Kinetic Induktance Charges and its Role in Classical Electrodynamics

By F. F. Mende

B.I. Verkin Institute, Ukraine

Abstract- The dielectric and magnetic constant of material media are the parameters, which are used during writing Maksvell equations. However, there is still one not less important material parameter, namely a kinetic inductance of charges, which has not less important role, than the parameters indicated. Unfortunately, importance and fundamentality of this parameter in the works on electrodynamics, until now, is not noted, and kinetic inductance is present in all equations of electrodynamics implicitly. This work is dedicated to the examination of the role of the kinetic inductance of charges in the electrodynamics of material media and to the restoration of its rights as the fundamental parameter, on the importance that not less meant than dielectric and magnetic constant.

Keywords: electrodynamics; maxwell equation; kinetic inductance of charges; dielectric constant; magnetic permeability.

GJRE-J Classification : FOR Code: 020302
Kinetic Inductance Charges and its Role in Classical Electrodynamics

F. F. Mende

Abstract: The dielectric and magnetic constant of material media are the parameters, which are used during writing Maxwell equations. However, there is still one not less important material parameter, namely a kinetic inductance of charges, which has not less important role, than the parameters indicated. Unfortunately, importance and fundamentality of this parameter in the works on electrodynamics, until now, is not noted, and kinetic inductance is present in all equations of electrodynamics implicitly. This work is dedicated to the examination of the role of the kinetic inductance of charges in the electrodynamics of material media and to the restoration of its rights as the fundamental parameter, on the importance that not less meant than dielectric and magnetic constant.

Keywords: electrodynamics; maxwell equation; kinetic inductance of charges; dielectric constant; magnetic permeability.

I. INTRODUCTION

In the existing scientific literature occurs only the irregular references about the kinetic the inductance of charges [1-3]. the most in detail physical essence of this parameter and its place in the description of electrodynamic processes in the conductors is examined in work [4]. In this work is introduced also the concept of surface kinetic and field inductance, which earlier was not introduced:

\[ L_K = \frac{1}{\omega |\mathbf{H}_T(0)|^2} \text{Im} \int_0^\infty \tilde{j}^* \mathbf{E} \, dz, \]

\[ L_H = \frac{1}{|\mathbf{H}_T(0)|^2} \int_0^\infty |\mathbf{H}_T|^2 \, dz, \]

Where \( L_K \) and \( L_H \) - surface kinetic and field inductance, \( \mathbf{E} \) - the tension of electric field, \( \tilde{j}^* \) - the complexly conjugate value of current density, \( \mathbf{H}_T \) - tension of magnetic field; \( \mathbf{H}_T(0) \) - the value of the tension of magnetic field on the surface; \( \omega \) - frequency of the applied field. These relationships are valid for the case of the arbitrary connection between the current and the field in the metals and the superconductors, and they reveal the physical essence of surface kinetic and field inductance.

a) Plasmo-like media

The equation of motion of electron takes the following form:

\[ m \frac{d\tilde{v}}{dt} = e\mathbf{E}, \]

where \( m \) - mass electron, \( e \) - electron charge, \( \mathbf{E} \) - tension of electric field, \( \tilde{v} \) - speed of the motion of charge. In the work [2] it is shown that this equation can be used also for describing the electron motion in the hot plasma.

Using an expression for the current density

\[ \tilde{j} = ne\tilde{v}, \]

from (1.1) we obtain the current density of the conductivity of the free electrons

\[ \tilde{j}_L = \frac{ne^2}{m} \int \mathbf{E} \, dt \]

in relationships (2.2) and (2.3) the value \( n \) represents the specific density of charges. After introducing the designation

\[ L_k = \frac{m}{ne^2}, \]

we find

\[ \tilde{j}_L = \frac{1}{L_k} \int \mathbf{E} \, dt \]

In this case the value of \( L_k \) presents the specific kinetic inductance of charge carriers [4-7]. Its existence connected with the fact that charge, having a mass, possesses inertia properties.

Pour on relationship (1.5) it will be written down for the case of harmonics:

\[ \tilde{j}_L = -\frac{1}{\omega L_k} E_0 \cos \omega t \]
For the mathematical description of electrodynamic processes the trigonometric functions will be here and throughout, instead of the complex quantities, used so that would be well visible the phase relationships between the vectors, which represent electric fields and current densities. From relationship (1.5) and (1.6) is evident that presents inductive current, since its phase is late with respect to the tension of electric field to the angle.

During the presence of summed current it is necessary to consider bias current
\[ j_{\Sigma} = e_0 \frac{\partial \vec{E}}{\partial t} = e_0 \vec{E}_0 \cos \omega t \]

Is evident that this current bears capacitive nature, since, its phase anticipates the phase of the tension of electrical to the angle \( \frac{\pi}{2} \). Thus, summary current density will compose
\[ j_{\Sigma} = e_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \int \vec{E} \, dt \]

or
\[ j_{\Sigma} = \left( \omega e_0 - \frac{1}{\omega L_k} \right) \vec{E}_0 \cos \omega t \quad (1.7) \]

Introducing the plasma frequency \( \omega_0 = \sqrt{\frac{1}{L_k e_0}} \), relationship (1.7) it is possible to rewrite
\[ j_{\Sigma} = \omega e_0 \left( 1 - \frac{\omega_0^2}{\omega^2} \right) \vec{E}_0 \cos \omega t \quad (1.8) \]

If in the conductor are ohmic losses, then total current density determines the relationship
\[ j_{\Sigma} = \sigma \vec{E} + e_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \int \vec{E} \, dt \]

where \( \sigma \) - conductivity of metal.

### Dielectrics

In the existing literature there are no indications that the kinetic inductance of charge carriers plays some role in the electrodynamic processes in the dielectrics. However, this not thus [7]. This parameter in the electrodynamics of dielectrics plays not less important role, than in the electrodynamics of conductors.

Let us examine the simplest case, when oscillating processes in atoms or molecules of dielectric obey the law of mechanical oscillator.
\[ \left( \frac{\beta}{m} - \omega^2 \right) \vec{r}_m = \frac{e}{m} \vec{E} \quad (2.1) \]

Where \( \vec{r}_m \) - deviation of charges from the position of equilibrium, \( \beta \)-coefficient of elasticity, which characterizes the elastic electrical binding forces of charges in the atoms and the molecules. Introducing the resonance frequency of the bound charges of \( \omega_0 = \beta / m \)

we obtain from (2.1):
\[ \vec{r}_m = -\frac{e}{m \left( \omega^2 - \omega_0^2 \right)} \vec{E} \quad (2.2) \]

is evident that in relationship (2.2) as the parameter is present the natural vibration frequency, into which enters the mass of charge. This speaks, that the inertia properties of the being varied charges will influence oscillating processes in the atoms and the molecules.

Since the general current density on Wednesday consists of the bias current and conduction current
\[ \text{rot} \vec{H} = \vec{j}_\Sigma = e_0 \frac{\partial \vec{E}}{\partial t} + ne \vec{v} \]

that, finding the speed of charge carriers in the dielectric as the derivative of their displacement through the coordinate
\[ \vec{v} = \frac{\partial \vec{r}_m}{\partial t} = -\frac{e}{m \left( \omega^2 - \omega_0^2 \right)} \frac{\partial \vec{E}}{\partial t} \]

from relationship (2.2) we find
\[ \text{rot} \vec{H} = \vec{j}_\Sigma = e_0 \frac{\partial \vec{E}}{\partial t} - \frac{1}{L_{kd} \left( \omega^2 - \omega_0^2 \right)} \frac{\partial \vec{E}}{\partial t} \quad (2.3) \]
But the value

\[ L_{kd} = \frac{m}{n e^2} \]

presents the kinetic inductance of the charges, entering the constitution of atom or molecules of dielectrics, when to consider charges free. Then relationship (2.3) can be rewritten

\[ \text{rot}\bar{H} = \jmath_{\Sigma} = \varepsilon_0 \left( \frac{1}{\varepsilon_0 L_{kd} (\omega^2 - \omega_0^2)} \right) \frac{\partial \bar{E}}{\partial t} \]  \hspace{1cm} (2.4)

But, since the value

\[ \frac{1}{\varepsilon_0 L_{kd}} = \omega_{pd}^2 \]

represents the plasma frequency of charges in atoms and molecules of dielectric, if we consider these charges free, then relationship (2.4) takes the form:

\[ \text{rot}\bar{H} = \jmath_{\Sigma} = \varepsilon_0 \left( 1 - \frac{\omega_{pd}^2}{(\omega^2 - \omega_0^2)} \right) \frac{\partial \bar{E}}{\partial t} \]  \hspace{1cm} (2.5)

Let us examine two limiting cases:

- If \( \omega = \omega_0 \), then from (2.5) we obtain

\[ \text{rot}\bar{H} = \jmath_{\Sigma} = \varepsilon_0 \left( 1 + \frac{\omega_{pd}^2}{\omega_0^2} \right) \frac{\partial \bar{E}}{\partial t} \]  \hspace{1cm} (2.6)

In this case the coefficient, confronting the derivative, does not depend on frequency, and it presents the static dielectric constant of dielectric. As we see, it depends on the natural frequency of oscillation of atoms or molecules and on plasma frequency. This result is intelligible. Frequency in this case proves to be such small that the inertia properties of charges it is possible not to consider, and bracketed expression in the right side of relationship (1.7) presents the static dielectric constant of dielectric. Hence immediately we have a prescription for creating the dielectrics with the high dielectric constant. In order to reach this, should be in the assigned volume of space packed a maximum quantity of molecules with maximally soft connections between the charges inside molecule itself.

- The case, when \( \omega \approx \omega_0 \), is exponential. Then

\[ \text{rot}\bar{H} = \jmath_{\Sigma} = \varepsilon_0 \left( 1 + \frac{\omega_{pd}^2}{\omega_0^2} \right) \frac{\partial \bar{E}}{\partial t} \]

and dielectric is converted in the plasma. The obtained relationship coincides with the case of plasma (1.8).

Now it is possible to examine the question of why dielectric prism decomposes polychromatic light into monochromatic components or why rainbow is formed. So that this phenomenon would occur, it is necessary to have the frequency dispersion of the phase speed of electromagnetic waves in the medium in question. If we to relationship (2.5) add the first equation of Maxwell, then we will obtain:

\[ \text{rot}\vec{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \]

\[ \text{rot}\bar{H} = \varepsilon_0 \left( 1 - \frac{\omega_{pd}^2}{(\omega^2 - \omega_0^2)} \right) \frac{\partial \bar{E}}{\partial t} \]  \hspace{1cm} (2.7)

That we will obtain the wave equation

\[ \nabla^2 \bar{E} = \mu_0 \varepsilon_0 \left( 1 - \frac{\omega_{pd}^2}{\omega^2 - \omega_0^2} \right) \frac{\partial^2 \bar{E}}{\partial t^2} \]

If one considers that

\[ \mu_0 \varepsilon_0 = \frac{1}{c^2} \]
where $C$ - speed of light, then no longer will remain doubts about the fact that with the propagation of electromagnetic waves in the dielectrics the frequency dispersion of phase speed will be observed. In the formation of this dispersion it will participate immediately three, which do not depend on the frequency, physical quantities: the self-resonant frequency of atoms themselves or molecules, the plasma frequency of charges, if we consider it their free, and the dielectric constant of vacuum.

II. Conclusion

This examination showed that this parameter as the kinetic inductance of charges characterizes electromagnetic processes in the conductors and the dielectrics and has the same fundamental value as the dielectric and magnetic constant of these media. Unfortunately, this important circumstance is not noted not only in the existing scientific literature, but also in the works of Maxwell.

References Références Referencias

1. Дж. Гудкайнд. Применение сверхпроводимости. УФН, том 106, 1972, вып.3.
5. Ф. Ф. Менде. Существуют ли ошибки в современной физике. Харьков, Константа, 2003, - 72 с.