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A New Expression of the Curve S-N in Fatigue based on the Concept of the "Weakest Link" of Weibull

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A New Expression of the Curve S-N in Fatigue based on the Concept of the "Weakest Link" of Weibull

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I. Introduction

The static essay, serving to determine the ultimate resistance to break, may be considered as a very particular case of a essay of fatigue, the number of cycles with break was equal at the most to 1. This ultimate resistance with break is the minimal value in which the break of the material occurs in a certain way.

The problem of dispersal of the trial results, as well in statics (for the determination of the ultimate resistance with break) that in fatigue (for the determination of the number of cycles with break or for the determination endurance limit) is the consequence of the structural heterogeneity of the defects since the elaboration of material [1]-[11]. The density of these defects vary from specimen to another, yet all taken from the same sample. In the case of fatigue, for a given stress level σ_i, one can also observe a dispersion value of the number of cycles to failure N_i [12]-[13]. The number of cycles to failure is related to the material structure and thus the defects that contain. That these defects are difficult to measure, it is advisable to use as a random variable [12], for a given stress level, the number of cycles to failure to explain the existence of defects by a probabilistic approach. We rely on the concept of the "weakest link" Weibull to formulate a new expression of the SN curve can take into account the actual volume of a flawless material considered. In the absence of a means of assessing defect in the structure, the results for two types of materials are comparable with those of Wöhler and Basquin. A simulation made by the variation of the tensile strength of a material that shows this new expression for describing the fatigue behavior of a hardened material.

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II. Concept of "Weakest Link" Weibull

According to this concept, for juxtaposed volumes, subjected to monotonic loading, the failure of one is independent of the other, if one of the volumes fails, the system is considered failed [14]. The survival probability of a system consisting of n elements (or volume) is given by:

$$R_n = 1 - e^{-\int f dv} \tag{1}$$

Where f is the probability density of failure and dv is the elementary volume constituting the reference volume V.

III. Specification of the Probability Density Function f

We will consider as a random variable the number of cycles to failure for a given stress level by the fact that the number of cycles determines the fatigue. Dependent on the dispersion, the number of cycles is a variable representing statistically the stress applied from the viewpoint fatigue (damage to a given level of stress). We will consider the failure probability density f as a function of number of cycles to failure; it is given in the following form:

$$f(N) = \beta N^m \tag{2}$$

Where β is a parameter to be determined, and the module m, within the meaning of Weibull [14].

The parameter β can be determined when we know a characteristic value of the number of cycles N_c to a level corresponding stress. At this number of cycles N_c characteristic density of default probability take the value 1. It then pulls:

$$\beta = N_c^{-m} \tag{3}$$

or,

$$f(N) = \left(\frac{N}{N_c}\right)^m \tag{4}$$

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In this case, the probability of survival given equation (1) becomes:

$$R_n = 1 - e^{-\int \left(\frac{N}{N_C}\right)^m .dv} \quad (5)$$

IV. Proposal for a New Expression of the S-N Curve

It is obvious that there is a relationship between the probability of failure P (function the number of cycles) and the level of responsible of this fatigue failure stress. We admit that there is a function g dependent on the characteristics of fatigue σ^D and static R_m linking the probability of failure P stress responsible for this failure.

Since σ^D and R_m are constants for a given material, then g is a constant function. A simple relation linking the stress at failure probability is that the stress is proportional to the probability of failure; with as the proportionality coefficient function g. This relationship is expressed by :

$$\sigma = g(\sigma^D, R_m) P(N) = g(\sigma^D, R_m)[1 - R(N)] \quad (6)$$

ie, from (5):

$$\sigma = g(\sigma^D, R_m) e^{-\int \left(\frac{N}{N_C}\right)^m .dv} \quad (7)$$

For a tension loading when the load is uniform, the expression (7) becomes :

$$\sigma = g(\sigma^D, R_m) e^{-V\left(\frac{N}{N_C}\right)^m} \quad (8)$$

Thus, two limiting cases are possible according to the value of the stress σ:

- For $\sigma = R_m$, ultimate resistance to break, N = 0, the relation (8) becomes:

$$R_m = g(\sigma^D, R_m) \quad (9)$$

- For $\sigma = s^D$, endurance limit, $N = N^D$, the relation (8) becomes :

$$\sigma^D = g(\sigma^D, R_m) e^{-V\left(\frac{N^D}{N_C}\right)^m} \quad (10)$$

ie,

$$g(\sigma^D, R_m) = \sigma^D e^{V\left(\frac{N^D}{N_C}\right)^m} \quad (11)$$

The equality of two expressions g, given by equations (9) and (11) gives R_m depending on σ^D

$$R_m = \sigma^D e^{V\left(\frac{N^D}{N_C}\right)^m} \quad (12)$$

The expression of the S-N curve in tension given by equation (8) becomes :

$$\begin{cases} \sigma = R_m e^{-V\left(\frac{N}{N_C}\right)^m} \\ \text{ie} \\ \sigma = \sigma^D e^{V\left\{\left(\frac{N^D}{N_C}\right)^m - \left(\frac{N}{N_C}\right)^m\right\}} \end{cases} \quad (13)$$

V. Experimental Study of the New Expression

To validate the sensitivity of the proposed fatigue behavior of a material model for V = 1, we have experimental data from two types of materials: steel martensite P220 and 100C6. A comparative study is made, from these data, with models of the most utilized S-N curves [15]-[16] such that Basquin, the Wöhler and the Stromeyer :

$$\begin{cases} \log(N) = -a_B \log(\sigma) + b_B & \text{: Basquin} \\ \log(N) = -a_W \sigma + b_W & \text{: Wöhler} \\ \log(N) = -a_S \log(\sigma - \sigma^D) + b_S & \text{: Stromeyer} \\ \log(N) = a_N \log\left\{\log\left(\frac{R_m}{\sigma}\right)\right\} + b_N & \text{: proposed} \end{cases} \quad (14)$$

Where, for the proposed model $\begin{cases} a_N = \frac{1}{m} \\ b_N = \log(N_C) \end{cases}$

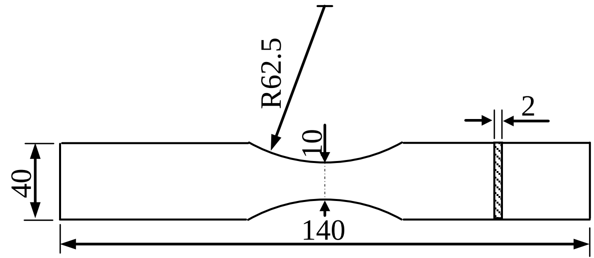
The curves parameters are determined for each fatigue testing each of the two steels result, by the method of least square.

a) Test Data

i. *P220 Steel*

The data are taken from the steel work performed LaMI (Laboratory of Mechanics and Engineering) in order to investigate the influence of cutting processes on the fatigue behavior of parts. Two procedures were used, laser and shears, it, with several execution qualities (good, standard and bad) corresponding to different sets of machines settings. Table.1 gives the geometry of the specimen and the fatigue test at constant amplitude tension-compression alternating statement.

Table 1 : Specimen geometry and fatigue testing of the P220 cut steel with the standard laser process result

Material: P220 Type of sollicitation: tension-compression at constant amplitude Solicitation frequency: 20Hz													
Dimensions and shape of the specimen													
													
□ (MPa)	344	321,8	319,9	319,8	311,5	311,1	310	309,5	290	280	275	270	235
N	10596,5	296828	281248	320467	371162	305503	204238	309191	520089	662397	3909947	5335707 (*)	7522596 (*)
(*) no rupture													

ii. *Martensite Steel 100C6*

The tableaux.2 and 3 below give, respectively, the chemical composition and the fatigue test result of the 100C6 steel [11].

Table 2 : Chemical composition (% by mass) of the martensite steel 100C6

Martensite steel 100C6								
Si	Mn	P	S	Cr	Cu	Ni	Mo	O
0,242	0,339	0,012	0,008	1,461	-	0,147	0,032	-

Table 3 : Fatigue test of martensite steel 100C6 result

Matereal: martensite steel 100C6 (E=210GPa, R_m=2300MPa, σ^D=850MPa et $N^D=6.0265x10^9$cycles). Solicitation frequency: 20KHz												
Fatigue test results												
σ (MPa)	975	960	950	940	930	900	890	880	870	860	850
Nmoy. $x10^6$	0,1183	0,1019	0,1639	9,806	167	466	664,4	542,1	990,75	1853,3	6026,5

The value of the ultimate resistance to break R_m of the steel P220 is between 600 and 800 MPa, it will take three values in this range for the simulation thus allowing to highlight the sensitivity of the proposed model.

b) Comparison

The comparison will be made by tracing curves in the plane (log (σ), log (N))). Parameter values, determined by the method of least squares, corresponding to different models, given by Equations (14), for the two steels, are given in the following Table.4 :

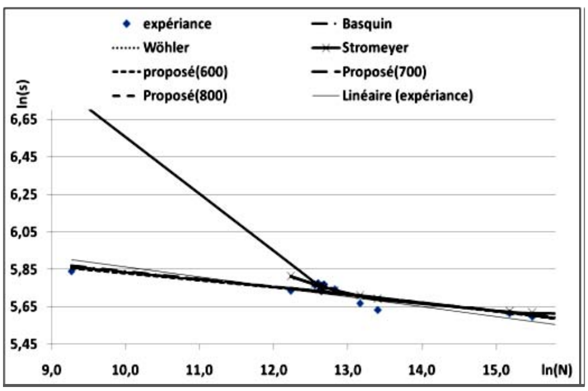
Table 4 : Value of parameters of the models used for the two steels

P220 steel	
Model	Parameters
Basquin	a_B=23.66 ; b_B=148.2
Wöhler	a_W=0.078 ; b_W=36.65
Stromeyer	a_S=1.52 ; b_S=17.73
Proposed Model	R_m=600MPa; σ^D=271.24MPa; m=0.062; N_c=214037564;
	a_N=16.13; b_N=19.18
	R_m=700MPa; σ^D=271.57MPa; m=0.051; N_c=15622128; a_N=19.61; b_N=16.56

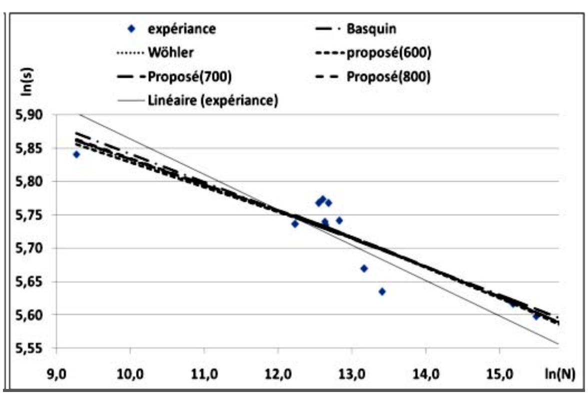
	R_m=800MPa ; σ^D=271.76MPa ; m=0.044; N_c=925359 ;a_N=22.73; b_N=13.74
Matensite steel 100C6	
Model	Parameters
Basquin	a_B=94.2 ; b_B=659.3
Wöhler	a_W=0.10 ; b_W=111.5
Stromeyer	a_S=4.02 ; b_S=32.2
Proposed Model	R_m=2300MPa ; σ^D=863MPa; m=0.0115; N_c=3.47x10^{10} ; a_N=86.96; b_N=24.27

VI. Results and Discution

- P220 steel



(a)



(b)

a) The four models and the linear trend of the experiment;

b) Models Basquin of Wölher those proposed and the linear trend of the experience.

Figure 1 : SN curves in the plane (ln (σ) ln (N)) of the models studied for P220 steel

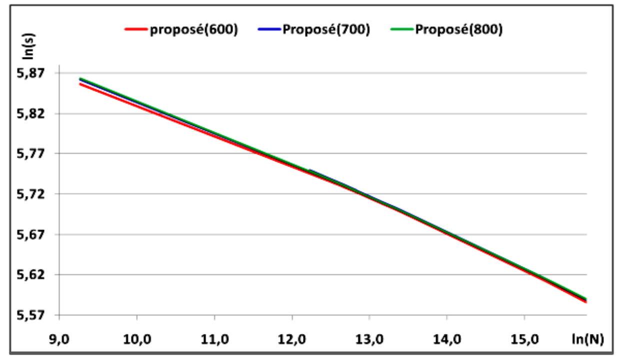
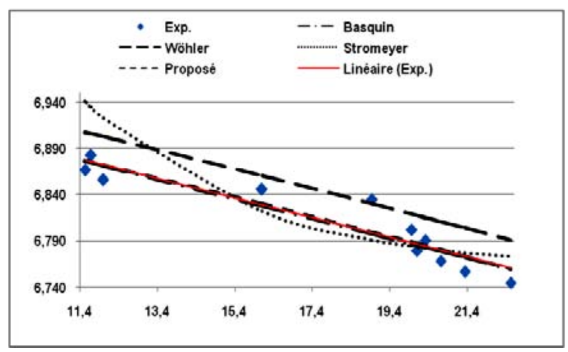


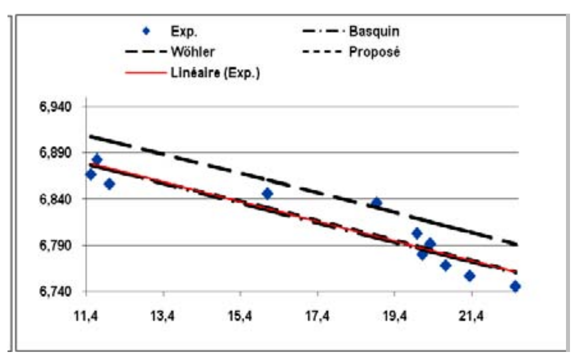
Figure 2 : Sensitivity of proposed model with the values of R_m

Here we can see clearly that the SN curve model Stromeyer is not linear and strongly deviates from the experimental curve in the oligocyclic domain (Figure.1.a). Basquin, Wöhler curves and that proposed for different values of R_m approximate linear trend given by experience. We find a correlation between the last three curves (Figure.1.b), the relative error on the stress values provided for by these curves do not generally exceed 5% compared to the values of the experiment.

- 100C6 steel



(a)



(b)

a) The four models and the linear trend of the experiment;

b) models Basquin of Wölher, he proposed and the linear trend of the experiment.

Figure 3 : SN curves in the plane (ln (σ) ln (N)) of the models studied, for steel 100C6

Basquin models and the proposed are approaching significantly from the linear trend of the experiment while that of Stromeyer does not follow this trend for large values of the constraint (figure.3.a). Wohler model, meanwhile, is shifted in translation relative to this trend (figure.3.a and b). The relative error on the values of constraint provided by the curves Basquin of Wölher and that proposed not exceeding 5% there too compared with the values of the experiment.

VII. Conclusion

The sensitivity of the proposed model to the values of R_m, is reflected in an angular shift in direction of rotation hands of a watch while R_m increases (figure.2). The limit of σ^D fatigue increases substantially, but the value of the number of cycles N_c dimunie characteristic significantly well as the modulus value m while R_m increases, the appreciable increase in the endurance limit with that of the breaking strength reflecting the inflection the S-N curve traced in the plane (σ; N), thus modifying the scope of limited field of fatigue.

The proposed model introduces a new parameter, the number of cycles characteristic N_c for which the failure probability density function takes the value 1and therefore the smallest number of cycles which the rupture occurs with certainty. It takes also into account the volume of the material. All the calculations are done by taking V=1. To take into account the defects can be considered as a fraction V corresponds to useful volume of the material constituting the piece to explain the variations of cycles number to failure for a given level of stress. The model allows, given its sensitivity to the change R_m, to track the fatigue behavior of work hardened materials, a phenomenon for which the value of its breaking strength R_m varies notably.

VIII. Acknowledgement

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Bibliography

1. Eric S. Lindquist. Strength of materials and the Weibull distribution. Probability Engineering Mechanics 9 (1994) 141-194.
2. Jason P. Petit, Robert H. Dodds Jr. Calibration of the Weibull stress scale parameter, σ_u using the Master curve. Engineering Fracture Machanics 72 (2005) 91-120.
3. G. Celeux, M. Porsz, J.N. Wandji, F. Perrot. Unsing Markov chain Monte Carlo methods to solve full Bayesian modeling of PWR vessel flow distributions. Reability Engineering and System Safety. 66 (1999) 243-252.
4. M.T. Todinov. Equation and a fast algorithm for determining the probability of failure intiated by flows. International Journal of Solid and Structures xxx (2002) xxx-xxx.
5. M.T. Todinov. Limiting of probability of failure for composants containing flows, Computational Materials Sciences (2005) 156-166.
6. M. Unosson, L. Olovsson, K. Simonsson. Failure modeling in finite elements analysis: Random material imperfections. Mechanics of Material 37 (2005) 1175-1179.
7. Robert Danzer. Some notes on the correlation between fracture and defect statistic: Are Weibull statistic valid for very small specimens? Journal of the European Ceramic Scociety.27 (2006) 3043-3049.
8. Martin Poncelet. Multiaxialité, hétérogénéité intrinsèque et structural des essais d'auto-échauffement et de fatigue à grnad nombre de cycles. Thèse. ENS Cachan/CNRS/Université de Paris 6. 2007-2008.
9. Martin Poncelet, Cédric Doudard, Sylvain Calloch, Francois Hild, Bastien Weber, André Galtier. Prediction of self-hearting measurements under propositional and non-proportinal multiaxial cyclic loadings. C. R. Mecanique 335 (2007) 81-86.
10. Cédric Doudard, M. Poncelet, S. Calloch, C. Boue, F. Hild, A. Galtier. Determination of an HCF criterion by thermal measurements under biaxial cycle loading.International Journal of Fatigue. 29 (2007) 748-757.
11. Gonzalo M. Dominguez Almaraz. Prediction of very high cycle fatigue failure for high strength steels, based on the inclusion geometrical properties. Mechanics of Material 40 (2008) 636-640.
12. B. Soh Fotsing, E. maronne, N. Nadjitonon, J.-L. Robert. Intégration d'une démarche fiabiliste dans l'exploitation des critères de fatigue multiaxiaux. 23ème Journée de Printemps de la Commission de Fatigue. Du 25 au 26 mai 2004.
13. Bastien WEBER. Fatigue multiaxiale des structures industrielles sous chargement quelconque. Thèse INSA Lyon, N) 99ISAL0056, 1999, 248p.
14. W. Weibull. A statical theory of the strength of materials. Ingeniors Vetenskaps Akademiens. Hamdlingar NR 151, 1939.
15. B. Tikri, N. Nadjitonon, J. L. Robert. Nouvelle loi non linéaire d'endommagement par fatigue basée sur la courbe de Bastenaire-IUT de Montluçon-Département Génie Mécanique et Productique-

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16. Ngargueudedjim K., Contribution à l'étude des lois d'endommagement en fatigue, Thèse de doctorat de l'INSA de Lyon N° 03 ISAL 0025, 2003.