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Relationship between New Types of Transitive and Chaotic Maps

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Abstract- The concepts of topological **γ**-transitive maps, **α**-transitive maps, **γ**-type chaotic and **α**-type chaotic maps were introduced by M. Nokhas Murad Kaki. In this paper, I study the relationship between two different notions of transitive maps, namely topological **α**- transitive maps, topological **γ**-transitive maps and investigate some of their properties in two topological spaces (X, **τα**) and (X, **τγ**), **τα** denotes the **α**–topology(resp. **τγ** denotes the **γ**–topology) of a given topological space (X, **τ**).. The two notions are defined by using the concepts of **α**-irresolute map and **γ**-irresolute map respectively Also, we study the relationship between two new types of chaotic maps, namely, **α**-type chaotic maps and **γ**-chaotic maps, and I will prove that the properties of **α**- transitive, **α**-type chaotic are preserved under **α**r-conjugacy and **γ**-transitive, **γ**-chaotic maps are preserved under **γ**r-conjugacy The main results are the following propositions:

1) Every topologically **γ**-type transitive map is a topologically **α**-type transitive map which implies topologically transitive map, but the converse not necessarily true..

2) Every **γ**-type chaotic map is **α**-chaotic map which implies chaotic map in topological spaces, but the converse not necessarily true..

Keywords: topologically γ- transitive, α - type chaotic, γ-type chaotic, α-dense.

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1) Every topologically γ-type transitive map is a topologically α-type transitive map which implies topologically transitive map, but the converse not necessarily true..

2) Every γ-type chaotic map is α-chaotic map which implies chaotic map in topological spaces, but the converse not necessarily true..

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I. Introduction

Recently there has been some interest in the notion of a locally closed subset of a topological space. According to Bourbaki [16] a subset S of a space (X, τ) is called locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [13] and [14] to define the concept of LC–continuity, i.e. a function $f : (X, \tau) \rightarrow (X, \sigma)$ is LC–continuous if the inverse with respect to f of any open set in Y is closed in X. The study of semi open sets and semi continuity in topological spaces was initiated by Levine [6]. Bhattacharya and Lahiri [8] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets which was introduced by Levine [5]. Throughout this paper, the word "space " will mean topological space The collections of semi-open, semi-closed sets and α–sets in (X, τ) will be denoted by $SO(X, \tau)$, $SC(X, \tau)$ and τ^{α} respectively. Njastad [7] has shown that τ^{α} is a topology on X with the following properties: $\tau \subseteq \tau^{\alpha}, (\tau^{\alpha})^{\alpha} = \tau^{\alpha}$ and $S \in \tau^{\alpha}$ if and only if $S = U \setminus N$ where $U \in \tau$ and N is nowhere dense $(i.e. Int(Cl(N)) = \varphi)$ in (X, τ). Hence $\tau = \tau^{\alpha}$ if and only if every nowhere dense (nwd) set in (X, τ) is closed, therefore every transitive map implies α-transitive. Also if every α-open set is locally closed then every transitive map implies α-transitive; and this structure also occurs if (X, τ) is locally compact and Hausdorff [36, p. 140, Ex. B]] and every α-open set is locally compact, then every α-open set is locally closed.

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In 1943, Fomin [27] introduced the notion of θ-continuous maps. The notions of θ-open sets, θ-closed sets and θ-closure where introduced by Veliˇcko [19] for the purpose of studying the important class of H-closed spaces in terms of arbitrary fiber-bases. Dickman and Porter [20], [21], Joseph [22] and Long and Herrington [31] continued the work of Veličko. We introduce the notions of θ-type transitive maps, θ-minimal maps and show that some of their properties are analogous to those for topologically transitive maps. Also, we give some additional properties of θ-irresolute maps. We denote the interior and the closure of a subset A of X by Int(A) and Cl(A), respectively. By a space X, we mean a topological space (X, τ) A point x ∈ X is called a θ-adherent point of A [19], if $A \cap Cl(V) \neq \phi$ for every open set V containing x. The set of all θ-adherent points of a subset A of X is called the θ-closure of A and is denoted by $Cl_{\theta}(A)$. A subset A of X is called - closed if $A = Cl_{\theta}(A)$. Dontchev and Maki [22] have shown that if A and B are subsets of a space (X, τ), then

$Cl_{\theta}(A \cup B) = Cl_{\theta}(A) \cup Cl_{\theta}(B)$ also

$Cl_{\theta}(A \cap B) = Cl_{\theta}(A) \cap Cl_{\theta}(B)$.

Note also that the θ-closure of a given set need not be a θ - closed set. But it is always closed. Dickman and Porter [20] proved that a compact subspace of a Hausdorff space is θ-closed. Moreover, they showed that a θ-closed subspace of a Hausdorff space is closed. Jankovi´[25] proved that a space (X, τ) is Hausdorff if and only if every compact set is θ-closed. The complement of a θ-closed set is called a θ-open set. The family of all θ-open sets forms a topology on X and is denoted by τ^{θ} *or* θ topology.

This topology is coarser than τ and that a space (X, τ) is regular if and only if $\tau = \tau^{\theta}$ [26]. Then we observe that every theta-type transitive maps is transitive if (X, τ) is regular. In general, $Cl_{\theta}(A)$ will not be the closure of A with respect to (X, τ^{θ}). It is easily seen that one always has $A \subseteq Cl(A) \subseteq Cl_{\delta}(A) \subseteq Cl_{\theta}(A) \subseteq \overline{A}^{\theta}$

where $\overline{A}^{\theta}$ denotes the closure of A with respect to (X,τ^{θ}). It is also obvious that a set A is θ-closed in (X, τ) if and only if it is closed in (X, τ^{θ}). The space (X,τ^{θ}) is called sometimes the semi regularization of (X, τ). A function $f: X \to Y$ is closure continuous [29] (θ continuous) at x ∈ X if given any open set V in Y containing f(x), there exists an open set U in X containing x such that $f(Cl(U)) \subseteq Cl(V)$.[29] In this paper, we will study the relationship between new classes of topological transitive maps called *γ type transitive* and α- type transitive, also, new classes of γ- type chaotic maps and α- type chaotic maps. We have shown that every γ-type transitive map is a α- type transitive map, but the converse not necessarily true and that every γ-type chaotic map is α-type chaotic map, but the converse not necessarily true we will also study some of their properties.

II. Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let X be a space and $A \subset X$. The intersection (resp. closure) of A is denoted by Int(A) (resp. Cl(A).

- *Definition 2.1* [6] A subset A of a topological space X will be termed semi- open (written S.O.) if and only if there exists an open set U such that $U \subset A \subset Cl(U)$.
- *Definition 2.2* [8] Let A be a subset of a space X then semi closure of A defined as the intersection of all semi–closed sets containing A is denoted by sClA.
- *Definition 2.3* [9] Let (X, τ) be a topological space and α an operator from τ to Ƥ(X) i.e α: η → Ƥ(X), where Ƥ(X) is a power set of X. We say that α is an operator associated with η if $U \subset a(U)$ for all U ϵ η
- *Definition 2.4* [10] Let (X,τ) be a topological space and α an operator associated with η. A subset A of X is said to be α-open if for each x ϵ X there exists an open set U containing x such that $a(U) \subset A$. Let us denote the collection of all α-open, semi-open sets in the topological space (X,τ) by τ^{α}, SO(τ), respectively. We then have $\tau \subseteq \tau^{\alpha} \subseteq$ *SO* (τ). A subset B of X is said to be α-closed [7] if its complement is α-open.
- *Definition 2.5* [9] Let (X, τ) be a space. An operator α is said to be regular if, for every open neighborhoods U and V of each x ϵ X, there exists a neighborhood W of x such that $a(W) \subset \alpha(U)\ a(V)$.. Note that the family τ^{α} of α –open sets in (X, τ) always forms a topology on X, when α is considered to be regular finer than τ.
- *Theorem 2.6* [30] For subsets A, B of a space X, the following statements hold:

  (1) $D(A) \subset D_{\theta}(A)$, where D(A) is the derived set of A

  (2) *If* $A \subset B$, *then* $D_{\theta}(A) \subset D_{\theta}(B)$

  (3) $D_{\theta}(A) \cup D_{\theta}(B) = D_{\theta}(A \cup B)$. And $D_{\theta}(A \cap B) \subset D_{\theta}(A) \cap D_{\theta}(B)$

Note that the family τ^{θ} of θ –open sets in (X,τ) always forms a topology on X denoted θ-topology and that θ-topology coarser than τ.

- *Definition 2.7* [4]: Let A be a subset of a space X. A point x is said to be an α- limit point of A if for each α- open U containing x, $U \cap (A \setminus x) \neq \phi$. The set of all α- limit points of A is called the α-derived set of A and is denoted by $D_{\alpha}(A)$.
- *Definition 2.8* [4] For subsets A and B of a space X, the following statements hold true:

  1) $D_{\alpha}(A) \subset D(A)$ where D(A) is the derived set of A

  2) if $A \subset B$ then $D_{\alpha}(A) \subset D_{\alpha}(B)$

  3) $D_{\alpha}(A) \cup D_{\alpha}(B) \subset D_{\alpha}(A \cup B)$

  4) $D_{\alpha}(A \cup D_{\alpha}(A)) \subset A \cup D_{\alpha}(A)$
- *Definition 2.9* [10]: The point x ϵ X is in the α–closure of a set A⊂X if α(U) ∩ A≠θ, for each open set U containing x. The α- closure of a set A is the intersection of all α -closed sets containing A and is denoted by $Cl_{\alpha}(A)$.
- *Remark 2.10:* For any subset A of the space X, $A \subset Cl(A) \subset Cl_{\alpha}(A)$
- *Definition 2.11* [10] Let (X, τ) be a topological space. We say that a subset A of X is α- compact if for every α-open covering Π of A there exists a finite subcollection $\{C_1, C_2, ..., C_n\}$ of Π such that $/A \subset \bigcup_{i=1}^{n} C_i$. Properties of α –compact spaces have been investigated by Rosa, E etc. and Kasahara, S [9,10].The following results were given by Rosas, E etc. [9].
- *Theorem2.12* Let (X, τ) be a topological space and α an operator associated with η. A⊂ X and K⊂A. If A is α–compact and K is α –closed then K is α –compact.
- *Theorem2.13* Let (X, τ) be a topological space and α be a regular operator on η. If X is α - 2 *T* (see Rosa, E etc. and Kasahara, S) [9,10] and K⊂X is α –compact then K is α – closed.
- *Definition 2.14* [10] The intersection of all α – closed sets containing A is called α –closure of A, denoted by $Cl_{\alpha}(A)$.
- *Remark 2.15* For any subset A of the space X, A $\subset Cl(A) \subset Cl_{\alpha}(A)$.
- *Lemma2.16* For subsets A and A_i (i ϵ I) of a space (X, τ) , the following hold:

1) A $\subset Cl_{\alpha}(A)$

2) $Cl_{\alpha}(A)$ closed; $Cl_{\alpha}(Cl_{\alpha}(A)) = Cl_{\alpha}(A)$

3) If A ⊂ B then $Cl_{\alpha}(A) \subset Cl_{\alpha}(B)$

4) $Cl_{\alpha}(\cap(A_i : i \in I)) \subset \cap(Cl_{\alpha}(A) : i \in I)$

5) $Cl_{\alpha}(\cup(A_i : i \in I)) = \cup(Cl_{\alpha}(A) : i \in I)$

- *Lemma 2.17* The collection of α –compact subsets of X is closed under finite unions. If α is a regular operator and X is an α- 2 *T* space then it is closed under arbitrary intersection.

- *Definition 2.18* Let (X, τ) be a topological any space, A subset of X, The $\operatorname{int}_\alpha(A)=\cup\{U : U$ is α–open and $U \subset A\}$.
- *Remark 2.19* A subset A is α –open if and only if $\operatorname{int}_\alpha(A) = A$.

Proof: The proof is obvious from the definition.

- *Definition 2.20* Let (X, τ) and (Y, ζ) be two topological spaces, a map $f : X \to Y$ is said to be α–continuous if for each open set H of Y, $f^{-1}(H)$ is α–open in X.
- *Theorem 2.21* [4]: For any subset A of a space X, $Cl_\alpha(A) = A \cup Cl_\alpha(A)$.
- *Theorem 2.22.*[4]: For subsets A, B of a space X, the following statements are true:

1) $\operatorname{int}_\alpha(A)$ is the largest α –open contained in A
2) $\operatorname{int}_\alpha(\operatorname{int}_\alpha(A)) = \operatorname{int}_\alpha(A)$
3) If A⊂B then $\operatorname{int}_\alpha(A) \subset \operatorname{int}_\alpha(B)$
4) $\operatorname{int}_\alpha(A) \cup \operatorname{int}_\alpha(B) \subset \operatorname{int}_\alpha(A \cup B)$
5) $\operatorname{int}_\alpha(A) \cap \operatorname{int}_\alpha(B) \supset \operatorname{int}_\alpha(A \cap B)$

- *Lemma 2.23* [7] For any α– open set A and any α-closed set C, we have
  1) $Cl_\alpha(A) = Cl(A)$
  2) $\operatorname{int}_\alpha(C) = \operatorname{int}(C)$
  3) $\operatorname{int}_\alpha Cl_\alpha A = \operatorname{int}(Cl(A))$
- *Remark 2.24.*[4]: It is not always true that every α-open set is an open set, as shown in the following example:
- *Example 2.25* Let X={a, b, c, d} with topology η={ϕ,{c,d}, X}. Hence α (τ)={ϕ,{c, d},{b, c, d}, {a, c, d},X} So {b, c, d} is α-open but not open.
- *Theorem 2.26* Let (*X*, *f*) *and* (*Y*, *g*) be two topological systems, if $f : X \to X$ and $g : Y \to Y$ are topologically αr-conjugate. Then

  (1) *f* is topologically α- transitive map if and only if *g* is topologically α-transitive map;

  (2) *f* is α-type chaotic map if and only if *g* is α-type chaotic map;

  (3) *f* is γ-type chaotic map if and only if *g* is γ-type chaotic map.

III. Transitive and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we mean a topological space *X* together with a continuous map $f : X \to X$. The space X is sometimes called the phase space of the system. A set $A \subseteq X$ is called $f-$*inveriant* if $f(A) \subseteq A$. A topological system (*X*, *f*) is called *minimal* if *X* does not contain any non-empty, proper, closed $f-$*inveriant* subset. In such a case we also say that the map *f* itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them.

Given a point x in X, $O_f(x) = \{x, f(x), f^2(x), ...\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω - limit set, i.e. the set of limit points of the sequence . $x, f(x), f^2(x), ...$ The following conditions are equivalent:

- (X, f) is γ-minimal (resp. α-minimal),
- every orbit is γ-dense (resp. α-dense) in X ,
- $\omega_f(x) = X$ for every x ∈ X.

A minimal map f is necessarily surjective if *X* is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system (X, f), a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of *A* has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f).

The basic fact discovered by G. D. Birkhoff is that in any compact system (X, f) there are minimal sets. This follows immediately from the Zorn's lemma. Since any orbit closure is invariant, we get that *any compact orbit closure contains a minimal set*. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) either are disjoint or coincide. A minimal set A is strongly $f-$ *inveriant*, i.e. $f(A) = A$. Provided it is compact Hausdorff.

Let (X, f) be a topological system, and $f : X \to X$ αr-homeomorphism of X onto itself. For A and B subsets of X, we let $N(A, B) = \{n \in \mathbf{Z}: f^n(A) \cap B \neq \phi\}$ We write $N(A,B) = N(x, B)$ for a singleton $A = \{x\}$ thus $N(x, B) = \{n \in \mathbf{Z}: f^n(x) \in B\}$ For a point $x \in X$ we write $O_f(x) = \{f^n(x) : n \in \mathbf{Z}\}$ for the orbit of x and $Cl_\alpha(O_f(x))$ for the α-closure of $O_f(x)$. We say that the topological system (X, f) is α-type point transitive if there is a point $x \in X$ with $O_f(x)$ α-dense. Such a point is called α-type transitive. We say that the topological systems (X, f) is topologically α-type transitive (or just α-type transitive) if the set $N(U, V)$ is nonempty for every pair U and V of nonempty α-open subsets of X.

- *Theorem 2.8* [37] Let (X, f) be a topological system where X is a non-empty locally γ-compact Hausdorff topological space and $f : X \to X$ is γ-irresolute map and that X is γ-type separable. Suppose that f is topologically γ-type transitive. Then there is an element $x \in X$ such that the orbit $O_f(x) = \{f^n(x) : n \in \mathbf{N}\}$ is γ-dense in X.

a) *Topologically α-Transitive Maps*

In [35], we introduced and defined a new class of transitive maps that are called topologically α-transitive maps on a topological space (X, τ), and we studied some of their properties and proved some results associated with these new definitions. We also defined and introduced a new class of α-minimal maps. In this paper we discuss the relationship between topologically α-transitive maps and θ-transitive maps. On the other hand, we discuss the relationship between α-minimal and θ-minimal in topological systems.

- *Definition 3.1.1* Let (X, τ) be a topological space. A subset A of X is called α-dense in X if $Cl_{\alpha}(A) = X$.
- *Remark 3.1.2* Any α-dense subset in X intersects any α- open set in X.

Proof: Let A be an α-dense subset in X, then by definition, $Cl_{\alpha}(A) = X$, and let U be a non-empty α- open set in X. Suppose that A∩U=ϕ . Therefore $B = U^c$ is α-closed and $A \subset U^c$=B. So $Cl_{\alpha}(A) \subset Cl_{\alpha}(B)$ i.e. $Cl_{\alpha}(A) \subset$ B, but $Cl_{\alpha}(A) = X$, so $X \subset$ B, this contradicts that U $\neq \varphi$

- *Definition 3.1.3* [12] A map $f: X \to Y$ is called α-irresolute if for every α- open set H of Y, $f^{-1}(H)$ is α- open in X.
- *Example 3.1.4* [35] Let (X, η) be a topological space such that X={a, b, c, d} and η ={ϕ, X ,{a, b}, {b}}. We have the set of all α-open sets is α(X, η)={ϕ, X, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}}and the set of all α-closed sets is αC(X, η)={ϕ, X, {c, d, {a, c, d}, {a, d}, {}a, c}, {d}, {c}}. Then define the map f : X→X as follows f(a)= a, f(b)= b, f(c)= d, f(d)= c, we have f is α-irresolute because {b} is α open and f-1({b})={b} is α-open; {a, b} is α-open and f- 1({a, b})={a, b} is α-open; {b, c} is α-open and f-1({b, c})={b, d} is α-open; {a, b, c} is α open and f-1({a, b, c})={a, b, d} is α-open; {a, b, d} is α-open and f-1({a, b, d})={a, b, c} is α-open so f is α-irresolute.
- *Definition 3.1.5* A subset A of a topological space (X,τ) is said to be nowhere α-dense, if its α-closure has an empty α-interior, that is, $\operatorname{int}_{\alpha}(Cl_{\alpha}(A)) = \phi$.
- *Definition 3.1.6* [35] Let (X, τ) be a topological space, $f: X \to X$ be α-irresolute map then *f* is said to be topological α-transitive if every pair of non-empty α-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$. In the forgoing example 3.1.4: we have f is α-transitive because b belongs to any non-empty α- open set V and also belongs to f(U) for any α-open set it means that f(U)∩V≠ ϕ so f is . α - transitive.
- *Example 3.1.7* Let (X, η) be a topological space such that X ={a, b, c}and η={ϕ, {a}, X}.Then the set of all α-open sets is ηα={ϕ, {a}, {a, b}, {a, c}, X}. Define f : X→X as follows f(a)=b, f(b)=b, f(c)=c. Clearly f is continuous because {a} is open and f({a})=ϕ is open. Note that f is transitive because f({a})={b} implies that f({a})∩{b}≠ϕ. But f is not α-transitive because for each n in N , fn({a})∩{a, c}=ϕ; since fn({a})={b} for every n ε N, and {b}∩{a, c}=ϕ. So we have f is not α-transitive, so we show that transitivity not implies α-transitivity.

- *Definition 3.1.8* Let (X, η) be a topological space. A subset A of X is called θ-dense in X if $Cl_{\theta}(A) = X$.
- *Remark 3.1.9* [38] Any θ-dense subset in X intersects any θ-open set in X.

Proof: Let A be a θ-dense subset in X, then by definition, $Cl_{\theta}(A) = X$, and let U be a non-empty θ-open set in X. Suppose that A∩U=ϕ. Therefore $B = U^c$ is θ-closed because B is the complement of θ-open and $A \subset U^c$=B. So $Cl_{\theta}(A) \subset Cl_{\theta}(B)$, i.e. $Cl_{\theta}(A) \subset$B, but $Cl_{\theta}(A) = X$, so $X \subset B$, this contradicts that U ϕ φ

- *Definition 3.1.10.[33]* A function $f: X \to X$ is called θ − irresolute if the inverse image of each θ − open set is a θ − open set in X.
- *Definition 3.1.11* A subset A of a topological space (X,τ) is said to be nowhere θ-dense, if its θ-closure has an empty θ-interior, that is, $\operatorname{int}_{\theta}(Cl_{\theta}(A)) = \phi$.
- *Definition 3.1.12* [34] Let (X, τ) be a topological space, and $f: X \to X$ θ.-irresolute) map, then *f* is said to be topologically θ-type transitive map if for every pair of θ- open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$

Associated with this new definition we can prove the following new theorem.

- *Theorem 3.1.13* [35]: Let (X, τ) be a topological space and $f: X \to X$ be α -irresolute map. Then the following statements are equivalent:
  (1) *f* is topological α-transitive map
  (2) For every nonempty α- open set U in X, $\overset{\infty}{\underset{n=0}{\cup}} f^n(U)$
  (3) For every nonempty α-open set U in X, $\overset{\infty}{\underset{n=0}{\cup}} f^{-n}(U)$
  (4) If $B \subset X$ is α- closed and B is f- invariant i.e. $f(B) \subset B$ then B=X or B is nowhere α-dense.
  (5) If U is α-open and $f^{-1}(U) \subset U$ then U is either empty set or α-dense in X.
- *Theorem 3.1.14* : [34] Let (X, τ) be a topological space and $f: X \to X$ be θ -irresolute map. Then the following statements are equivalent:
  (1) f is θ-type transitive map
  (2) $\overset{\infty}{\underset{n=0}{\cup}} f^n(D)$ is θ-dense in X, with D is θ-open set in X.
  (3) $\overset{\infty}{\underset{n=0}{\cup}} f^{-n}(D)$ is θ- dense in X with D is θ- open set. in X.
  (4) If $B \subset X$ is θ-closed and $f(B) \subset B$. then B = X or B is nowhere θ-dense
  (5) If and D is θ-open in X then D=ϕ or D is θ-dense in X.

IV. Alpha-Minimal Functions

We introduced a new definition on α-minimal[35] (resp. θ-minimal[34]) maps and we studied some new theorems associated with these definitions. Given a topological space X, we ask whether there exists α-irresolute (resp. θ-irresolute) map on X such that the set $\{f^n(x) : n \geq 0\}$, called the orbit of x and

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denoted by $O(x)$ f, is α-dense(resp. θ-dense) in X for each x ϵ X.. A partial answer will be given in this section. Let us begin with a new definition.

- *Definition 4.1* (α-minimal) Let X be a topological space and f be α-irresolute map on X with α-regular operator associated with the topology on X. Then the dynamical system (X, f) is called α-minimal system (or f is called α-minimal map on X) if one of the three equivalent conditions [35] hold:

  1) The orbit of each point of X is α-dense in X.

  2) $Cl_\alpha(O_f(x)) = X$ for each x ϵ X

  3) Given x ϵ X and a nonempty α-open U in X, there exists nϵ N such that $f^n(x) \in U$

- *Theorem 4.2* [35] For (X, f) the following statements are equivalent:

  (1) f is an α-minimal map.

  (2) If E is an α-closed subset of X with $f(E) \subset E$, we say E is invariant. Then E= ϕ or E=X.

  (3) If U is a nonempty α-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

V. Topological Systems and Conjugacy

In this section, I introduce and define θr-conjugated topological systems (*X, f*) *and* (*Y, g*), where X and Y are almost regular topological spaces. First I will define θr-homeomorphism and then I will prove new theorem associated with these new definitions:

- *Definition 5.1* A map.is said to be - homeomorphism if is bijective and thus invertible and both and are θr-irresolute
- *Definition 5.2* Two topological systems (*X, f*) and (*Y, g*) are said to be almost regular systems if X and Y are almost regular topological spaces.
- *Definition [38] 5.3* Let (*X, f*) and (*Y, g*) be two almost regular systems, then $f: X \to X$ and $g: Y \to Y$ are said to be topologically θr-conjugate if there is θr-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$. We will call h a topological θr-conjugacy. Thus, the two almost regular topological systems with their respective function acting on them share the same dynamics

VI. New Types of Chaos of Topological Spaces

We will give a new definition of chaos for γ-irresolute (resp. α-irresolute) self map $f: X \to X$ of a locally compact Hausdorff topological space X, so called γ-type chaos (resp. α-type chaos). These new definitions imply John Tylar definition which coincides with Devanney's definition for chaos when the topological space happens to be a metric space, but not conversely.

- *Definition 4.1* Let (*X, f*) be a topological system, the dynamics is obtained by iterating the map. Then, f is said to be γ-type chaotic (resp. α-type chaotic) on X provided that for any nonempty γ-open (resp. α-open) sets U and V in X, there is a periodic point $p \in X$ such that $U \cap O_f(p) \neq \phi$ and $V \cap O_f(p) \neq \phi$.
- *Proposition 4.2* Let (*X, f*) be a topological system. The map f is γ-type chaotic (resp.α-type chaotic) on X if and only if f is γ-type transitive (resp. α-type transitive) and the set of periodic points of the map f is *γ dense* (*resp. α −dense*) in X.

  Let us prove only for γ-type chaotic

  Proof: If f is γ-type chaotic on X, then for every pair of nonempty γ-open sets U and V, there is a periodic orbit intersects them; in particular, the periodic points are γ-dense in X. Then there is a periodic point p and $x, y \in O_f(p)$ with x ∈ U and y ∈ V and some positive integer n such that $f^n(x) = y$, so that $y = f^n(x) \in f^n(U)$ therefore $f^n(U) \cap V \neq \phi$ that is, f is γ-type transitive map.

The γ-type transitivity of f on X implies that for any nonempty γ-open subsets U, V ⊂ X, there is n such that for some x ∈ U, $f^n(x) \in V$ Now define

$W = f^{-n}(V) \cap U$. Then W is γ-open and nonempty with the property that $f^n(W) \subset V$.

But since the periodic points of f are γ-dense in X, there is a p ∈ W such that $f^n(p) \in V$. Therefore, $U \cap O_f(p) \neq \phi$ and $V \cap O_f(p) \neq \phi$, so that f is γ- type chaotic map.

VII. Conclusion

We have the following results

- *Proposition 7.1.* Every topologically γ-type transitive map is a topologically α-type transitive map which implies topologically transitive map, but the converse not necessarily true..
- *proposition 7.2.*Every γ-minimal map is α-minimal map which implies minimal map, but the converse not necessarily true..
- *Theorem 7.3* Let (*X, f*) *and* (*Y, g*) be two topological systems, if $f: X \to X$ and $g: Y \to Y$ are topologicallyαr-conjugate. Then

  (1) *f* is topologically α- transitive map if and only if *g* is topologically α-transitive map;

  (2) *f* is α-type chaotic map if and only if *g* is α-type chaotic map;

  (3) *f* is γ-type chaotic map if and only if *g* is γ-type chaotic map.

- *Proposition 7.4* Let (*X, f*) be a topological system. The map f is γ-type chaotic (resp.α-type chaotic) on X if and only if f is γ-type transitive (resp. α-type transitive) and the set of periodic points of the map f is *γ − dense* (*resp. α − dense*) in X.

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