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Simulation of Gear Dynamics by Circuit Theory Methods

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I. Introduction

The classical method of Lagrange equation (Genkin and Grinkevich, 1961) and the methods of dynamic stiffness or admittance (Airapetov et al. 1975) are used in studies of gear dynamics. These methods are cumbersome and laborious. In order to automate composition of equations of motion, the bond graph (Karnopp and Rosenberg, 1972) and some other methods are also used. However, the use of these methods presupposes presentation of a dynamic model as a system with concentrated parameters. Also, the bond graph of a relatively simple model, as, for example, a planetary gear transmission, is very cumbersome (Allen, 1979). On the other hand, electrical analogy method (Skudrzyk, 1968) and linear graph method (Mason and Zimmermann, 1960) allow us to model dynamic systems with both concentrated and distributed parameters (Podzharov, 1983, 1987, Sasa et al., 2004, Wojnarowski, 2006, Kalous, 2009).

In this paper a methodology of study of gear dynamics with the aid of the electric circuit theory methods is presented. The analogy between the force and electric tension is used to compose equivalent electric circuits for dynamic gear systems.

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Table 1 : Equivalent Parameters

No.	Mechanical System	Electrical System
1	Force	Tension
2	Speed	Electric current
3	Displacement	Electric charge
4	Mass	Inductance
5	Flexibility	Capacitance
6	Absorber	Electrical resistance

Nomenclature

J_i - moment of inertia of the mass m_i ,

k_i - torsional stiffness

C_{S2}, C_{S3} - support stiffnesses,

C_3 - stiffness of tooth engagement,

$T_1(t), T_2(t)$ - variable torsion moments,

μ_i - moment of inertia reduced to the mass moving on the line of action,

C_i - torsion stiffness reduced to linear stiffness on the line of action,

r_{bi} - base radius of a gear,

$F_i(t)$ - torsional moment reduced to a force applied in the line of action of gear engagement,

$S_i(t)$ - kinematic error in the gear engagement,

y_{ij} - element of the matrix of mechanical conductance,

Y - matrix of mechanical conductance,

Z - mechanical impedance,

T_{ij} - transmission in a graph between the points i and j,

f - frequency,

$j = \sqrt{-1}$.

II. Modelling One-Stage Gear Transmission

We shall now consider the use of electrical analogy for the automation of composition of equations of motion and their analysis in the example of one-stage gear transmission with flexible supports and coupled masses. The mechanical model of the transmission is shown in Fig.1 and in Fig. 2, presenting the equivalent electrical circuit. Here, the parameters of the torsion

system reduced to the parameters of a linear system are determined as follows:

$$\mu_i = J_i / r_{bi}^2,\ C_{ki} = k_i / r_{bi}^2,\ F_i(t) = T_i(t) / r_{bi} \qquad (1)$$

This system has 10 independent elements and 11 resonances and antiresonances (Skudrzyk, 1968), including zero and infinite frequencies. When the circuit is excited by variable tensions (forces) $F_1(t)$ and $F_2(t)$ the contours $C_1\mu_1$ and $C_3\mu_4$ act as low frequency filters filtering out high frequency components. Thus, the existence of large coupled masses linked to the gears by relatively low stiffness shafts means that the gears are dynamically isolated from external excitation in medium and high frequency ranges.

Let us consider stationary vibrations and assume that the gear is a linear dynamic system. Then, the solution of this system with periodic force or kinematic excitation can be found as a sum of harmonics. In this case the equations of motion can be composed as Kirchhoff equations of the equivalent electric circuit.

The total conductance between the points *a* and *b* will be equal to the sum of the conductances of parallel branches (Skudrzyk, 1968).

$$Y_{ab} = Y_0 + Y_1 + Y_2 + Y_3 + Y_4, \qquad (2)$$

where

$$Y_0 = j\omega / C_2,\ Y_2 = \left(j\omega m_2 + C_{S2} / j\omega\right)^{-1},$$

$$Y_1 = \left[\left(\frac{1}{j\omega\mu_1} + \frac{j\omega}{C_1}\right)^{-1} + j\omega\mu_2\right]^{-1},$$

$$Y_3 = \left(j\omega m_3 + C_{S3} / j\omega\right)^{-1},$$

$$Y_4 = \left[\left(\frac{1}{j\omega\mu_4} + \frac{j\omega}{C_3}\right)^{-1} + j\omega\mu_3\right]^{-1}. \qquad (3)$$

The input impedance between points *a* and *b* can be found as inverse of Y_{ab}

$$Z_{ab} = 1/Y_{ab}. \qquad (4)$$

Substituting the equations (3) in the equation (2) and (4) and making Z_{ab} equal zero, we can find a frequency characteristic of Z_{ab}, which is shown in Fig. 3 for the transmission with the following parameters:

$J_1 = 0.012\ kg\,m^2$, $J_2 = 0.000686\ kg\,m^2$,
$J_3 = 0.00471\ kg\,m^2$, $J_4 = 0.02\ kg\,m^2$,
$m_2 = 1.56\ kg$, $m_3 = 3.8\ kg$,
$C_{S2} = 0.455\cdot10^8\ N/m$, $C_{S3} = 0.101\cdot10^8\ N/m$,
$C_{S2} = 0.27\cdot10^9\ N/m$, $k_1 = 4270\ N\cdot m$,
$k_3 = 2000\ N\cdot m$, $r_{b2} = 0.0383\ m$, $r_{b3} = 0.0634\ m$.

It was calculated neglecting the damping in the system and, according to the Foster theorem (Skudrzyk, 1968), it has a monotonous character. We can find from the curve that the poles are at frequencies 55 Hz, 209 Hz, 300 Hz, 794 Hz and 5200 Hz. The zeros are at the frequencies 115 Hz, 259.5 Hz, 408 Hz and 859.5 Hz.

The poles f_{pi} and zeros f_{sj} can also be found approximately from the concepts of parallel and successive resonances or resonances of tensions and currents:

$$f_{p1} = \frac{1}{2\pi}\sqrt{\frac{C_1}{\mu_4}} = 50\ Hz, \qquad (5)$$

$$f_{p2} = \frac{1}{2\pi}\sqrt{\frac{C_2 + C_{S2}}{\mu_3 + m_3}} = 232\ Hz,$$

$$f_{p3} = \frac{1}{2\pi}\sqrt{\frac{C_{S1}}{m_2 + \mu_1 + \mu_2}} = 336\ Hz, \qquad (7)$$

$$f_{p4} = \frac{1}{2\pi}\sqrt{\frac{C_1 + C_{S2}}{\mu_{23} + m_2}} = 778\ Hz, \qquad (8)$$

$$f_{p5} = \frac{1}{2\pi}\sqrt{C_2\left(\mu_2^{-1} + \mu_3^{-1} + m_2^{-1} + m_3^{-1}\right)} = 5162\ Hz \qquad (9)$$

$$f_{S1} = \frac{1}{2\pi}\sqrt{\frac{C_3}{\mu_3}} = 104\ Hz, \qquad (10)$$

$$f_{S2} = \frac{1}{2\pi}\sqrt{\frac{C_{S3}}{m_3}} = 259.5\ Hz, \qquad (11)$$

$$f_{S3} = \frac{1}{2\pi}\sqrt{\frac{C_1}{\mu_2}} = 397\ Hz, \qquad (12)$$

$$f_{S4} = \frac{1}{2\pi}\sqrt{\frac{C_{S2}}{m_2}} = 859.5\ Hz, \qquad (13)$$

As we see, the approximate and exact frequencies are very similar to each other. Therefore, the formulas (5) – (13) can be used for the analysis of resonances.

Now, let us use the linear graph method (Mason and Zimmermann, 1960) to obtain a general form for this model. A linear graph for the circuit in Fig. 2 is constructed in Fig. 4. This graph illustrates the relations between forces F_i (upper nodes) and velocities v_i (lower nodes). The lines between them are transmissions, which in this case are mechanical

admittances y_i or mechanical impedances z_i. Thus, in this graph we use relations

$$v_i = y_i \cdot F_i \quad \text{and} \quad F_i = z_i \cdot v_i \tag{14}$$

In order to simplify the graph, the number of nodes can be reduced retaining only the nodes which we need to determine. Further simplification of the graph can be implemented by adding parallel transmissions and excluding the nodes, which we do not need to determine, by splitting them. Hence, splitting the nodes $F_{\mu i}$, v_{Ci}, $v_{\mu i}$ and excluding the loops l_i, we can obtain the transformed graph presented in Fig. 5.
Here,

$$y_{\mu i} = 1/(j\omega\mu_i),\ y_{Ci} = j\omega/C_i,$$
$$y_{Si} = 1/(j\omega\mu_i + C_i/(j\omega)) \tag{15}$$

The transmissions of this graph can be determined by the following formulas

$$T_{11} = \frac{y_{\mu 1}}{y_{Ci}(1-l_1)};\ T_{12} = \frac{y_{\mu 2}}{y_{Ci}(1-l_1)},$$

$$T_{21} = \frac{y_{\mu 2}}{y_{C2}(1-l_2)},\ T_{22} = \frac{1}{y_{C2}(1-l_2)},$$

$$T_{23} = \frac{y_{\mu 3}}{y_{C2}(1-l_2)},\ T_{32} = \frac{y_{\mu 3}}{y_{C3}(1-l_3)},$$

$$T_{33} = \frac{y_{\mu 4}}{y_{C3}(1-l_3)}. \tag{16}$$

Where

$$l_1 = -(y_{\mu 1} + y_{\mu 2})/y_{C1},\ l_3 = -(y_{\mu 3} + y_{\mu 4})/y_{C3},$$
$$l_2 = -(y_{\mu 2} + y_{\mu 3} + y_{S2} + y_{S3})/y_{C2}. \tag{17}$$

This graph can also be described by the equations determining the nodes in relation to adjacent nodes and transmissions that link them:

$$\begin{aligned} F_{C1} - T_{12}F_{C2} &= T_{11}F_1(t) \\ -T_{21}F_{C1} + F_{C2} - T_{23}F_{C3} &= T_{22}\dot{S}_2(t) \\ -T_{32}F_{C2} + F_{C3} &= T_{33}F_4(t) \end{aligned} \tag{18}$$

Substituting (16) and (17) in (18) and multiplying each of the ith equation (18) by $y_{Ci}(1-l_i)$, we have

$$\begin{aligned} y_{11}F_{C1} - y_{12}F_{C2} &= y_{\mu 1}F_1(t) \\ -y_{21}F_{C1} + y_{22}F_{C2} - y_{23}F_{C3} &= \dot{S}_2(t) \\ -y_{32}F_{C2} + y_{33}F_{C3} &= F_4(t) \end{aligned}$$

Where $y_{11} = y_{C1} + y_{\mu 1} + y_{\mu 2}$, $y_{12} = y_{21} = y_{\mu 2}$,

$$y_{22} = y_{C2} + y_{\mu 2} + y_{\mu 3} + y_{S2} + y_{S3},\quad (20)$$

$$y_{23} = y_{32} = y_{\mu 3},\ y_{33} = y_{C3} + y_{\mu 3} + y_{\mu 4}.$$

As we can see from equation (19) and (20), equation (19) has the form

$$Y \times F_c = P, \tag{21}$$

Here, the matrix of the mechanical conductance Y is symmetrical; each itch diagonal element is positive and equal to the sum of the input mechanical conductances of the elements, which enter the itch node. Each non-diagonal element is negative and equal to the transition conductance of the elements, which locate between itch and j-th nodes. This type of matrix is known in the circuit theory as matrix of node equations (Karni, 1966).

The excitation term in the right-hand part of the i-th equation is equal to the velocity of cinematic error in the gear engagement, in the case of cinematic excitation. In the case of force excitation, it equals to the product of the exciting force and the transition conductance between the point of application of the force and the itch node.

Therefore, it is not necessary to compose equivalent electric circuits and graphs. Instead, following the rules explained above, we can directly compose the matrix of mechanical conductance and the vector of the right-hand part of the equations.

The above formulated rules can be extended to more complicated systems and systems with distributed parameters.

A dynamic calculation of the gear with the parameters mentioned above was implemented using the equations (15) – (20). The damping in elastic elements was considered by introducing complex stiffness (Skudrzyk, 1968).

$$\overline{C_i} = C_i(1 + j\eta_i), \tag{22}$$

Where η_i - loss factor in i-th elastic element.

The results of calculation of dynamical forces F_{Ci} in elastic elements are presented in Fig.6; the amplitude of kinematic excitation $\dot{S}_2 = 5\ \mu m$ and the loss factor in all of the elastic elements was taken equal to *0.1.* As we see from the graph, in the low frequency range the dynamic forces in each elastic element are the same. The resonance at the frequency 55 Hz corresponds to the frequency of natural vibrations of mass μ_4 at the stiffness C_3, and the high frequency resonance at 5200 Hz is the natural frequency of

vibrations of all masses at the stiffness C_2 of the gear engagement. The origin of all other resonances can be checked with the aid of equations (5) – (9).

The measurement of noise and vibration of this gear shows that it has high levels at frequency 5200 Hz.

III. Conclusions

a) The use of electric analogy allows us to avoid the derivation of equations of motion and to make a frequency analysis without solving the equations.

b) Equations analogous to node equations known in the circuit theory can be used in the gear dynamics for the systems with concentrated and distributed parameters.

c) There is no need to compose any electric circuit and linear graph to obtain linear equations of motion of a dynamic system. They can be composed as node equations directly for a dynamic model.

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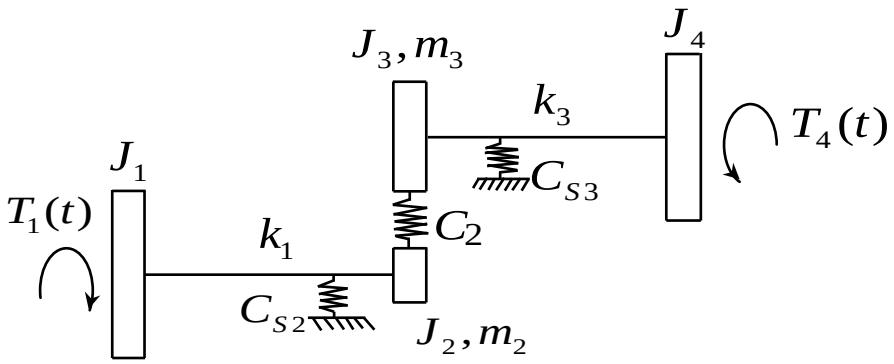

Fig. 1 : Gear dynamic model

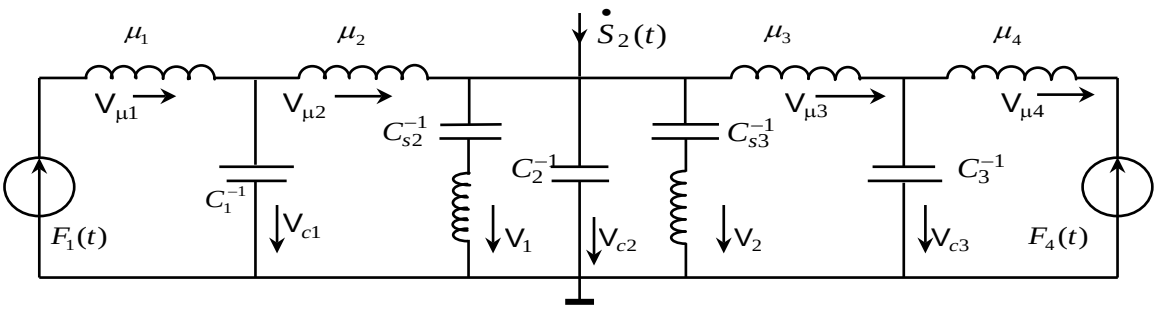

Fig. 2 : Equivalent electrical circuit

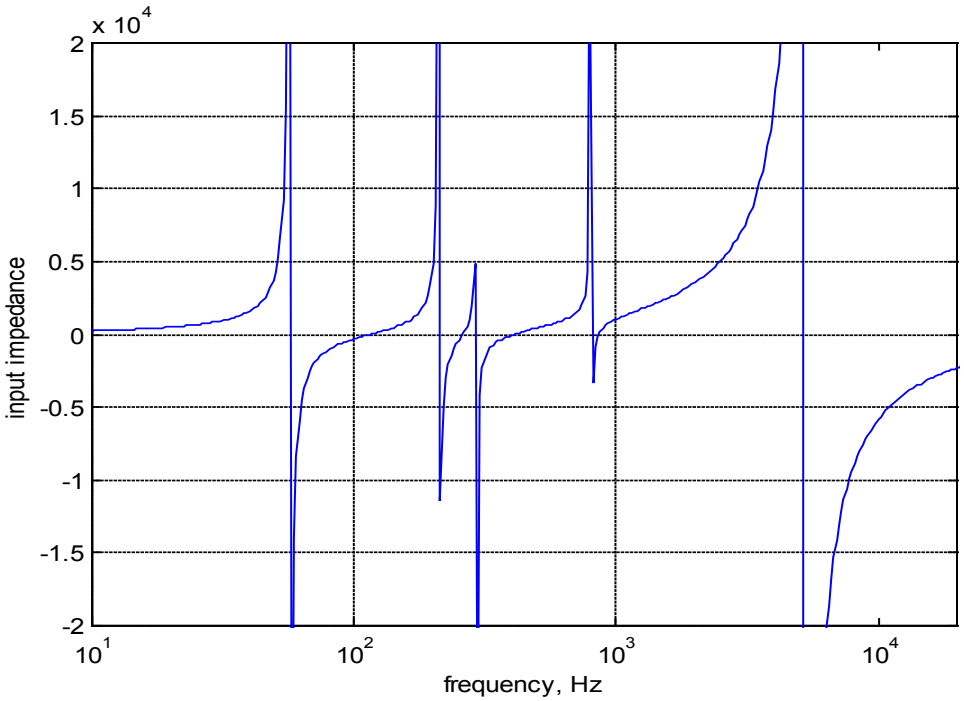

Fig. 3 : Frequency characteristic of input impedance

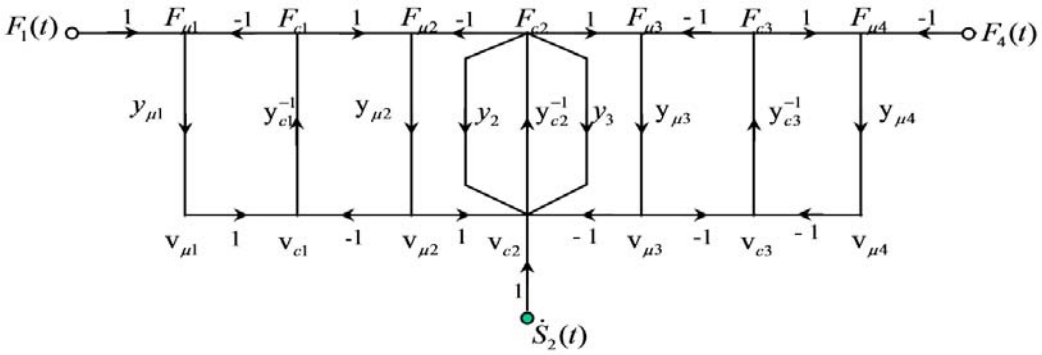

Fig. 4 : Linear graph of gear dynamic model

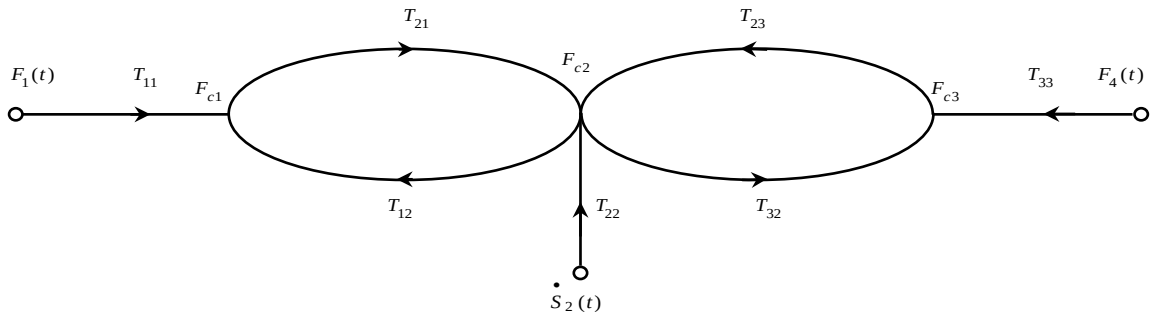

Fig. 5 : Transformed linear graph of gear dynam

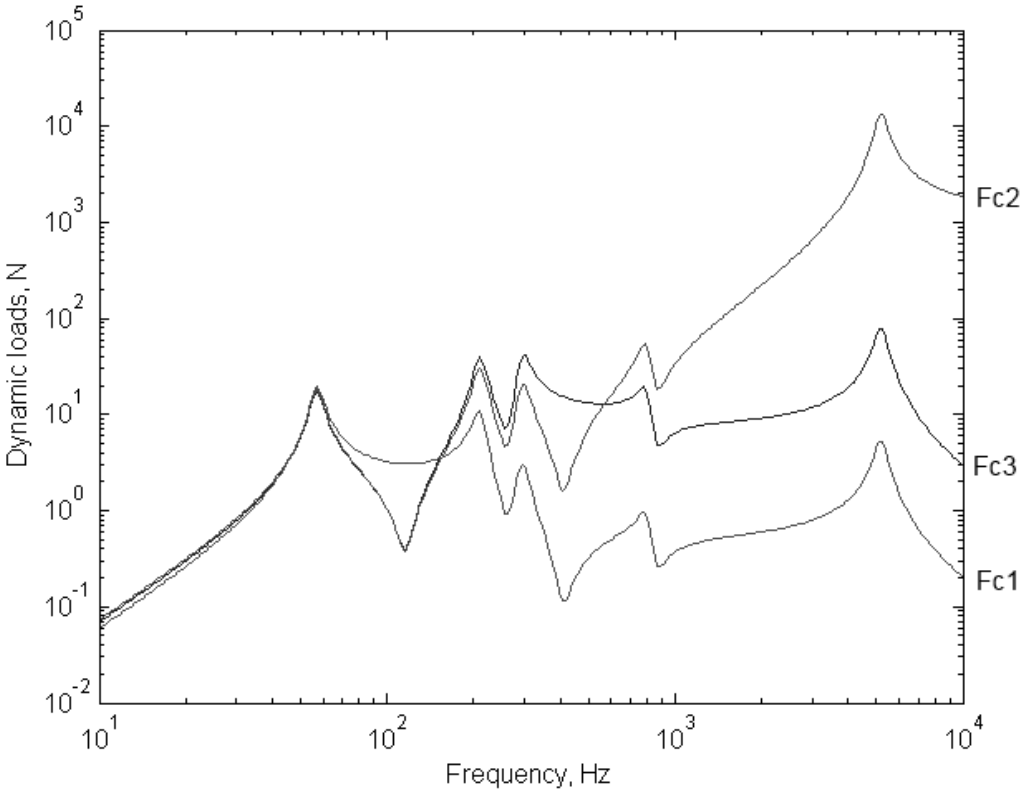

Fig. 6 Frequency characteristic of dynamic loads in the gear:
Fc1 - in the gear support, Fc3 – in the pinion support,
Fc2 – in the gear engagement

Fig. 6 : Dynamic loads in the gear