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An Inhomogeneous Gravitational Field and the Body without Center of Gravity

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I. Introduction

Concept of homogeneity (or inhomogeneity) of the gravitational field is a rather specific one. Namely, according to the Newton`s law of gravitation, either intensity or direction, or each of these two characteristics of the gravitational force acting on the body, depend on its position in the gravitational field. So, in fact, for the body as a whole, the gravitational field is always an inhomogeneous one. Such a classification makes sense only if it is restricted to the active part of the gravitational field, that is, on the part occupied by the body.

II. Homogeneous and Inhomogeneous Gravitational Field

Consider a body of mass *m* moving in the gravitational field of the dominant gravitational center of mass m*. Assume that m/m*«1, so the gravitational field is a stationary one. Gravitational noice, as well as the gravitational anomalies, are excluded.

The motion of the body is composed: while moving in its orbit, it revolves about its principal central axis of inertia (1), which is perpendicular to the orbital plane. The mass center C of the body is chosen to be at the origin of two moving frames of reference xCy and ξCη. The first one is related to the geometry of the orbit, Cx being oriented toward the gravitational center, that is toward its mass center C*. The orbital angle is ψ. The second frame is related to the geometry of mass of the body, Cξ having direction of the principal axis (3) and Cη – direction of the axis (2) of the ellipsoid of inertia. Position of the second reference frame with respect to the first one is defined by the angle of relative rotation φ (Fig.1).

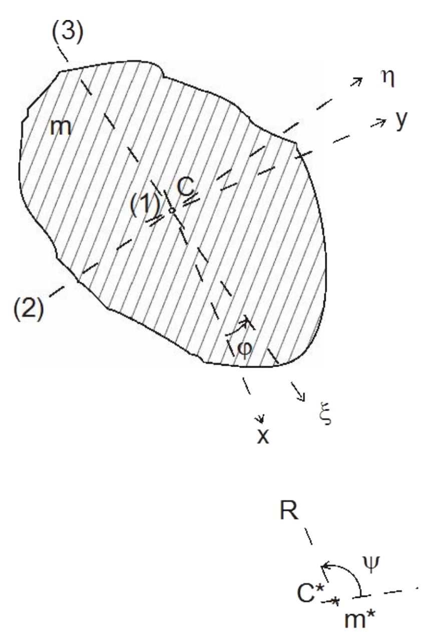

Fig. 1 : Body in the gravitational field and two frames of reference

In a homogeneous gravitational field, intensities and directions of the elementary gravitational forces acting on the body`s particles depend on the position $(\overline{CC^*} = R, \psi)$ of the mass center in the gravitational field, solely. They are functions neither of

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the positions of the particles (ξ,η) within the body, nor of the angle of relative rotation ϕ. All these forces have the same direction, that is, they are parallel to the coordinate axis Cx and their sum, the "weight" of the body, coincides with that line, regardless of the relative position of the body in the frame of reference xCy. In a homogeneous gravitational field the resultant of the ellementary gravitational forces always passes through the mass center of the body and the gravitational moment does not exist. In fact, the mass center of the body in a homogeneous gravitational field represents the center of gravity, as concieved by Archimedes some 2,5 centuries B.C. (Fig. 2).

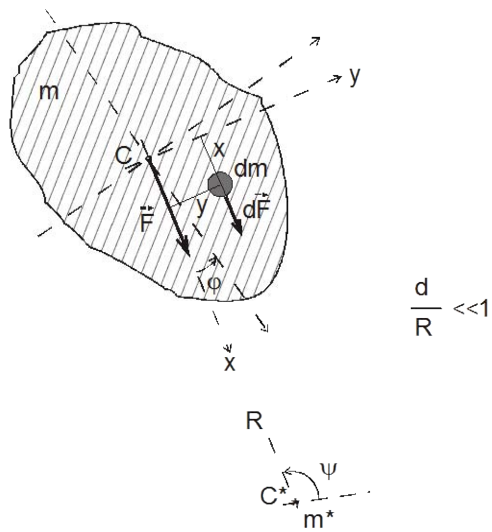

Fig. 2 : Homogeneous gravitational field

Such an interpretation of the gravitational field acting on the body is only posible if the dimensions of the body are negligible compared to the distance between the mass center of the body and of the gravitational center. If the largest dimension of the body is d, invariability of intensities and directions of the gravitational forces acting on the body`s particles imply d/R≈0 compared to unity in the approximate calculus of the gravitational load $\vec{F}, \overrightarrow{M}^{C}$.

On the other hand, if the gravitational field is inhomogenous one, at least the first power of the fraction d/R has to be retained in the expression containing unity in the course of calculus of the gravitational load. Depending on the retained power of d/R it is possible to speak of the first, the second, or the higher order inhomogeinity of the field. In the inhomogeneous gravitational field intensities and directions of the ellementary forces depend on the position of the mass center in the gravitational field, but also on the position of the particles within the body, as well as on the relative position of the body.

$$d\vec{F} = d\vec{F}(R, \Psi, \xi, \eta, \varphi)$$

All these forces converge toward the gravitational center and so does their resultant.

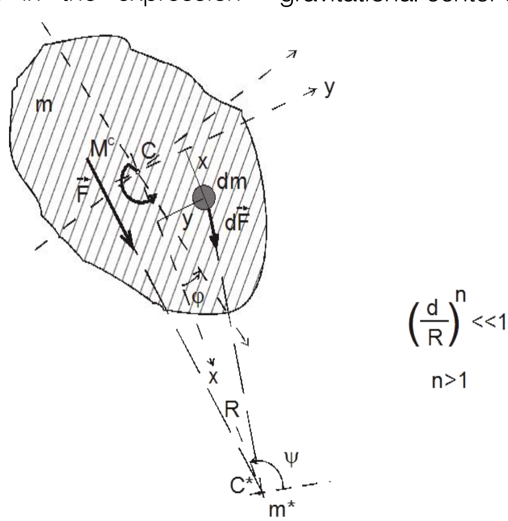

Fig. 3 : Inhomogeneous gravitational field.

Generally, this resultant doesn`t pass through the mass center of the body, so it has to produce the gravitational moment (Fig. 3).

Within the first order inhomogeneity case, the gravitational force is the same as in the homogeneous gravitational field

$$F = G\frac{m \cdot m^*}{R^2}$$

Where G is the gravitational constant and the gravitational moment for the described motion is equal

$$M^C = -\frac{3}{2}G\frac{m^*(I_2 - I_3)}{R^3}\sin 2\varphi\ ,$$

Where I_2 and I_3 are the medium and the minimum principal moments of inertia for the mass center of the body. Obviously, the gravitational moment is a harmonic function of the double angle of relative rotation, with the amplitude depending on the mass of the gravitational center, on the distance between the body and this center and finally, on the mass distribution in the body.

When the small body goes around the large body in a closed orbit, its orbital and rotational motions gradually become resonant just because of the gravitational torque existence (/3/,/4/). For example the Moon circulates around Earth and rotates around its axis in the 1/1 resonance

The first characteristic of inhomogeneity of the gravitational field is the existence of the gravitational moment, in the general case. The second one is the absence of the center of gravity of the body.

III. Body without Center of Gravity

Authors of many of textbooks are not quite precise about that point. Having, probably, in mind the existence of the gravitational moment in such a field, they often claim implicitly, or even explicitly, that in an inhomogeneous gravitational field the center of gravity doesn't coincide with the mass center of the body. This is definitely wrong, of course, because such an assertion may lead the reader to the false conclusion that there exists something like a "moving center of the gravity" in the rotating body exposed to the inhomogeneous gravitational field. There is no such a point in, or in the vicinity of the body, satisfying the definition of the center of gravity, if the body is exposed to the field of the convergent gravitational forces. The "weight" vector of the body has to pass through the point toward which converge all its components and that is the gravitational center. The relative rotation of the body produces tilting of this vector about this center (Fig. 4).

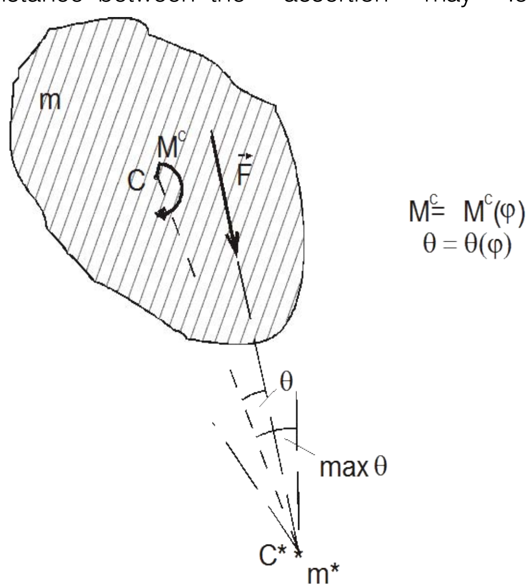

Fig. 4 : Oscillation of the gravitational force around the gravitational center

The angle between the gravitational force and the direction CC* is equal

$$\theta = \frac{3}{2}\frac{I_2 - I_3}{mR^3}\cdot \sin 2\varphi\,.$$

IV. Conclusion

We have stressed the distinction between homogeneous and inhomogeneous gravitational fields. A homogeneous gravitational field is marked by the existence of the center of gravity and the absence of the gravitational moment acting on the body. On the other hand, in an inhomogeneous gravitational field the body has no center of gravity and the gravitational moment, generally, exist. Concerning the calculus of the gravitational load, one has to adopt d/R≈0, compared to unity, for the homogeneous and at least d/R≠0, compared to unity, for an inhomogeneous gravitational field.

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