}{\left[TBP(P,T)+TVP(n)\right]} = 3256\times\frac{2901}{(2901+236)} = 3011$$

$$\text{Supplier's profit } = \pi(n,T)\times\frac{TVP(n)}{\left[TBP(P,T)+TVP(n)\right]} = 3256\times\frac{236}{(2901+236)} = 245$$

Table 3 shows that the total annual profit under joint decision \$3256 (=\$2690+\$567) which is greater than the total profit under independent decision \$3137 (=\$2901+\$236). It establishes that joint decision is advantageous to both the players. The last row of table 3 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.

VI. Conclusion

A single-vendor single-buyer inventory policy is analyzed. The demand is considered to be price-sensitive stock-dependent. The vendor offers order-dependent credit time to settle the account for the purchases made. The computational algorithm is outlined to maximize the joint total profit per unit time with respect to number of transfers from the vendor to the buyer, retail price of the product to be set by the buyer and cycle time.

Based on the results, it is established that longer credit period helps buyer while out- flow risk is for vendor. To entice the buyer for the co-ordinate decision, vendor should set proper threshold for pre-specified order units. It is observed that the order - dependent trade credit attracts the buyer for larger order and thereby saving in transportation cost which is part of the ordering cost.

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Appendix A

The rate of change of inventory at any instant of time can be discussed by differential equation

$$\frac{dI(t)}{dt} = -\left(\alpha + \beta I(t)\right)s^{-\eta}, 0 \le t \le T$$

with $I(0)=Q$ and $I(T)=0$. Using $I(T)=0$, the solution of the differential equation is

$I(t)=\frac{\alpha}{\beta}\left(e^{\beta s^{-\eta}(T-t)}-1\right),\ 0\le t\le T$ The units to be purchased $Q=I(0)=\frac{\alpha}{\beta}\left(e^{\beta s^{-\eta}T}-1\right)$.

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