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Calculation of the Strength Reliability of Parts under Random Loading

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This paper proposes a new approach to the solution of the problem, which is based on the application of the mathematical apparatus of nonparametric statistics. The considered approach of calculating the probability of a failure and the quantile estimates of the safety factor of machine parts are universal. They allow estimation of the strength reliability of items regardless of the complexity of the laws of distribution of random values of the actual and ultimate stresses.

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Calculation of the Strength Reliability of Parts under Random Loading

V. N. Syzrantsev [α], K. V. Syzrantseva [σ] & L. A. Chernaya [ρ]

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I. Introduction

At present, when solving the problems associated with an increase in the manufacturing efficiency, improvement of the diagnostics of diseases, statistical data processing in insurance and financial mathematics, one has to deal with experimental data. The distribution density functions (DDFs) for these data are most frequently unknown and are not described by the laws of distribution of random quantities that were developed in the theory of mathematical statistics. Therefore, the main trend in the development of the statistical science involves the elaboration of methods for processing experimental data that allow the actual laws of distribution of random quantities to be taken into account. In the second half of the last century, an approach to estimation of many functionals on the basis of a nonparametric estimate of the probability density was proposed in [1, 2, 3]. To date, owing to the development of the computer engineering, this approach has gained significant development for solving various problems in economics and medicine [4, 5, 6, 7]. Nonparametric methods became widespread in solving identification and regression-analysis problems [8, 9, 10].

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The most important characteristics of numerous engineering objects are their strength and reliability. Up to now, these characteristics are determined on the basis of the laws that are considered in the theory of parametric statistics [11, 12]. At the same time, it was shown in [13, 14, 15] that the DDFs of the actual and ultimate stresses, on the basis of which the probability of no-failure operation of an item is determined, are seldom described by the laws that were studied in the statistical theory. This study considers the solution of the problem of calculating the probability of no-failure operation of several engineering objects on the basis of applying methods of nonparametric statistics.

II. Statement of the Problem

Calculated estimates of the strength reliability of parts are currently obtained using two fundamentally different approaches. According to the first one [11, 12], the probability of a failure of a part is calculated as

$$Pr[y = (\sigma - s) \geq 0] \qquad (1)$$

where σ - are the effective stresses (MPa) at a hazardous place of the part,

s - the permissible stresses (MPa) for its material.

Problem (1) requires knowledge of the distribution density function (DDF) $f_\sigma(\sigma)$ of the random quantity and the DDF $f_S(s)$ of the random quantity s. If the functions $f_\sigma(\sigma)$ and $f_S(s)$ are known to within parameters, the solution of problem (1) reduces to the calculation of the integral

$$Q = \frac{1}{F_\sigma \cdot F_s} \int_0^\infty \left(\int_0^\infty f_\sigma(u+t) \cdot f_s(t) dt \right) du \qquad (2)$$

where $F_\sigma = \int_0^\infty f_\sigma(u) du \;\; F_s = \int_0^\infty f_s(u) du$.

It is conventionally assumed that the density functions $f_\sigma(\sigma)$ and $f_S(s)$ are distributed according to a normal law, thus allowing the problem (1) to be solved on the basis of tables of the normal distribution. Papers [11, 12] presents the solutions of problem (1) for several laws of distribution of the random quantities σ and s that were studied in the theory of parametric statistics.

Despite the versatility of this approach, it is not always possible to obtain a quantitative estimate of the strength reliability of a studied part within its framework.

This is confirmed by Fig. 1, which shows the functions $f_\sigma(\sigma)$ and $f_S(s)$ for one of the studied parts.

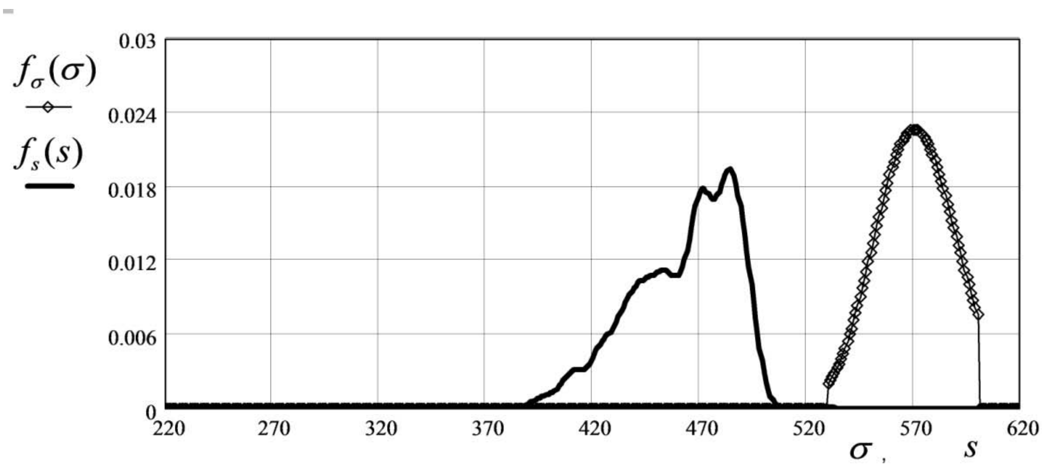

Fig. 1 : Density functions $f_\sigma(\sigma)$ and $f_S(s)$

It can be easily seen in Fig. 1 that the calculation of the probability of failure using formula (1) results here in the zero value of the probability of a failure. In this case, the problem of estimating the technical state of a part can be solved via realization of the second approach, which implies the calculation of quantile (n_σ^α) estimates of the safety margin (n_σ) at a specified probability α via the numerical solution of the equation

$$\int_0^{n_\sigma^\alpha} f_n(n_\sigma)dn_\sigma = \alpha \qquad (3)$$

with respect to n_σ^α .

Here, $f_n(n_\sigma)$ is the DDF for n_σ , which is calculated from the dependence

$$n_\sigma = s/\sigma \qquad (4)$$

As a rule, when the safety margin is calculated, the random character of σ and s is disregarded and only their average values are used. However, for a number of reasons, determining the characteristics of the random quantity n_σ on the basis of formula (4) is not a trivial problem [13]. For actual conditions of the use of parts, the random quantity σ is not described by the laws that were considered within the framework of parametric statistics. The analysis of the results of processing experimental data (yield stress, ultimate stress of pipe steels), which are used to calculate allowable stresses, shows that the use of a normal distribution law is not always correct here and more "flexible" laws should be used, e.g. the Gram-Charlier law. Because the samples $\sigma_i, i=\overline{1,m}$ and $\sigma_i, i=\overline{1,m}$ always have finite lengths, the left- and right-censored density functions $f_\sigma(\sigma)$ and $f_S(s)$ must be used in calculations using expression (4). The law of distribution of the random quantity n_σ is known only for some particular cases. For example, if the functions $f_\sigma(\sigma)$ and $f_S(s)$ obey a normal law, the distribution density function $f_S(s)$ corresponds to the Cauchy distribution, for which a mean value and a variance are generally absent. For the reasons that were presented above, problem (3) can be solved using conventional methods of parametric statistics only under serious assumptions. As a result, the correct calculation results are not guaranteed. Here, more powerful algorithms that operate regardless of the complexity of the functions $f_\sigma(\sigma)$, $f_s(s)$, and $f_s(s)$ must be applied. Exactly such algorithms, the possibility of realization of which is provided only by the achievements of the modern computer engineering and computer simulation methods, were developed within the framework of the theory of nonparametric statistics [13, 14, 15].

III. Used Theoretical Methods

For determination of probability of part failure in accordance with equation (2) it is necessary to solve two auxiliary problems.

Problem 1: Reconstruction of an unknown DDF on the basis of a sample of values of a random quantity

According to [13], on the basis of a sample of stresses $\sigma_i, i=\overline{1,m}$, the estimate of a left ($\sigma_{\min} = \min_i\{\sigma_i\}$) and right ($\sigma_{\max} = \max_i\{\sigma_i\}$) censored unknown DDF for stresses is represented in the form of the expansion (Parzen-Rosenblatt estimate with a normal kernel):

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$$f_\sigma(\sigma)=\frac{1}{m\cdot h_\sigma\cdot\sqrt{2\cdot\pi}}\sum_{i=1}^{m}\exp\left[-0{,}5\left(\frac{\sigma-\sigma_i}{h_\sigma}\right)^2\right]\cdot\frac{1}{c_\sigma} \quad (5)$$

$$c_\sigma=\frac{1}{m\cdot h_\sigma\cdot\sqrt{2\cdot\pi}}\int_{\sigma_{\min}}^{\sigma_{\max}}\sum_{i=1}^{m}\exp\left[-0{,}5\left(\frac{\sigma-\sigma_i}{h_\sigma}\right)^2\right]d\sigma$$

in which

$$c_\sigma=\frac{1}{m\cdot h_\sigma\cdot\sqrt{2\cdot\pi}}\int_{\sigma_{\min}}^{\sigma_{\max}}\sum_{i=1}^{m}\exp\left[-0{,}5\left(\frac{\sigma-\sigma_i}{h_\sigma}\right)^2\right]d\sigma$$

and the value of the spreading parameter h_σ corresponds to the maximum of the information functional:

$$\max_{h_\sigma} J=\max_{h_\sigma}\left\{\frac{1}{m}\sum_{i=1}^{m}\ln\left[\frac{1}{(m-1)h_\sigma}\sum_{j\neq i}^{m-1}\frac{1}{\sqrt{2\cdot\pi}}\exp\left(-0{,}5\left(\frac{\sigma_i-\sigma_j}{h_\sigma}\right)^2\right)\right]\right\} \quad (6)$$

The solution of problem (6) allows determination of all parameters that are included in (5) and, thus, reconstruction of the function $f_\sigma(\sigma)$.

For a kernel function with a normal kernel, a close-to-optimal value of the parameter is defined from the dependence

$$h_\sigma=D_\sigma\cdot m^{-\frac{1}{5}} \quad (7)$$

where D_σ is the sample variance that is calculated on the basis of the available sample of values $\sigma_i,\ i=\overline{1,m}$:

$$D_\sigma^2=\frac{1}{m-1}\sum_{i=1}^{m}\left(\sigma_i-\frac{1}{m}\sum_{i=1}^{m}\sigma_i\right) \quad (8)$$

Problem 2: Generation of a random-quantity sample in accordance with a known DDF.

This algorithm is a nonparametric generator of a random quantity. Let there be a random-quantity sample $s_j,\ j=\overline{1,n}$, on whose basis the DDF $f_s(s)$ is defined. As an example, let us assume that the random quantity obeys the left ($s_{\max}=\max_j\{s_j\}$) and right ($s_{\min}=\min_j\{s_j\}$) censored Gram-Charlier law:

$$f_s(s)=\frac{1}{\lambda_2\sqrt{2\pi}}\exp\left[-\frac{(u_s)^2}{2}\right]\times \quad (9)$$

$$\times\left\{1+\frac{\lambda_3}{6}\left[(u_s)^3-(u_s)\right]-\frac{\lambda_4}{24}\left[(u_s)^4-5(u_s)^2+3\right]\right\}\cdot\frac{1}{c_s}$$

where

$$c_s=\int_{s_{\min}}^{s_{\max}}\left\{\frac{1}{\lambda_2\sqrt{2\pi}}\exp\left[-\frac{(u_s)^2}{2}\right]\times\right.$$

$$\left.\times\left\{1+\frac{\lambda_3}{6}\left[(u_s)^3-(u_s)\right]-\frac{\lambda_4}{24}\left[(u_s)^4-5(u_s)^2+3\right]\right\}\right\}ds$$

$u_s=\dfrac{s-\lambda_1}{\lambda_2}$; λ_1 and λ_2 are the mean value and the standard deviation of the random quantity s, respectively; $\lambda_3=\frac{1}{n-1}\sum_{j=1}^{n}(s_j-\lambda_1)^3$ and $\lambda_4=\frac{1}{n-1}\sum_{j=1}^{n}(s_j-\lambda_1)^4-3$ are, respectively, the asymmetry and excess for the random quantity s.

Let us consider the algorithm for extending the sample s_j to a length $m>n$.

Let us specify a random quantity V with a normal distribution law. To obtain the random quantity s with the distribution function $F_s(s)$, it is necessary to use the equation [13]:

$$F_s(s)=V \quad (10)$$

Because $F_s(s)=\int_0^s f_s(s)ds$, on the basis of dependence (9), we obtain

$$F_s(s)=P_s(s)/c_F\ , \quad (11)$$

Where $c_F=\int_{s_{\min}}^{s_{\max}}P_s(s)ds$; $u_s=\dfrac{s-\lambda_1}{\lambda_2}$;

$$P_s(s)=\frac{1}{\lambda_2\sqrt{2\pi}}\int_{s_{\min}}^{s}\exp(-0{,}5u_s^2)ds-\frac{\lambda_3}{6}\left[u_s^2-1\right]\frac{1}{\lambda_2\sqrt{2\pi}}\times$$

$$\times\exp\left(-0{,}5u_s^2\right)+\frac{\lambda_4}{24}\left(u_s^3+3u_s\right)\frac{1}{\lambda_2\sqrt{2\pi}}\exp\left(-0{,}5u_s^2\right)$$

By solving transcendent equation (10) at a fixed value of the random quantity $V=const$ from the range [0, 1], we determine a new value of the random quantity s with the DDF $f_s(s)$. This procedure is repeated, and the sample s is extended to the required size.

The algorithm for generating a sample of a random quantity that, e.g., has the DDF in the form of (5) is constructed quite analogously. This algorithm is called the nonparametric random-number generator [13, 14,15].

IV. Computer Experiments Realizing Developed Approach

Example 1. It is required to determine the probability of a failure of a pipe that is exposed to an internal pressure and a temperature during operation. The pipe diameter is 1420 mm, its wall thickness is 16.5 mm, the pipe material is 17GS steel, and the permissible stresses for the pipe material obey a normal distribution law.

In order to reconstruct the DDF $f_\sigma(\sigma)$, samples of the pressure $p_i,\ i=\overline{1,m}$ and temperature

t_i , $i = \overline{1, m}$, which were registered every day during a year ($m = 365$), were used. By realizing the algorithm of the *auxiliary problem* 2, we reconstructed the pressure and temperature DDFs $f_p(p)$ and $f_t(t)$) (see Fig. 2).

For each pair of values p_i ,t_i, $i = \overline{1, m}$ calculations of the effective stress σ_i in the pipe were performed and allowed us to obtain the sample σ_i , $i = \overline{1, m}$, using which the DDF for the acting stresses $f_\sigma(\sigma)$ was reconstructed.

The function $f_s(s)$ for steel 17GS is taken in the form (9) with the following parameters: λ_1 = 570.9 MPa, λ_2 = 19.3 MPa, λ_3 = 0.1480, λ_4 = 0.0209, $S_{\min}$ = 530 MPa, and $S_{\max}$ = 600 MPa. The graphic illustration of the functions $f_\sigma(\sigma)$ and $f_s(s)$ is shown in Fig. 3, from which it follows that, in this case, the probability of a failure, i.e., the solution of problem (2), is zero (Q = 0).

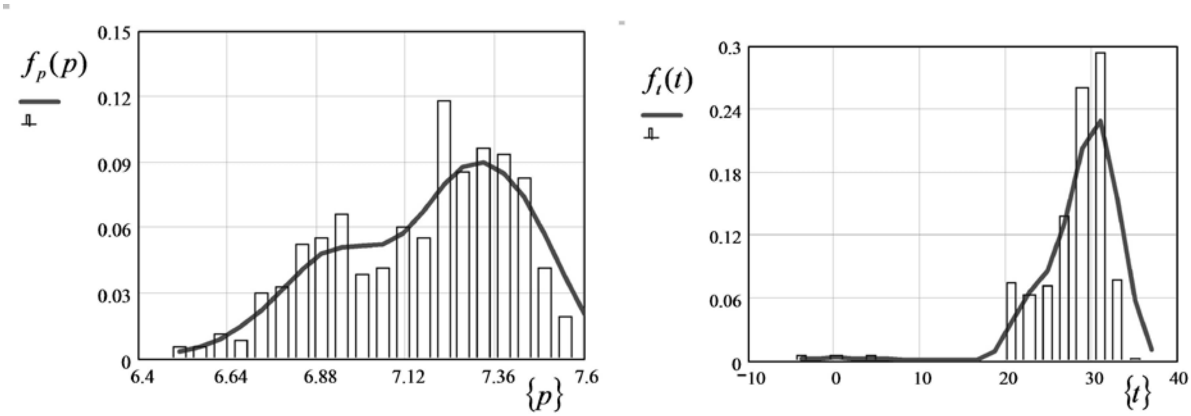


Fig. 2 : Functions $f_p(p)$ and $f_t(t)$.

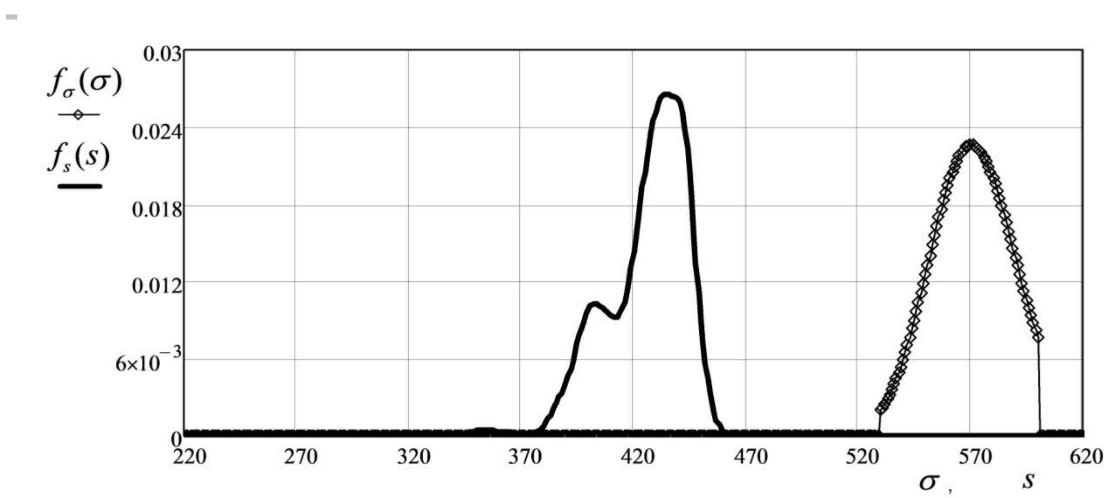


Fig. 3 : Functions $f_\sigma(\sigma)$ and $f_s(s)$

Example 2. For the data of example 1, it is required to determine quantile estimates of the safety margin.

Let us use the sample σ_i , $i = \overline{1, m}$. For the known function $f_s(s)$, the sample s_i , $i = \overline{1, m}$ is obtained using the algorithm of *auxiliary problem 2*. If the values of σ_i and s_i are known, formula (4) is used to calculate the sample $n_{\sigma i}$, $i = \overline{1, m}$. By realizing the algorithm of auxiliary problem 1, we determine the density function $f_n(n_\sigma)$. This function is shown in Fig. 4.

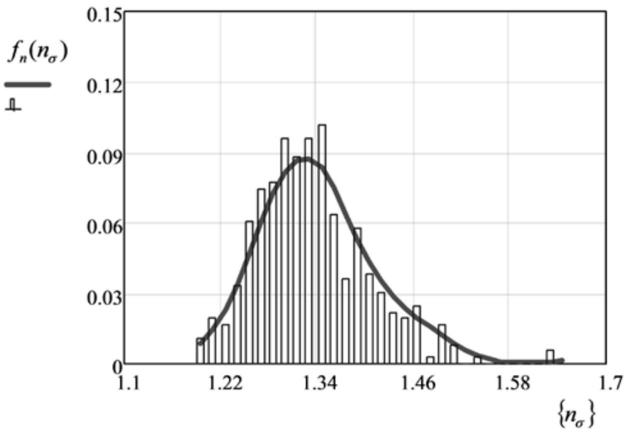


Fig. 4 : Density function $f_n(n_\sigma)$

In order to calculate quantile ($\alpha = 0{,}01;\ 0{,}05;\ 0{,}50$) estimates of the safety margin (n_σ^α), it is necessary to solve equation (4). The resulting values are $n_\sigma^{0,01}$ =1,19417; $n_\sigma^{0,05}$ =1,23122; $n_\sigma^{0,50}$ =1,33403. If it is required to estimate the probability that the safety margin is <1.2, it is sufficient to calculate the integral

$$Pr[n_\sigma \le 1{,}2] = \int_0^{1,2} f_n(n_\sigma)dn_\sigma = 0{,}01348. \qquad (12)$$

Example 3. It is required to determine the no-failure operation of a helical gearing. The torque at the pinion gear (T_{1H}^* , Н·м) changes in accordance with the β -distribution (heavy-duty operation), in accordance with the γ -distribution (light-duty operation), and according to a bimodal law.

The dependence for calculating the contact stresses σ_H (MPa) that act in the engagement of teeth of helical gearings has the form

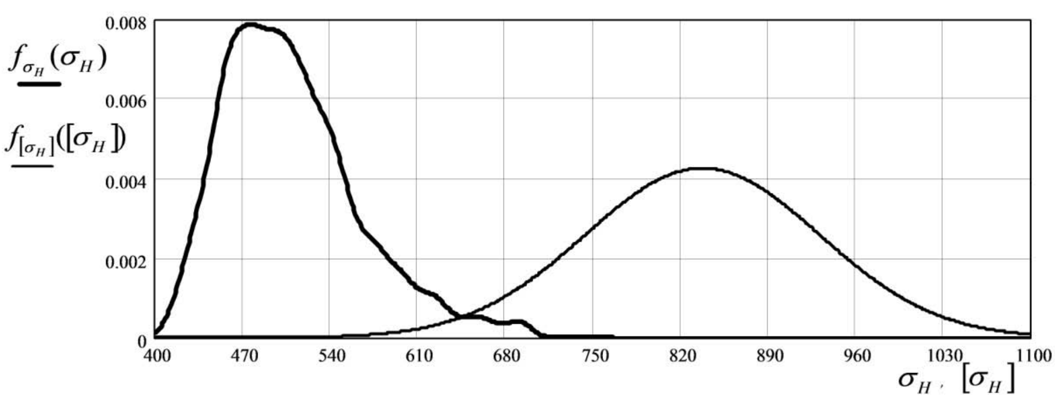
$$\sigma_H = 6{,}13 \cdot 10^3 \cdot Z_H \frac{1}{a_W} \sqrt{\frac{T_{1H}^*}{b_W} \cdot \frac{(u+1)^3}{u} \cdot K_{H\Sigma}} \ , \qquad (13)$$

where Z_H is the coefficient that accounts for the shapes of the mated surfaces; a_W is the interaxial distance of the helical gearing (mm); b_W is the working width of the gear rim (mm); u is the gear ratio; and $K_{H\Sigma}$ is the load factor, which is related to T_{1H}^* via a nonlinear dependence.

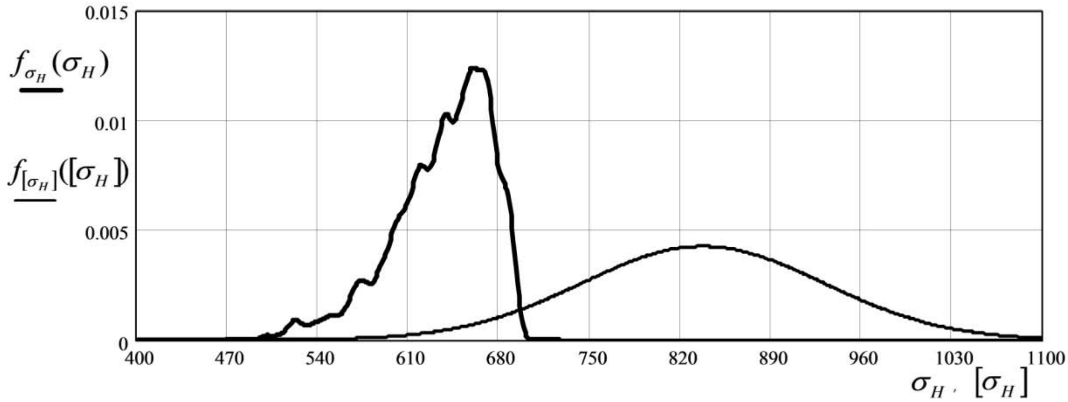
It follows from (13) that the dependence of σ_H on T_{1H}^* is essentially nonlinear. Thus, even if the random quantity T_{1H}^* obeys a normal law, the law of σ_H distribution of cannot be determined.

The results of calculating the DDFs for the actual σ_H and permissible $[\sigma_H]$ stresses (a normal distribution law for $[\sigma_H]$ was adopted in the calculations) using the above-considered algorithms are presented in Fig. 5.

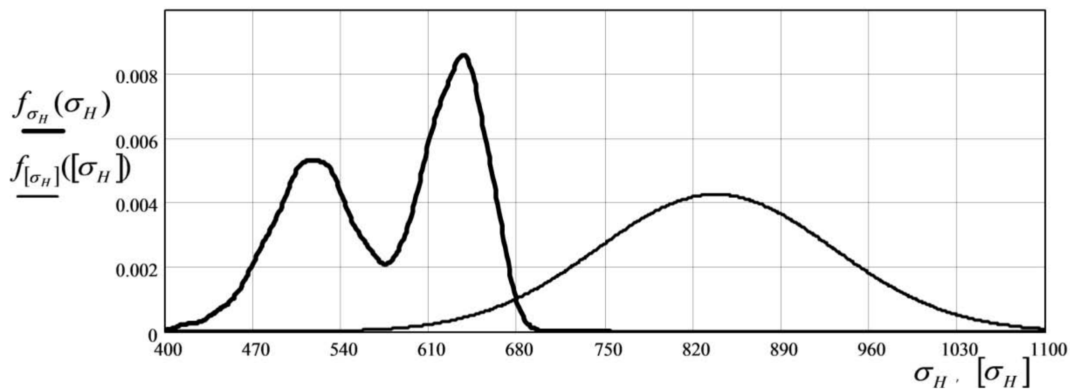
The gearing parameters are as follows: the number of teeth of the gear Z_1 = 32, the number of teeth of the wheel Z_2 = 64, the coefficient of displacement of a gear tooth χ_1 = 0, the coefficient of displacement of a wheel tooth χ_2 = 0, the pitch m = 5 mm, the width of the gear rim b_W = 60 mm, and the tilt angle of the tooth trace β = 16°15’. For light-duty operation, heavy-duty operation, and torque changes according to a bimodal law, the probabilities of no-failure gearing service are 0.9980, 0.9780, and 0.9911, respectively.



a) light-duty operation



b) heavy-duty operation



c) the torque changes according to a bimodal law

Fig. 5 : Distribution density functions σ_H and $[\sigma_H]$

V. Results and Discussion

In the conventional approach to the solution of the considered problems for each random quantity using the fitting criteria (chi-square, omega-square, Kolmogorov-Smirnov), a distribution law must be selected. However, this law can be adopted only with a certain probability. The value of this probability is not a priori known. In this case, there is a risk of adopting a distribution law that is actually not realized (error of the second kind). Thus, the reliability of the result of solving the problem is an uncertain value.

The use of methods of nonparametric statistics for solving problems makes it possible to eliminate the aforementioned uncertainty.

VI. Conclusion

The approach considered in this study and the mathematical apparatus for calculating the probability of no-failure operation or a failure and quantile estimates of the safety margin of machine components and structures is universal. It allows estimation of the strength reliability of articles regardless of the complexity of the laws of distribution of random values of actual and limiting stresses.

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