Global Journal of Researches in Engineering
Industrial Engineering
Volume 13 Issue 3 Version 1.0 Year 2013
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 Print ISSN:0975-5861

Water Distribution as a no Cooperative Game

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Abstract - The water distribution problem of the Mexican Valley is modeled as a three-person no uncooperative Game in which agriculture, industry, and domestic water users are the players and the total water Amounts supplied to the users are the payoff functions. The equilibrium is determined by solving a nonlinear optimization problem, which can be derived based on the Kuhn-Tucker necessary Conditions. All constraints are linear and the objective function is quadratic, so standard solution Algorithm and software can be used.

GJRE-G Classification : FOR Code: 090509

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I. Introduction

The limited amount of natural resources creates conflicts between the users, since any user can Increase its supply only in the expense of the others. As it is well known, water shortage is one of the most worrying problems of our society. According to recent predictions, by 2010 we will need about 17% more water than available to feed the world. Therefore efficient usage of water and optimal water distribution schemes become necessities.

The Mexican Valley is one of the most critical areas, since Mexico City with its 19 million in habitants is the most populated city in the world, and agriculture is the main economic activity in the region which requires large amount of irrigation water. In addition there are industrial users, and the development of industrial employers is crucial for the welfare of the population. The water supply is divided into surface water, groundwater and treated water. The groundwater supply has the best quality, and treated water has the worst. So, treated water can have only limited usage.

The ground water resources are over exploited at a rate of 100% or more, which has the direct Cones quinces of drying springs and sinking ground up to 0.4m/year in some areas.

The current water shortage situation raises the necessity of importing surface and ground water from the neighboring watershed, which also might raise dispute over water resources.

The water distribution problem of the Mexican Valley is modeled as a three person game. Where the three users are the players and the total water amounts supplied to the users are the payoff functions.

There are many earlier works dealing with similar problems. The survey paper of Hype (1992) and the more recent paper of Kapelanet al. (2005),Donevska et al. (2003),Coppola and Szidarovszky(2004), Salazar et al. (2010) can be mentioned among others. As the game theoretical concepts and methods are concerned the books of Szidarovszky et al.(1986) and Forgo et al. (1999) give comprehensive summaries.

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II. Mathematical Model

The three players are agriculture (k=1), industry (k=2) and domestic users (k=3). For each player the Decision variables are

s_k =surface water supply from local source
g_k =ground water supply from local source
t_k =treated water supply
s_k =imported surface water supply
g_k =imported ground water supply

So, the straegy of player k is the 5-dimensional vector $\underline{x}_k = (s_k, g_k, t_k, s_k^*, g_k^*)$ The strategy set of the players are determined by feasibility constraints. The three users have two common constraints:

$$s_k + g_k + t_k + s_k^* + g_k^* \le D_k \qquad (1)$$

and

$$s_k + g_k + t_k + s_k^* + g_k^* \ge D_k^{\min} \quad (k=1, 2, 3), \qquad (2)$$

Where D_k^{min} is the minimum necessary amount and D_k is the total demand of player k. Constraint (1) is requested to avoid wasting water. In addition to these constraints each user has its own conditions.

Agriculture (k=1) has two special water quality related constraints. Some crops can use only ground water, since they are very sensitive to the quality of the water they are irrigated with. If 1 is the ratio of the water need of these sensitive crops to the total water amount used for irrigation, then it is required that

$$\frac{g_1 + g_1^*}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \ge \alpha_1 . \qquad (3)$$

Some less sensitive crops can tolerate treated water, which has the worst quality. If 1 is ratio of the Water need of these less sensitive crops to the total water amount used for irrigation, then we have to assume that since treated water cannot be used to irrigate crops which cannot tolerate treated water.

$$\frac{t_1}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \le \beta_1 \qquad (4)$$

Industry (k=2) has very similar constraints, since there are certain usages which can be supplied only by groundwater as well as some other usages can tolerate treated water:

$$\frac{g_2 + g_2^*}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \geq \alpha_2 \qquad (5)$$

and

$$\frac{t_2}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \leq \beta_2 \,. \qquad (6)$$

Domestic users (k=3) have limitations only on the amount of treated water, since it can be used only for limited purposes (such as irrigating parks, golf courses, etc). Hence this constraint can be given. As

$$\frac{t_3}{s_3 + g_3 + t_3 + s_3^* + g_3^*} \leq \beta_3 \qquad (7)$$

Notice that all constraints (3)-(7) can be rewritten into linear forms.

The total water available in both local and imported surface and groundwater resources can be represented by the additional constraints:

$$s_1 + s_2 + s_3 = S_s \qquad (8)$$

$$g_1 + g_2 + g_3 = S_g \qquad (9)$$

$$s_1^* + s_2^* + s_3^* \leq S_s^* \qquad (10)$$

$$g_1^* + g_2^* + g_3^* \leq S_g^* \qquad (11)$$

Where the right hand sides are the maximum available amounts supplied from the different resources. We require equality in constraints (8) and (9) to be sure that all local resources have to be used before water is imported from other watersheds.

The payoff function for each player is the water supply:

$$\text{Maximize * *} \quad s_k + g_k + t_k + s_k^* + g_k^* \,. \qquad (12)$$

Hence we have a three-player noncooperative game with linear payoffs and linear constraints defining the strategy sets of the players.

III. Solution Methodology

In our case the number of players is n=3, the strategy of player k is the five-dimensional vector x_k, and the strategy set of this player is defined by the individual constraints

$$\underline{A}_k \underline{x}_k \quad \underline{a}_k \qquad (13)$$

Obtained from (1) through (7) and by the joint constraints (8) through (11):

$$\underline{B}_1 \underline{x}_1 + \underline{B}_2 \underline{x}_2 + \underline{B}_3 \underline{x}_3 \leq \underline{b} \,. \qquad (14)$$

The payoff of player k can be written in general as

$$\text{Maximize} \quad \underline{c}_k^T \underline{x}_k \,. \qquad (15)$$

Since the constraints are linear, the Kuhn-Tucker regularity conditions are satisfied, so there are Nonnegative vectors v_k and w_k such that

$$\begin{aligned} & \underline{A} \underline{x}_k \quad \underline{a}_k \\ & \underline{B}_1 \underline{x}_1 + \underline{B}_2 \underline{x}_2 + \underline{B}_3 \underline{x}_3 \leq \underline{b} \\ & \underline{c}_k^T - \underline{v}_k^T \underline{A}_k - \underline{w}_k^T \underline{B}_k = \underline{0}^T \\ & \underline{v}_k^T (\underline{a}_k - \underline{A}_k \underline{x}_k) + \underline{w}_k^T (\underline{b} - \underline{B}_1 \underline{x}_1 - \underline{B}_2 \underline{x}_2 - \underline{B}_3 \underline{x}_3) = 0. \end{aligned} \qquad (16)$$

Because of the other constraints the left hand side of the last condition is always nonnegative, so its minimal value is zero. Consider now the following quadratic programming problem with linear constraints:

$$\text{Minimize} \sum_{k=1}^{3} \underline{v}_k^T (\underline{a}_k - \underline{A}_k \underline{x}_k) + \left(\sum_{k=1}^{3} \underline{w}_k^T \right) (\underline{b} - \underline{B}_1 \underline{x}_1 - \underline{B}_2 \underline{x}_2 - \underline{B}_3 \underline{x}_3) \qquad (17)$$

The Nikaido-Isoda theorem implies the existence of at least one equilibrium, so there is at least one solution of system (16). So the optimal objective function value is zero, and all optimal solutions satisfy the Kuhn-Tucker necessary conditions (16). Because of the linearity of the game, conditions (16) are also sufficient, implying that all optimal solutions of problem (17) are Nash-equilibria of the three-person game. For the numerical solution of (17) standard methodology and software is available.

IV. Numerical Results

The data for our case study were given by a research group of the Universidad Autonoma Chapingo, who investigated the same problem by using different solution concept and methodology (Salazar et al., 2007).The input data are given in Table 1. In addition, the available water supplies from the

Table 1

	k=1	k=2	k=3
D_k^{min}	594	177	1092.8
D_k	966	230	2123
α_k	0.41	0.066	-
β_k	0.33	0.20	0.06

Different sources are limited as given in constraints (8) through (11) with S_s =58, S_g =1702, S^*_s =453 and S_s* =169. These quantities and min D_k and D_k are given in mill m3/year, the constants a_k and β_k are ratios, unit less quantities. The equilibrium was computed by solving the optimization problem (17).

The results are presented in Table 2. The objective function at the optimum was zero showing.

Table 2 : Numerical Results

	k=1	k=2	k=3	Total
s_k	0	0	58	58
g_k	966	205.353	530.647	1702
t_k	0	0	75.702	75.702
s^*_k	0	24.647	428.353	453
g^*_k	0	0	169	169
Total	966	230	1261.702	

That global optimum was reached. All demands of agriculture and industry can be satisfied, since there ceiled amounts are their total demands. The domestic demand can be satisfied only partially, on 59.43% level. All surface and groundwater resources are used. The restrictive constraints on treated water usage make the use of more treated water impossible. Importing the large amount of surface and groundwater into the Maxi can Valley will raise severe conflicts with the neighboring regions. In addition, a larger amount of treated water has to be used which raises the important issue of water quality, since under the given constraints the only way to increase water supply is to increase the treated water usage. In order to have larger water supply and avoid serious social conflicts further developments are needed in combination with more efficient water usage by the three sectors. May be a market-driven water pricing policy would give incentives to the users.

V. Conclusions

The water distribution problem in the Maxi can Valley was modeled as a three-person no cooperative game. The three users were the players, the supplied water amounts the payoffs, and the strategy set were determined by supply and water quality constraints. The Nash equilibrium of this game was determined by solving a special quadratic optimization problem with linear constraints, which was derived based on the Kuhn-Tucker conditions.

The numerical results indicate that a combination of investment for further developments in the infrastructure is needed in combination with more efficient water usage in order to avoid serious social conflicts and water shortages.

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