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The Shewhart-Ewma Automatic Control Chart

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The Shewhart-Ewma Automatic Control Chart

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I. Introduction

Statistical process control (SPC) is a powerful tool whose primary purpose is process stabilization based on the identification of the assignable causes of a significant process change and appropriate corrective action. A side benefit of SPC is that the process is "improved". That is the process becomes more predictable.

Automated process control (APC) is an advanced tool whose primary purpose is to analyze the data coming from the process, apply the appropriate heuristics rules, and signal if the process behavior changes significantly, and/or automatically adjust the process input variables to maintain control. The value of APC is that it replaces the human being in analyzing the data and it assures that the heuristic analysis rules are consistently applied. In addition, it allows exception reporting (i.e., receiving a notification only when there is a significant change in the process occurs).

The utilization of these tools adds substantially to the ability to effectively monitor and control processes at minimum cost. If the two tools are combined, then even greater benefits can be achieved. However, the effectiveness of the SPC-APC system is a function the strength of the SPC control procedure and the soundness of the APC heuristic algorithms applied to the data. The goal is thus to marry a powerful control procedure with a robust heuristic program for data analysis and inference generation.

II. Methodology

The exponential weighted moving average is an extremely effective SPC control procedure that has been used in industry for years. It is a procedure that has a number of design features that make it a highly desirable choice in selecting a control methodology.

- It is very sensitive to small process changes and thus allows the practitioner to detect changes early and respond to them.
- It is robust against non-normal data. This means it can be applied to distributions where the data is skewed or not bell shaped and it will still provide reasonably accurate results (see the appendix).

a) The Exponentially Weighted Moving Average Control Chart for the Process Mean

Roberts introduced the EWMA control chart in 1959. See also Crowder (1989), and Lucas and Saccucci (1990) for a good discussion on the EWMA control chart. The exponentially weighted moving average is defined as follows:

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1}$$

where $0 < \lambda \leq 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that:

$$z_0 = \mu_0$$

Sometimes the average of preliminary data is used as the starting value of the EWMA so in that case $z_0 = \bar{x}$.

- *Formula for the EWMA Control Limits*

$$\mathrm{CLz} = \mu_0$$

$$\mathrm{UCLz} = \mu_0 + \mathrm{K}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}$$

$$\mathrm{LCLz} = \mu_0 - \mathrm{K}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]}$$

where K is the distance to the control limit.

- *Example of and EWMA Control Chart*

Given that $\mu_0 = 10$ and $\sigma = 1$ the EWMA using $\lambda = .1$ and $K = 2.7$ for the following set of observations is given in Table 1:

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Table 1 : EWMA Example

Obs No.	Obs X	Predicted EWMA
1	9.45	9.9450
2	7.99	9.7495
3	9.29	9.7036
4	11.66	9.8992
5	12.16	10.1253
6	10.18	10.1307
7	8.04	9.9217
8	11.46	10.0755
9	9.20	9.9880
10	10.34	10.0232
11	9.03	9.9238
12	11.47	10.0785
13	10.51	10.1216
14	9.40	10.0495
15	10.08	10.0525
16	9.37	9.9843
17	10.62	10.0478
18	10.31	10.0740
19	8.52	9.9186
20	10.84	10.0108
21	10.90	10.0997
22	9.33	10.0227
23	12.29	10.2495
24	11.50	10.3745
25	10.60	10.3971
26	11.08	10.4654
27	10.38	10.4568
28	11.62	10.5731
29	11.31	10.6468
30	10.52	10.6341

The Graph of the EWMA Control Chart

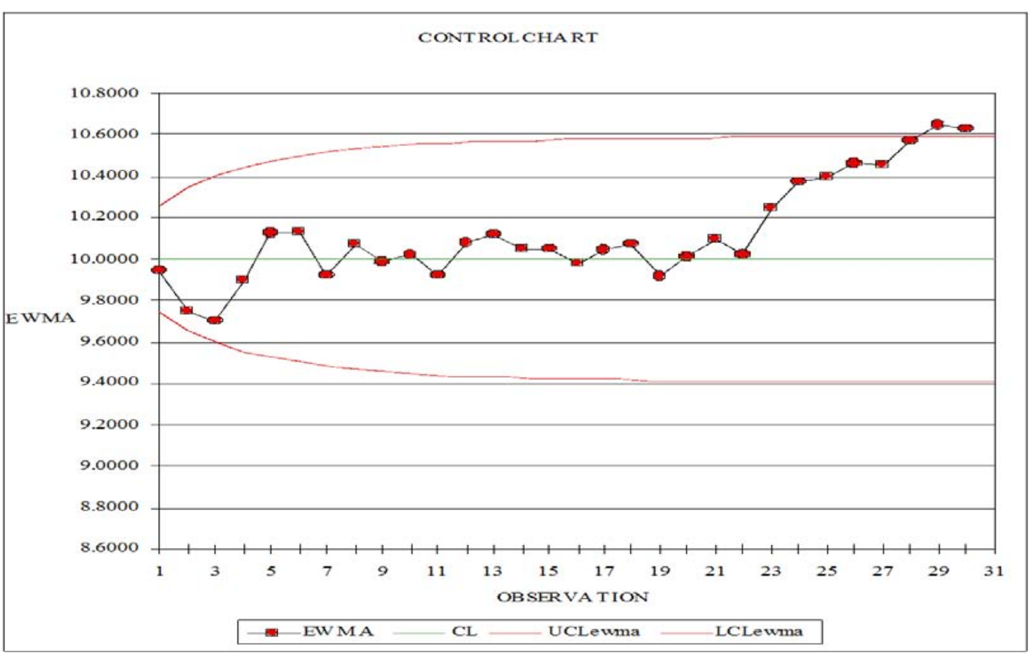

Figure 1 : EWMA Control Chart

b) Control Chart Design

One of the critical performance measures for any control procedure is its Average Run Length (ARL). That is, how long it takes to signal when a significant change occurs. More precisely, if the process experiences a change in its mean or variance, then the Average Run Length is the average number of samples that will be evaluated before a warning signal is generated indicating a change has occurred. The ARL can also be thought of as the time that it takes to signal once a change of a given magnitude has occurred, assuming a constant time interval between samples. Using this metric, good chart performance means that the ARL is a large number when the process is stable (i.e., no change has occurred (denoted ARL_0). A signal in this case would be a false alarm), and a small number one when the process is unstable (i.e., when there has been a change, and thus providing quick detection). Control system design goals seek to find procedures that maximize ARL_0 and simultaneously minimize ARL_x for $x > 0$.

The ARL_0 for a Shewhart individuals control chart using the single test, one point outside the control limits is 370, whereas for an EWMA with $\lambda = .1$ and $K = 2.8$ it is 500 (see Figure 2). This is a much better false alarm rate than the Shewhart individuals control chart. Further, if the K value were set to 3, the typical value, then the EWMA false alarm rate would be even better.

Examining Figure 2 it can be seen that the EWMA control chart is very effective in detecting small process shifts but not large changes. The Shewhart chart, on the other hand, is insensitive to small changes but sensitive to large ones. If we could combine the procedures, we would have an even better control procedure.

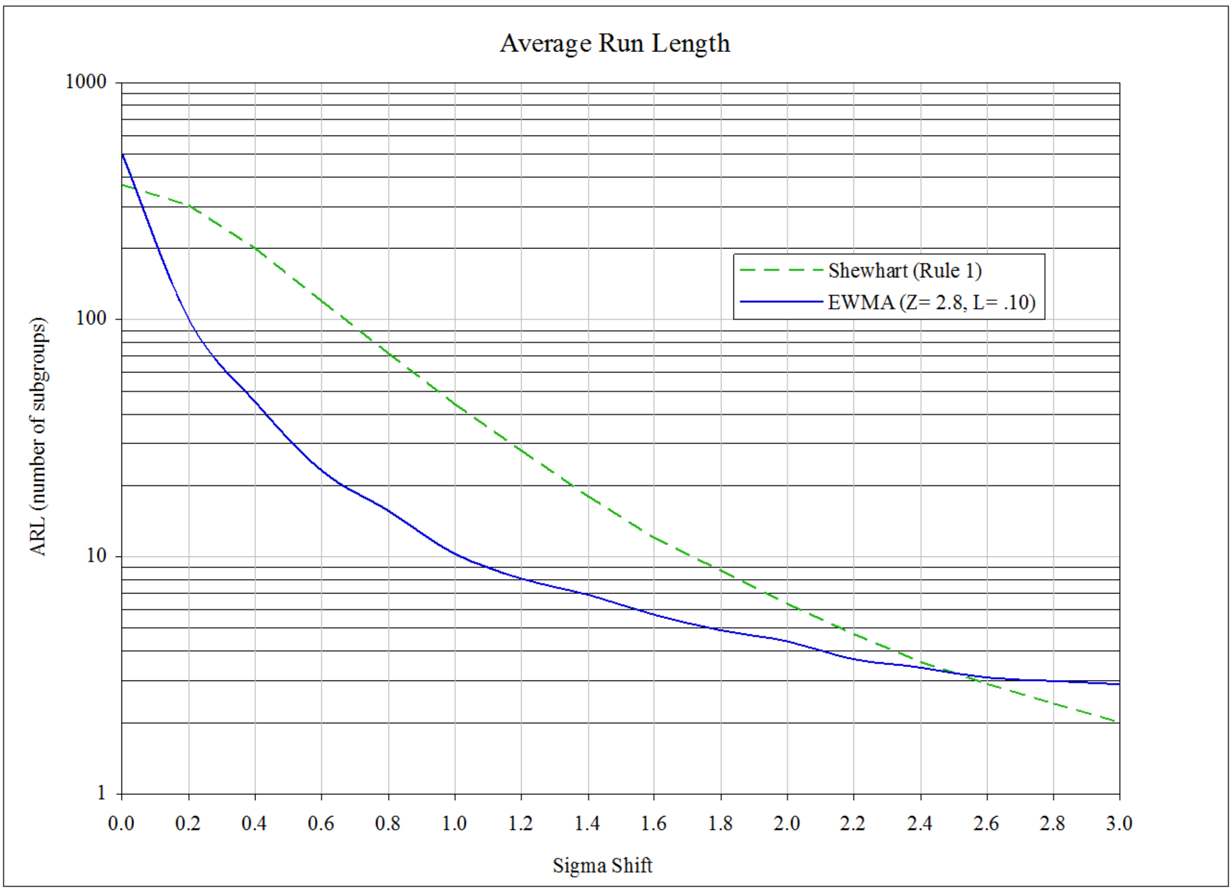

Figure 2 : ARL for a Shewhart and EWMA Control Procedure

c) The Shewhart-EWMA Control Chart

Since the EWMA chart performs well in detecting small shifts but does not react to large shifts as quickly as the Shewhart chart. A good way to further improve sensitivity of the control procedure to large shifts without sacrificing the ability to detect small shifts quickly is to combine a Shewhart chart with the EWMA. These combined Shewhart-EWMA control procedures are effective against both large and small process changes. When using such schemes, it may be helpful to use slightly wider than usual limits on the Shewhart chart (e.g., 3.2 sigma is recommended by Montgomery, 2001). The reason for this is to prevent the false alarm rate from increasing too much when we add the Shewhart chart.

It is also possible to plot the observed value x and the EWMA on the same control chart along with both Shewhart and EWMA limits. This produces one chart for the combined control procedure that operators quickly become adept at interpreting because there is only one test–a single point outside the control limits on either chart is an out-of-control signal. When the plots are computer generated, different colors or plotting symbols can be used for the two sets of control limits and statistics. In the automated case when a signal is generate it can be plotted as a large red dot or an X on the control chart.

Graph of a Shewhart-EWMA Control Chart

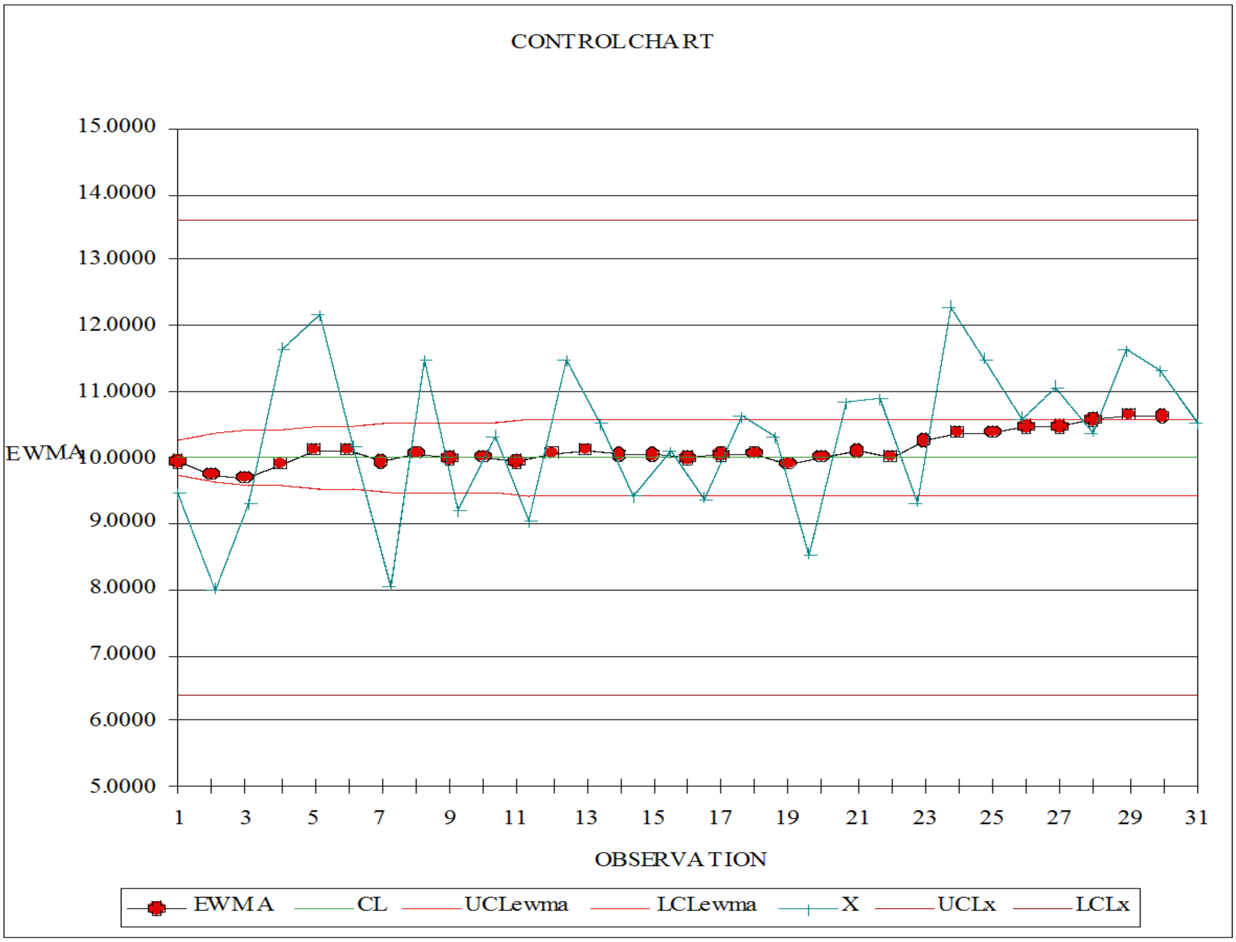

Figure 3 : Shewhart-EWMA Control Chart

d) Automating the Shewhart-EWMA

The control chart establishment process consists of two phases.

- *Phase I :* The center line, and control limits are determined from the mean and variance of a process that is in a state of statistical control, or from the data that has been edited to remove signals and corrective action has been taken prevent reoccurrence.
- *Phase II :* The centerline, and control limits are used as standards for incoming data after Phase I. They are used until such time as the process parameters (mean and/or variance) have statistically been determined to have changed. If the process parameters have changed, then the practitioner must take the appropriate actions, which may include going back to Phase I and re-computing a new center line and control limits.

The automation program proposed here attempts to mimic this procedure algorithmically.

- First it establishes the baseline performance (the number of samples used is adjustable and if a signal occurs within the window, then the baseline program restarts). The program would not go on to Phase II if the initial baseline were unstable.
- Once the initial baseline parameters are established, the automation algorithm takes over. Any point that is outside the Shewhart limits or the EWMA limits generates a signal. If four points in a row signal on the EWMA chart, then the program assumes the process has shifted. If this happens, the old parameters are reset and the program goes

back to step 1 and attempts to establish a new baseline. The number of points in a row required to declare a shift is selectable.

The assumptions and algorithm rules for the procedure are listed below:

i. *EWMA design rules (λ, K_1)*

Initially set: $\lambda = .10$ and $K_1 = 2.58$ (this gives an alpha of .01 or 99% control limits)

ii. *Shewhart design rules (K_2)*

Initially set: $K_2 = 3.0$ (Montgomery suggests using 3.2 to reduce the combined alpha risk of having two charts.)

iii. *Initial process baseline computation rules*

Readings are taken until there are eight (8) in a row that are all within the EWMA control limits. The control parameters (mean and variance) are computed from these eight readings and are used to compute the center line (CL), the Upper Control Limit (UCL) and Lower Control Limit (LCL) for both the EWMA and Shewhart control procedures.

iv. *Signal rules*

If a single point falls outside the control limits on either the EWMA or Shewhart control chart, then a warning signal is generated and an email notification is sent to the responsible authority.

v. *Process change rules*

If there are four (4) points in a row outside the EWMA control limits, then the process is assumed to have changed (i.e., the mean has changed) and the program resets. The probability of this happening assuming that the process was stable is about 1 in 2000 for $\lambda = .10$ based on simulation studies. The EWMA false reset rate depends on the autocorrelation induced in the EWMA data by the λ value selected.

vi. *Revised baseline computation rules*

If a process change is detected, then all historical data prior to the four signal points is no longer used and the new control chart parameters are generated following the procedure listed in step 3 above.

vii. *Communication and response rules*

If a signal is generated, then an email is sent to the responsible authority. Receipt of the email must be acknowledged. If it is not acknowledged within one (1) day, then an email is sent to the designated alternate. This email also requires acknowledgement.

The Automated Shewhart-EWMA Control Chart

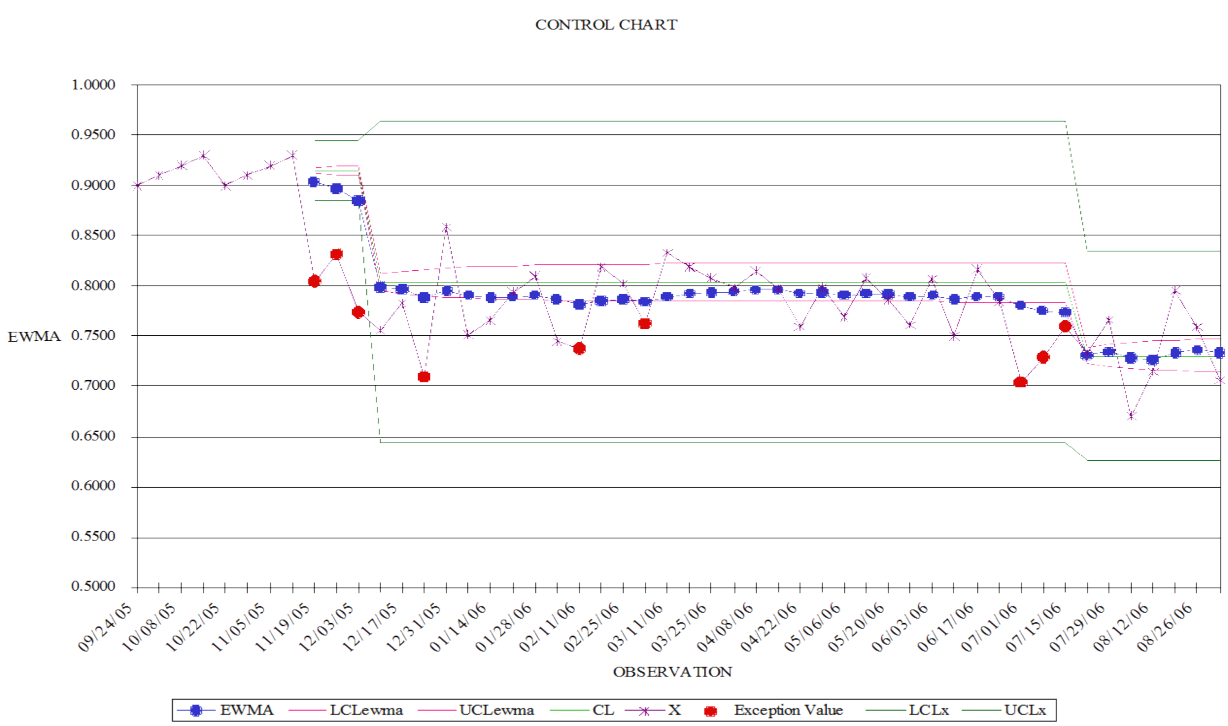

Figure 4 : Automated Shewhart-EWMA Control Chart with Shift Rule of Three in a Row

III. Summary

There are infinitely many ways a process may exhibit unstable behavior. Therefore, it is impossible to define a finite set of detections rules that would be able to detect all the possible types of instability. Hence, there is no perfect control chart as each one has its define a finite set of detections rules that would be able to detect all the possible types of instability. Hence, there is no perfect control chart as each one has its strengths and weaknesses. The Shewhart individuals control chart offers a good graphical representation of the process performance, capability, and is sensitive to large process shifts, but it not robust against nonnormality. The EWMA control chart does not give the practitioner a good view of individual observations, process performance, or capability, but it is fairly robust against nonnormality and it is sensitive to small process changes. The Shewhart-EWMA control procedure resolves most of these issues but is designed to detect changes in the mean and not the variance unless we apply more runs rules to the Shewhart x-chart (e.g., 5 points in a row outside ± 1 sigma).

Also, when the Shewhart-EWMA is automated a host of additional issues present themselves. These include the reasonableness of the following design decisions:

EWMA design rules (λ, K_1)

Shewhart design rules (K_2)

Initial baseline computation rules

Signal rules

Process change declaration rules

Revised baseline computation rules

Communication and response rules

The value of an automated process control procedure rests on its ability to accurately and precisely perform its detection and communication function in a timely manner. Since the goal is to reduce the labor content of the process monitoring activity via automation and exception reporting. The authors have attempted to make rational design choices, but in the final analysis the procedure must work in practice, or as Dr. Shewhart noted:

> *The fact that the criterion, which we happen to use, has a fine ancestry of powerful statistical theorems does not justify its use. Such justification must come from empirical evidence that it works.*
>
> *Walter A. Shewhart*

IV. Acknowledgement

I wish to thank Mr. Kip Rapp for programming the algorithms and providing the process behavior chart used in Figure 4.

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