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Numerical Methods

Rule Induction System

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Highlights

Fourier Spectral Methods

Kuramoto-Sivashinsky Equation

Discovering Thoughts, Inventing Future

VOLUME 14

ISSUE 1

VERSION 1.0



GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: I

NUMERICAL METHODS

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NUMERICAL METHODS
VOLUME 14 ISSUE 1 (VER. 1.0)

OPEN ASSOCIATION OF RESEARCH SOCIETY

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$P \neq NP$ Proof (Millennium Prize Problem Solved using the Proof of $X \pm Y = B$ at System 1)

By Martins Kolawole Alabi

Abstract- The proof of $x \pm y = b$ at system 1 is the premise that was used to prove that $P \neq NP$ where b is the total sum of input in the subset sum problem. The proof of $x \pm y = b$ systems are forms of $x \pm y = b$ that was derived from the coexistence of three quantities denoted by n , $n + 1$, $n + 2$ where n represents any positive integer. The proof of $x \pm y = b$ at system 1 is a computable function definable by an algorithm where n is the argument (input value) of the function to the corresponding output value b . System 1 can be defined as a system which the proof of $x \pm y = b$ belongs when $n = 1$. The proof of $x \pm y = b$ at system 1 is the proof of a mathematical method that proves something can evolve from nothing and its graph shows that the shape of the universe is a cone and this can further be mapped with an expanding universe or universes to locate the point of the big bang .i.e. a hypothetical point in space where the universe began. See the reference list for details.

Keywords: input, systems, subset sum problem, algorithm, $P \neq NP$, the proof of $x \pm y = b$.

GJRE-I Classification : FOR Code: 010301



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P \neq NP Proof (Millennium Prize Problem Solved using the Proof of $X \pm Y = B$ at System 1)

Martins Kolawole Alabi

Abstract The proof of $x \pm y = b$ at system 1 is the premise that was used to prove that $P \neq NP$ where b is the total sum of input in the subset sum problem. The proof of $x \pm y = b$ systems are forms of $x \pm y = b$ that was derived from the coexistence of three quantities denoted by $n, n + 1, n + 2$ where n represents any positive integer. The proof of $x \pm y = b$ at system 1 is a computable function definable by an algorithm where n is the argument (input value) of the function to the corresponding output value b . System 1 can be defined as a system which the proof of $x \pm y = b$ belongs when $n = 1$. The proof of $x \pm y = b$ at system 1 is the proof of a mathematical method that proves something can evolve from nothing and its graph shows that the shape of the universe is a cone and this can further be mapped with an expanding universe or universes to locate the point of the big bang .i.e. a hypothetical point in space where the universe began. See the reference list for details.

Keywords: input, systems, subset sum problem, algorithm, $P \neq NP$, the proof of $x \pm y = b$.

I. SUBSET SUM PROBLEM

Consider the subset sum problem, an example of a problem that is easy to verify, but whose answer may be difficult to compute. Given a set of integers, does some nonempty subset of them sum to 0? For instance, does a subset of the set $\{-2, -3, 15, 14, 7, -10\}$ add up to 0? The answer "yes, because $\{-2, -3, -10, 15\}$ adds up to zero" can be quickly verified with three additions.

There is no known algorithm to find a subset that sum to 0 in polynomial time. Thus it takes a very long time to find a subset that sum to 0 as the complexity of the problem grows on a deterministic Turing machine given the computer's present state and any inputs (there is only one possible action that the computer might take) and *sequential* (it performs actions one after the other). Literally, polynomial time means that as the complexity of the problem grows, the difficulty in solving it doesn't grow too fast. Therefore, if there is a known algorithm to find a subset that sum to 0 in polynomial time then $P = NP$ but if there is no known algorithm to find a subset that sum to 0 in polynomial time and it can be proved then $P \neq NP$.

II. PREMISE

Computers are deterministic and they cannot identify subset that sum to 0 or subset that do not sum

to 0 without cause (a thing that makes something happen). It can be observed that there is no other information given to the computer rather than the input themselves. Therefore, for us to have a solution to the P vs NP problem the input must equate to something that is provable. Otherwise, the input is meaningless.

The proof of $x \pm y = b$ at system 1 is the premise that was used in this paper to prove that $P \neq NP$ where b is the total sum of input. The proof of $x \pm y = b$ at system 1 gives a solution when integers that sum to 0 combines with one or two integers that do not sum to 0 regardless of input size.

III. ASSERTION

If $P = NP$. Then a determinant would be found in polynomial time.

Determinant (a factor that causes something) here is referring to an integer n that gives indication or hint about a subset that sum to 0 when the proof of $x \pm y = b$ at system 1 is established in the subset sum problem. They can be classified as a cause for subset that sum to the total sum of input.

Given that $x \pm y = b$ where b is the solution to any given number x and y for which x is number i or j .

So that, $i + y = b$.

Also that, $j - y = b$.

Let $i = (nb + n) / n + 1$.

Let $y = (b - n) / n + 1$.

Let $j = (nb + 2b - n) / n + 1$.

Therefore the proof of $x \pm y = b$ system is given as:

$(nb + n) / n + 1 + (b - n) / n + 1 = b$.

$(nb + 2b - n) / n + 1 - (b - n) / n + 1 = b$.

System 'n' is a system which $x \pm y = b$ belongs for every given positive integer n . For instance, when $n = 1$.

Let $i = (b + n) / 2$.

Let $y = (b - n) / 2$.

Let $j = (3b - n) / 2$.

Therefore the proof of $x \pm y = b$ at system 1 is given as:

$(b + n) / 2 + (b - n) / 2 = b$.

$(3b - n) / 2 - (b - n) / 2 = b$.

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$f(n) = b$ where n is a function of b and b is the total sum of input in the subset sum problem.

IV. SEARCH FOR DETERMINANT

Given a set of integers $\{-2, -3, 15, 14, 7, -10\}$ does some nonempty subset of them sum to 0?

Let i denote each integer in the set.

Since input $\{-2, -3, 15, 14, 7, -10\}$ equals 21. Then $b = 21$.

The proof of $x \pm y = b$ at system 1 is implicit. When we know i and b we can know n by making use of the formula $n = (2 \times i) - b$ derived from the proof of $x \pm y = b$ at system 1.

For $i = -2$, $n = -25$.

For $i = 15$, $n = 9$.

For $i = 14$, $n = 7$.

For $i = 7$, $n = -7$.

For $i = -3$, $n = -31$.

For $i = -10$, $n = -41$.

$i = 14$ when $n = 7$ and $i = 7$ when $n = -7$.

Since $7 + (-7) = 0$. Then $n = 7$ is a determinant and $n = -7$ is a determinant.

First hint: If the sum of equal and opposite values for n equals 0 outputs all integers in the set except integers that reference equal and opposite values for n .

Therefore the answer is subset $\{-2, -3, 15, -10\}$ which adds up to zero.

Given a set of integers $\{-2, -3, 15, 14, -10\}$ does some nonempty subset of them sum to 0?

Let i denote each integer in the set.

Since input $\{-2, -3, 15, 14, 7, -10\}$ equals 14. Then $b = 14$.

By using the formula $n = (2 \times i) - b$ derived from the proof of $x \pm y = b$ at system 1.

For $i = -2$, $n = -18$.

For $i = -3$, $n = -20$.

For $i = 15$, $n = 16$.

For $i = 14$, $n = 14$.

For $i = -10$, $n = -34$.

$i = 14$ when $n = 14$.

Since $n = 14$. Then 14 is a determinant.

Second hint: If n is equal to the total sum of input output all integers in the set except the integer that has equal value with the total sum of input. Therefore the answer is subset $\{-2, -3, 15, -10\}$ which adds up to zero.

V. COMPLEXITY

The complexity of the problem grows as we increase the length of random input. For instance, given

a set of integers $\{-2, -3, 15, 14, 7, -10, 50\}$ does some nonempty subset of them sum to 0?

Let i denote each integer in the set.

Since input $\{-2, -3, 15, 14, 7, -10, 50\}$ equals 71. Then $b = 71$.

By using the formula $n = (2 \times i) - b$ derived from the proof of $x \pm y = b$ at system 1.

For $i = -2$, $n = -75$.

For $i = -3$, $n = -77$.

For $i = 15$, $n = -41$.

For $i = 14$, $n = -43$.

For $i = 7$, $n = -57$.

For $i = -10$, $n = -91$.

For $i = 50$, $n = 29$.

No answer could be found because of absence of a determinant. Therefore no hint is applicable.

a) Comment

The algorithm presented in this paper outputs subset that sum to 0 quickly if we enter integers that sum to 0 as many as possible with one or two integers that do not sum to 0 via the program below e.g. a set of 200 integers like the set $\{-2, 2, -3, 3, -4, 4, \dots, 500\}$ or $\{-2, 2, -3, 3, -4, 4, \dots, 500, 9000\}$.

b) The algorithm

```
import javax.swing.*;
public class Proof {
    public static void main(String args[]) {
        double total = 0; int colum; int coum = 1;
        double x, q, lo = 1; int vb;
        String input, inp;
        double t = 0; long start = 0; long end = 0;
        String output = "Subscript\tValue\n";
        String seval = "", String seva = "", String seal = "";
        input = JOptionPane.showInputDialog("How many input do you want?");
        vb = Integer.parseInt(input);
        double[] array = new double[vb];
        double[] array = new double[vb]; double[] n = new double[vb]; double[] c1n = new double[vb];
        for (int counter = 0; counter < array.length; counter++) {
            inp = JOptionPane.showInputDialog("Follow the procedure and enter your values");
            q = Double.parseDouble(inp);
            array[counter] = q;
            total += array[counter];
            seval = seval + q + ", ";
        }
```



```

        start = System.currentTimeMillis();
    }
    for ( int counter = 0; counter < array.length;
counter++ )
    {
        arry[ counter ] = 2 * array[ counter ];
        seva = seva + arry[ counter ] + ", ";
    }
    for ( int counter = 0; counter < arry.length;
counter++ )
    {
        //start = System.currentTimeMillis();
        n[ counter ] = arry[ counter ] - total;
        seal = seal + n[ counter ] + ", ";
        if( n[ counter ]==total ){
            String answer=" ";
            for ( int i = 0; i < array.length; i++ ){
                if( array[i] != n[ counter ] )
                    answer+=array[i]+" ";
            }
        }
    }
    JTextArea outputArea = new JTextArea();
    end = System.currentTimeMillis();
    long res = end - start;
    JOptionPane.showMessageDialog( null,"Start
time was: "+start+"milliseconds. End Time was:
"+end+"milliseconds. Total time was:
"+res+"milliseconds. Result is: " + answer, "P is not
NP", JOptionPane.INFORMATION_MESSAGE );
    System.exit( 0 );
}
}

for ( int counter = 0; counter < n.length; counter++ )
{
    double cou = n.length;
    double fir = n[counter];
    for ( column = coum; column < cou;
column++ )
    {
        String hj="";
        double sec = n[column];
        c1n[column] = fir + n[column];
        if(c1n[column] == 0)
        {
            for ( int columns = 0; columns <
array.length; columns++ )
            {

```

```

        column;

        if (columns != cons)
        {
            if (columns != colu)
            {
                hj = hj + array[columns] + ", ";
            }
        }

        JTextArea outputArea = new JTextArea();
        end = System.currentTimeMillis();
        long res = end - start;
        JOptionPane.showMessageDialog( null,
"Start time was: "+start+"milliseconds. End Time was: "+end+"milliseconds. Total time was: "+res+"milliseconds. Result is: " + hj, "P is not NP",
JOptionPane.INFORMATION_MESSAGE);
        System.exit( 0 );
    }}
    coum = coum + 1;
    String outpt = "Subscript\tValue\tColumn\n";
}
for ( int counter = 0; counter < c1n.length;
counter++ )
{
    double chk = c1n[counter];double len =
c1n.length;

    if(chk != 0)
    {
        JOptionPane.showMessageDialog(
null,"No result. Program terminated!", "P is not NP ",
JOptionPane.INFORMATION_MESSAGE);
        System.exit( 0 );    }}
    System.exit( 0 );
}
}
}

```

VI. CONCLUSION

Subset sum problem is NP-complete and the proof of $x \pm y = b$ at system 1 has been established in the subset sum problem and each number is equally probable. This justifies that each integer in the set of integers are in conjunction with 'b'. Therefore, the algorithm and the assertion presented in this paper can be dependent on as reliable.

The program outputs subset that sum to 0 quickly when integers that sum to 0 combines with one or two integers that do not sum to 0 regardless of input size. As the complexity of the problem grows the program terminates quickly without an answer.

Is $P = NP$? This is the real question we are concerned with. Is the set of problems in P actually the same as the set of problems in NP ? The program is a P algorithm; an answer in polynomial time is called P . Questions for which an answer can be verified in polynomial time is called NP . The program terminates and those NP -complete problems do not exist in P therefore, $P \neq NP$.

Thus it is concluded that the existence of an algorithm exists in polynomial time (P) only if the algorithm runs in polynomial time regardless of input size.

- Here's a P program that does solve the problem
- It does solve the problem because the problem does not exist in P .
- Therefore $P \neq NP$.

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Numerical Simulation of Vertical Axis Wind Turbine at Low Speed Ratios

By Ion Mălăel, Horia Dumitrescu & Vladimir Cardoso

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Abstract- A renewed interest in vertical axis wind turbines (VAWT) has been seen recently, in particular at relatively low Reynolds number ($Re \approx 10^5$) appropriate to the urban applications. From this perspective, the Computational Fluid Dynamics (CFD) is regarded as a promising technique for aerodynamic studies of VAWT. The paper presents a computational investigation on a particular dynamic stall phenomenon associated with unsteady flow around the NACA 0018 airfoil of a three straight bladed rotor, at high angle of attack (AOA). Two airfoil flows with angle of attack higher than 45° of an isolated blade and a confined blade in rotor at low speed ratios (TSR), are numerically simulated using CFD. It is concluded that the quasi-steady prediction used in previous models is in disagreement with experimental and numerical data because the unsteadiness generated by spinning rotor, though very important for the self-starting of VAWT, in the past were ignored.

Keywords: *dynamic stalls; low reynolds number, CFD; vawt.*

GJRE-I Classification : *FOR Code: 230116*



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Numerical Simulation of Vertical Axis Wind Turbine at Low Speed Ratios

Ion Mălăeș^a, Horia Dumitrescu^σ & Vladimir Cardoso^p

Abstract- A renewed interest in vertical axis wind turbines (VAWT) has been seen recently, in particular at relatively low Reynolds number ($Re \approx 10^5$) appropriate to the urban applications. From this perspective, the Computational Fluid Dynamics (CFD) is regarded as a promising technique for aerodynamic studies of VAWT. The paper presents a computational investigation on a particular dynamic stall phenomenon associated with unsteady flow around the NACA 0018 airfoil of a three straight bladed rotor, at high angle of attack (AOA). Two airfoil flows with angle of attack higher than 45° of an isolated blade and a confined blade in rotor at low speed ratios (TSR), are numerically simulated using CFD. It is concluded that the quasi-steady prediction used in previous models is in disagreement with experimental and numerical data because the unsteadiness generated by spinning rotor, though very important for the self-starting of VAWT, in the past were ignored.

Keywords: *dynamic stalls; low reynolds number, CFD; vawt.*

I. INTRODUCTION

The depletion of fossil energy resource and global warming trends has lead to the recognition of a low carbon economy as an international strategy for sustainable development. Among several green and renewable energy resources, wind energy has seen a rapid growth worldwide and will play an increasingly important role in the future economy.

Wind turbines are typical devices that convert the kinetic energy of wind into electricity. From the perspective of urban applications, where the wind is very turbulent and unstable with fast changes in direction and velocity, vertical axis wind turbines have several advantages over the widely used horizontal axis wind turbines. However VAWT suffer from many complicate aerodynamic problems, of which dynamic effects are inherent phenomena when they operating at low values of tip speed ratio (TSR) $\lambda < 2$, and this has a significant impact on their self-start capabilities, i.e. without external assistance. Therefore, it is crucial to have a good understanding of the starting process, in particular at relatively low Reynolds number ($Re \approx 10^5$) appropriate to the urban applications of VAWT, which remains to this day incomplete.

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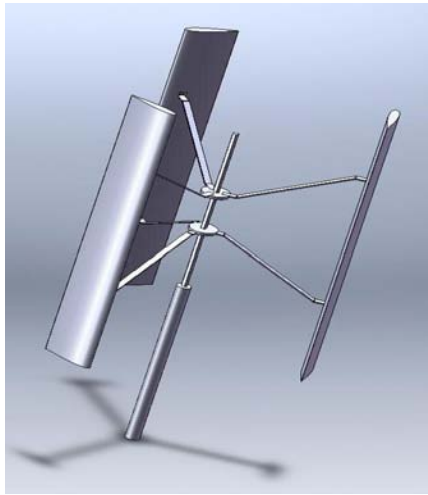
Traditionally, dynamic stall is a term used to describe the delay in the stall on wings and airfoils that are rapidly pitched with angle of attack, α , significantly beyond the static stall angle and normally can generate a substantially larger lift for a short period of time than can be obtained quasi-statically [1],[2]. The VAWT blade operating at $\lambda \geq 2$ perceives a cyclic variation in the relative wind speed and the angle of attack which is very similar to what would be experienced with a sinusoidal pitching blade in a stationary frame of reference.

On the basis of this similarity the dynamic stall on VAWT blades was investigated using the simpler motion of oscillating airfoils [3]. However at low speed ratios, $\lambda < 2$, the motion has both pitch component and plunge component and the blade frequently experiences high angles of attack beyond the stall value.

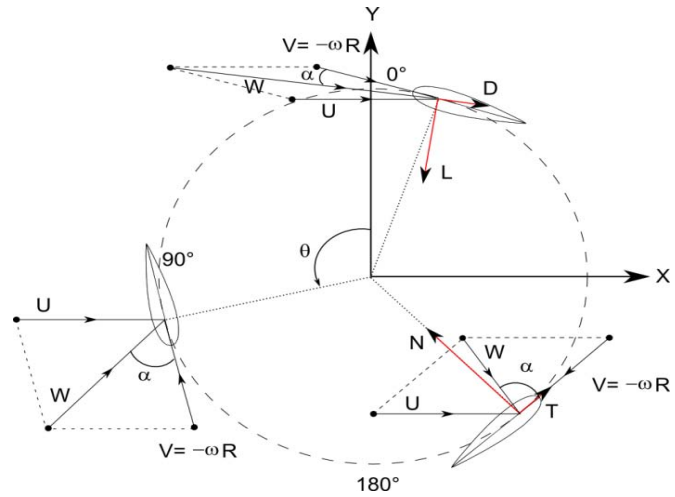
Particularly at very low TSR that often occurs in the starting process, the maxim AOA is far beyond the stall angle. Therefore a good representation of high AOA flows is essential in the correct prediction of the aerodynamics and VAWT performance.

II. MOTION AND AERODYNAMICS OF VAWT

Figure 1a is a schematic of a straight bladed fixed-pitch VAWT which is the simplest but typical form of the Darrieus type VAWT. Despite the simplicity, its aerodynamic analysis is still quite complex. One feature is that the relative velocities perceived by the blade always change as the blade moves to different azimuth positions. Figure 1b illustrates typical flow velocity around a rotating VAWT blade at a given azimuth angle as well as the aerodynamic forces perceived by the blade. The azimuth angle is set to be 0 when the blade is at the top at the flight path and it increases in a counter-clock wise direction. It should be noted that, even disregarding the variation of the induced local flow velocity U_{local} , both the magnitude and the direction of the effective velocity perceived by the blade, U_{eff} , change in a cyclic manner as the blade rotates through different azimuth angles. This kind of motion is called the Darrieus motion [4]. As a result, the aerodynamic loads exerted on the blade change cyclically with θ .



a)



b)

Figure 1 : Basics of VAWT: a) sketch of a fixed-pitch straight-bladed VAWT; b) typical flow velocities in Darrieus motion

From the vectorial description of velocities, Fig. 1b, we can obtain the following expression that establishes the relationship between angle of attack α_D

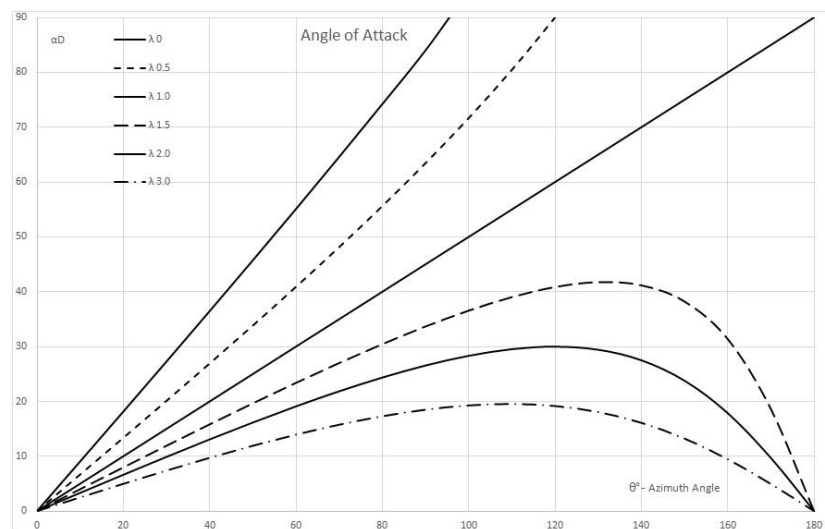
the tip speed ratio λ and azimuth angle θ of a blade in Darrieus motion (without the velocity induction).

$$\tan \alpha_D = \frac{U_\infty \sin \theta}{\Omega R + U_\infty \cos \theta} = \frac{\sin \theta}{\lambda + \cos \theta} \text{ or } \alpha = \tan^{-1} \left(\frac{\sin \theta}{\lambda + \cos \theta} \right) \quad (1)$$

Another important parameter is the reduced frequency which governs the level of unsteadiness. The reduced frequency k , defined as $k = \frac{\omega c}{2U_{eff}}$ where ω is the angular frequency of the unsteadiness, c is the blade chord and U_{eff} is the effective velocity of the blade, can be expressed in terms of TSR as:

The variation of angle of attack α_D , its normalized values $\frac{\alpha_D}{\alpha_{Dmax}}$, and the reduced frequency are evaluated from eqs. (1) and (2) as function of the azimuth angle θ for various values of λ , as shown in Fig. 2.

$$k = \left(\frac{c}{D} \right) \frac{\lambda}{\sqrt{\lambda^2 + 2\lambda \cos \theta + 1}} \quad (2)$$



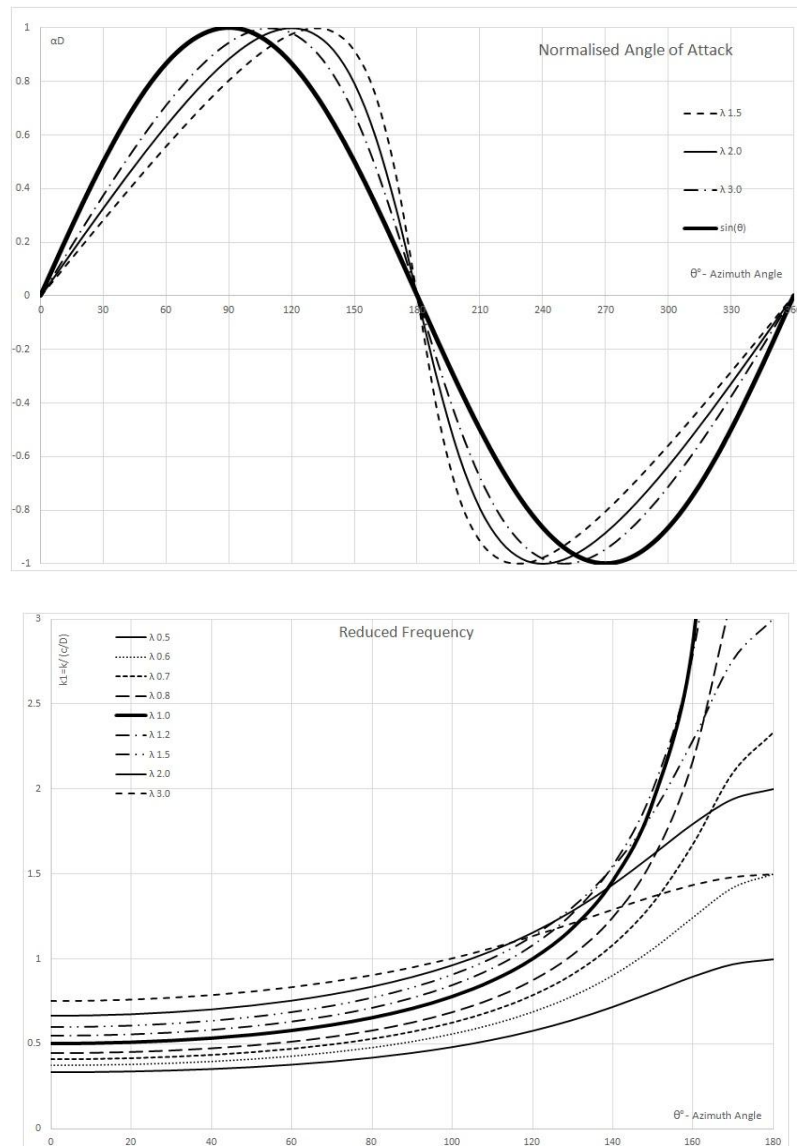


Figure 2 : Flow conditions for Darrieus motion: a) variation of angle of attack; b) normalized angle of attack and sine-curve; c) reduced frequency as a function of λ and the azimuth angle θ

This figure shows some specific features of the Darrieus motion which behaves differently in terms of TSR as:

- The variation of angle of attack for $\lambda \approx 1$ presents a strong discontinuity-like type instability in the point of maximum incidence corresponding to the change of incidence direction at the downstream passage ($\theta = 180^\circ$); for the values of $\lambda > 1$ the passage point is an inflectional instability of incidence variation when the angle of attack becomes zero, (Fig 2a). The unsteady flow phenomena caused by the instabilities of Darrieus motion is a complex mechanism which leads to formation of an intense leading-edge vortex, post-stall vortex shedding, reattachment of flow, and wake capture.
- The variations of normalized angles of attack α at the high values of ($\lambda \geq 2$) are very similar to the

sine-curve, i.e. $\alpha = \sin \theta$ with their peaks at about same azimuth angle of 90° ; unlike these variations, the variations at the low values of λ ($\lambda < 2$) contain elements of plunging motion and have their peaks at different azimuth angles (130° for $\lambda = 1.5$) (Fig. 2b).

- This difference is typically termed the phase shift which is an important parameter for generation of thrust.
- The variation of the reduced frequency k shows the existence of a band of tip speed ratios about $\lambda = (0.7 - 1.5)$ with the rough increase of frequency like a discontinuity. (Fig. 2c) (0.7 - 1.5).

According to the diagram from Fig.2c, the unsteadiness associated with the flow field and VAWT operating state can be classified into three levels:

- *Zero level* is the distributed unsteadiness, when $k/(c/D) \leq 1.0$ and $\lambda \leq 0.5$; commonly this effect is neglected and a quasi-steady assumption is used;
- *First level* is the located unsteady phenomenon of dynamic stall with lift increment at low angle of attack ($\alpha \approx 25^\circ$), occurring at $\theta = 90^\circ$ when $k/(c/D) \leq 2.0$ and $\lambda \geq 2$; its effect is similar to a sinusoidal pitching airfoil;
- *Second level* is the located unsteady phenomenon of dynamic stall with drag reduction at high angle of attack ($\alpha > 45^\circ$), occurring at $\theta = 180^\circ$ when $k/(c/D) > 2.0$ and $\lambda = 0.7-1.5$; to this day it is still unknown.

The two types of dynamic stall address to different portions of the static lift characteristic which for the VAWT blades operating at low TSR presents a particular double peak characteristic, with two peak values, C_{L1S} at low AOA ($\alpha_{1S} \approx 10^\circ$) and C_{L2S} at high AOA $\alpha_{2S} \approx 45^\circ$, Fig.3. The main difference between these two types of stall is dependence upon

the Reynolds number: first stall is much dependent on Reynolds and second stall is practically with no-effect on it.

The first phenomenon is a lift dynamic stall similar to airfoils rapidly pitching with the angle of attack α , significantly beyond the static stall angle, α_{1S} , and normally can generate a substantially larger lift than can be obtained quasi-statically. This phenomenon is well documented [5], [6], and results from the combination of the unsteady motion of the airfoil and the separation of the boundary layer, when the stall process can be divided into four key stages, i.e. attached flow, development of the leading- edge vortex, post-stall vortex shedding and the reattachment of the flow. Therefore, in the present paper we only focus on the Darrieus motion of confined blades at low TSR which experience high values of the angle of attack, $\alpha_D > 45^\circ$, and can trigger the drag dynamic stall phenomenon less influenced by Reynolds number.

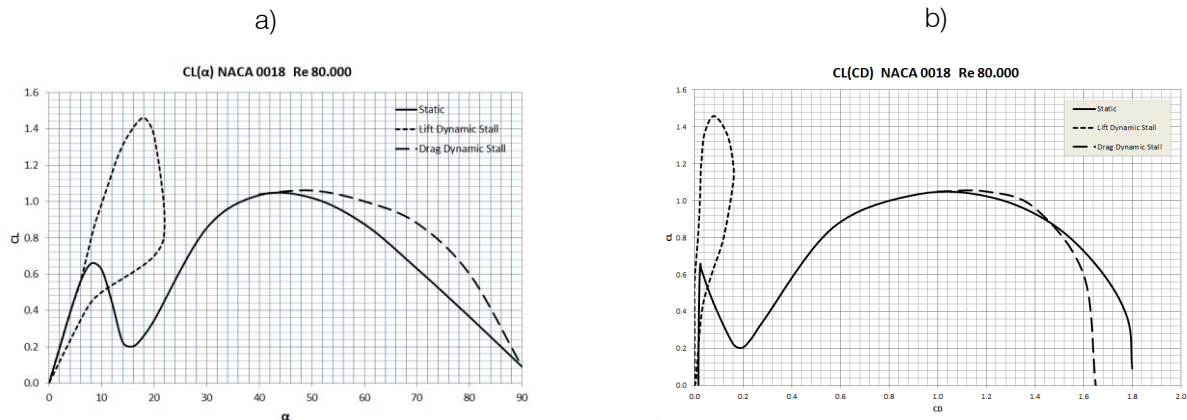


Figure 3 : Static and dynamic lift and drag coefficients of NACA0018 airfoil at $Re_c = 80,000$; a) $C_L(\alpha)$; b) $C_L(C_D)$

The drag dynamic stall occurs only on blades operating in a closed flow field in which the rotor is acting as a pump on the separated volume of air forcing it to move radially towards the blade.

The drag dynamic stall is a term used to describe the delay in the drop of the second static stall lift coefficient C_{L2S} on the blade passing in the downwind ($\theta = 180^\circ$) and which can generate simultaneously little lift and significant drag reduction for a short period of time when TSR is of order unity. The drag stall dynamic stall, occurring at low TSR in the range of $\lambda = 0.7-1.5$ (Fig. 2c), characterizes the shift of the operating modes from mixed lift-drag driving to full lift driving which is important for the continuous thrust-producing, i.e. the self-starting of rotor. Therefore, the objective of this paper is to correctly simulate drag dynamic stall which is found in VAWTs and make a contribution towards a better understanding of the flow physics of this unknown phenomenon directly related to the self-starting of VAWT intended for the built and urban environment in the future.

III. CFD SIMULATIONS

The CFD simulation of airfoil flow with an AOA higher than 45° is rarely discussed in the literature. However blades encounter a very high AOA as they rotate at a low TSR (as shown in fig 2a). The aerodynamic data of static airfoil at AOAs ranging from 0° to 180° is the fundamental input of a double-multiple stream tube (DMS) model that is one widely accepted method for evaluating the power of VAWTs in engineering practice [7]. But, the unsteadiness generated by the rotor operating at low TSR is inevitable in assessment VAWT starting performance. So that, in this section it's contribution due to a confined airfoil into a three straight-bladed rotor is examined and compared with the data from a single static airfoil.

a) CFD simulation of single static airfoil

Airfoil NACA 0018 is one of commonly used blade section in VAWTs. In this investigation the aerodynamic data for a full range of AOAs published by Scheldahl and Klimas [8] is used. These experimental

data offer a good opportunity to examine the capacity of CFD at very high AOA's. The commercial CFD software Fluent was employed in flow computational. Fluent is based on the finite volume method which discretizes the computational domain into some small volumes and has been tested in many applications. The detailed computational treatments and algorithms are explained as follows:

i. *Mesh geometry and boundary conditions*

Figure 4a shows the geometric scheme and boundary conditions in the CFD model of single NACA

0018 airfoil. The far-field boundary was set as velocity inlet. It was located $30c$ away from the airfoil where c is the chord length of the airfoil, in order to avoid wave reflection. The oncoming flow velocity was 10 m/s , which, together with chord length, results in a Reynolds number of 10^5 .

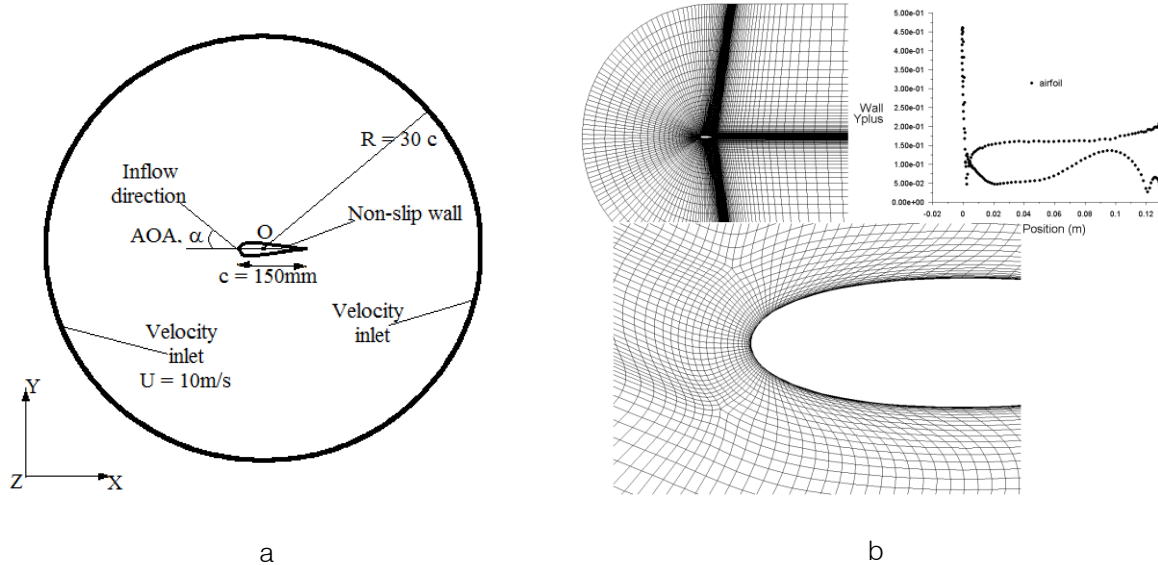


Figure 4 : Computational setup: for a single NACA0018 airfoil: a) model geometry and boundary conditions; b) mesh structure and y^+ value

Both O and C-grid mesh topologies can minimize the skewness of a near-wall mesh and converge fast under a high-order discretization scheme. In this study, the O-grid topology was adopted because it can reduce grid number and avoid high aspect ratios of grids in the far wake. In order to resolve the laminar sub layer directly, the first grid spacing on the airfoil was determined to make y^+ less than 1. Grid-stretching was limited to less than 1.08 in both streamwise and cross flow directions to ensure numerical stability. Figure 4b shows the final mesh in 2D model.

ii. *Transition Model*

The incompressible Navier-Stokes equations are appropriate for solving the VAWT aerodynamics,

- transport equation for the intermittency, γ :

$$\rho \frac{\partial \gamma}{\partial t} + \rho \frac{\partial (U_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial t} \left[\left(\mu + \frac{\mu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right] \quad (3)$$

- transport equation for the momentum thickness Reynolds number, $\text{Re}_{\theta t}$:

$$\rho \frac{\partial \text{Re}_{\theta t}}{\partial t} + \rho \frac{\partial (U_j \text{Re}_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j} \left[\sigma_{\theta t} (\mu + \mu_t) \frac{\partial \text{Re}_{\theta t}}{\partial x_j} \right] \quad (4)$$

because the resultant flow velocity is generally smaller than 0.3 times the Mach number. Stall, either static or dynamic, may occur in a rotating VAWT and both are dominated by vortex separation and involve flow unsteadiness. Therefore, an unsteady fluid solver is necessary to investigate such kinds of flow.

The choice of transitional models influences the computational results and the required computation resource. The transition model uses two transport equations, one for intermittency and one for a transition onset criterion in terms of momentum thickness Reynolds number [9].

iii. Simulation setup

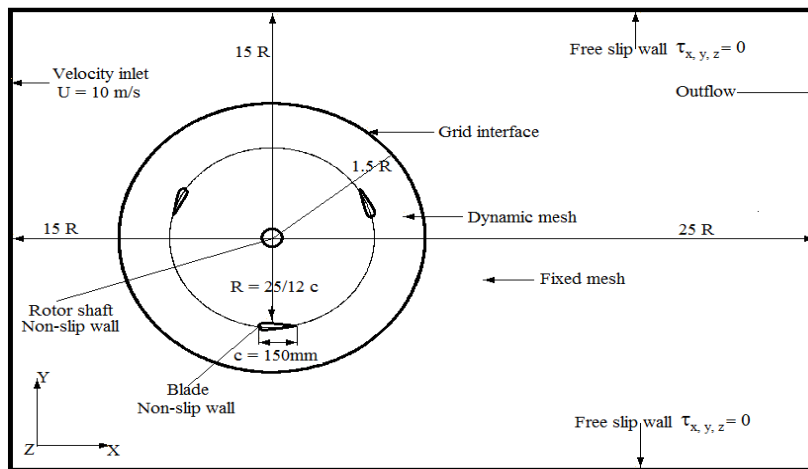
The 2D Unsteady Reynolds-Averaged Navier-Stokes (URANS) approach was selected to solve the discretized continuity and momentum equation, and a second order implicit formula was used for the temporal discretization. The SIMPLEC scheme was used to solve the pressure-velocity coupling. SIMPLEC converges faster than SIMPLE. Time step size is a crucial parameter in unsteady flow simulations. To get accurate results of an airfoil beyond stall, Sorensen et al. [10] and Travin et al. [11], suggested the non-dimensional time steps $\tau = \frac{\Delta t U_\infty}{c}$ to be 0.01 and 0.025 respectively.

$\tau = 0.01$ (according to the real time step $\Delta t = 0.00015$) was applied in the simulations of the single airfoil.

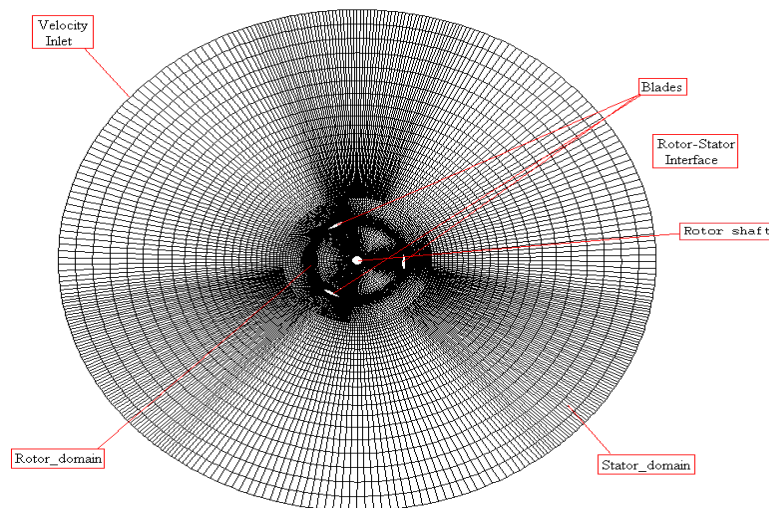
b) CFD simulation of three straight-bladed VAWT

Figure 5a shows the geometry and the flow conditions of the VAWT model in the CFD simulation.

The radius of the VAWT was $R = 0.3125\text{m}$. A velocity inlet with a constant wind speed $U = 10\text{ m/s}$ was located at 4.5 m (i.e. $14R$) in front of the turbine. The applied outflow condition combined a zero diffusion flux of all flow variables in the normal direction to the exit plan and an overall mass balance correction. The side boundaries were 4.5 m ($14R$) from the turbine center to minimize the blocking effect. A free-slip all boundary conditions was applied where the normal velocity components and the normal gradients of all velocity components were assumed to be zero. The mesh configurations used in the CFD model of the single static airfoil were transferred into the VAWT model with new boundary adaptation as is shown in Fig. 5b.



a)



b)

Figure 5 : Computational setup for 3 straight-bladed VAWT: a) model geometry and boundary conditions; b) mesh structure for whole domain

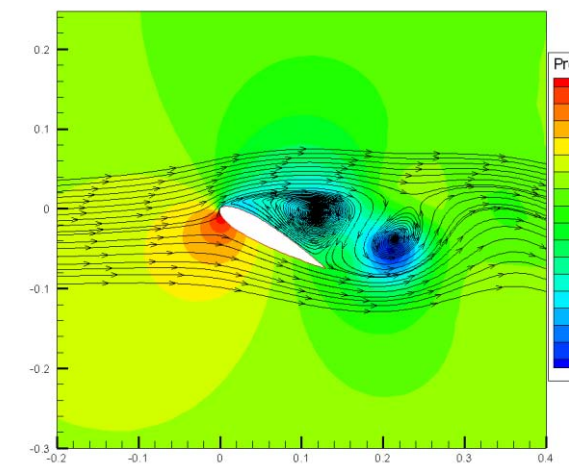
The dynamic effects of the blade influence the energy extraction process of VAWT, and thus, the determination of the time step should consider amplitude, frequency and far field velocity. In the present study, the reduced frequency was $k = 0.24$, the physical time step was $0.5^0 \Omega^{-1}$ and the corresponding non-dimensional time step was $\tau = 0.02$ [12].

IV. RESULTS AND DISCUSSIONS

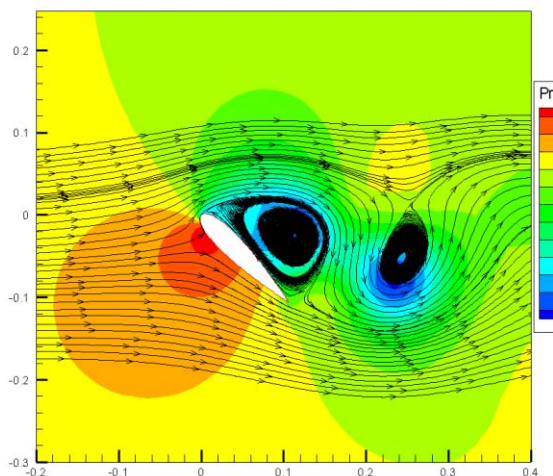
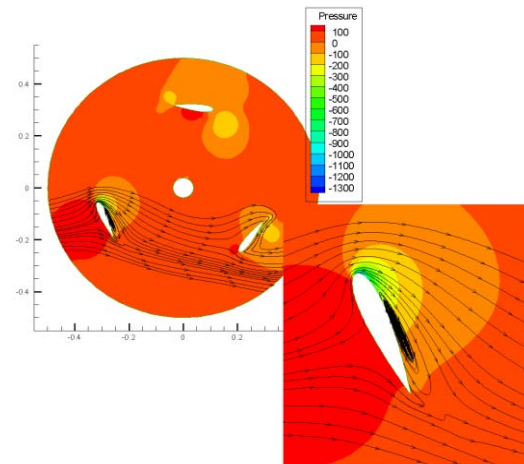
The flow in the VAWT operating at wind speed of 10 m/s was simulated using 2D URANS. Results for a single airfoil blade in high angle of attack flow were compared with the results obtained by a three straight-bladed rotor at low TSR.

The CFD simulation of airfoil flow with an AOA higher than 45° is rarely discussed in the literature where the flow is dominated by the dynamics of the interacting vortices generated by the separating boundary layer. Figure 6a, b shows the pressure field superimposed on the instantaneous streamlines computed for both single

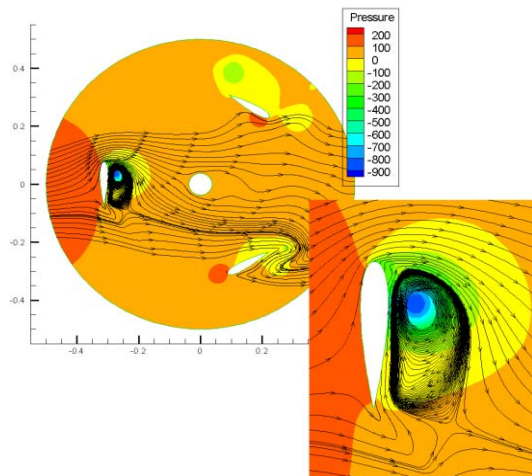
blade and three-confined blades. The single airfoil blade in high angle of attack flow produces a different wake structures as the AOA increases, Fig.6 a. In the near wake there are two main vortices: leading edge vortex (LEV) and trailing edge vortex (TEV). The position of vortices change in terms of the AOA as : when the AOA is $\alpha = 30^\circ$, the LEV is stronger than TEV and this is the last shed vortex (SV) so that the flow produces a classical von Karman vortex street; when the AOA is of order $\alpha \approx 45^\circ$, the both LEV and TEV have comparable strengths, SV becomes nearer the airfoil and the flow produce pairs of counter-rotating vortices of equal strengths, called viscous (weak) vortex doublet (VD); and when the AOA exceeds the value $\alpha \geq 60^\circ$, noticeably in the flow pattern near the airfoil there are a cluster of the two main vortices and the formation of a quasi-potential (strong) vortex doublet (QPD). Considering Reynolds number circumstances in this study, these vortex structures forming slowly rotating QPD street are perceived as large separation bubbles shedding from the blade.



$\alpha = 30^\circ$



$\alpha = 45^\circ$



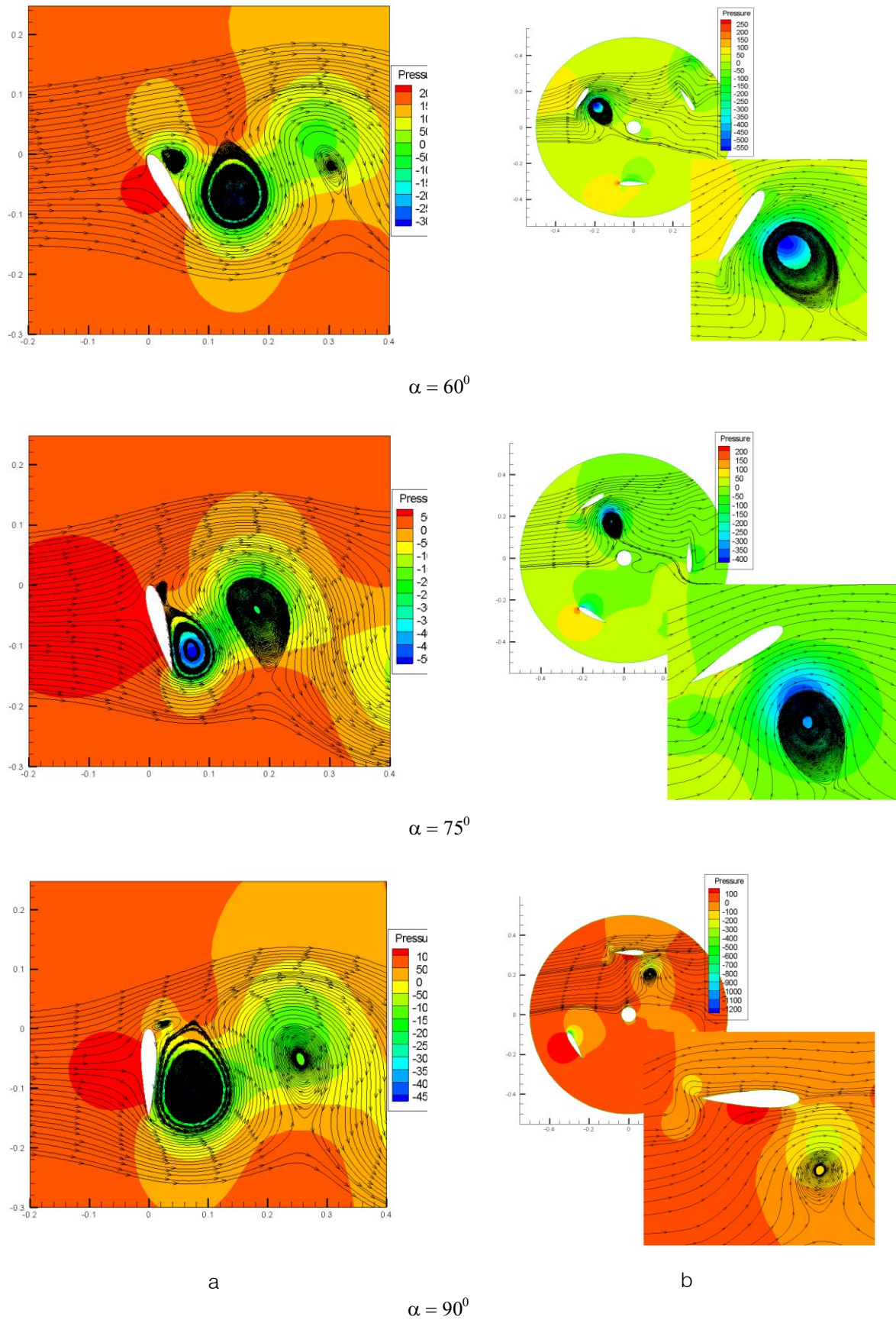


Figure 6 : Pressure field superimposed on the instantaneous streamlines computed using the transition SST model for NACA0018 airfoil at $Re_c = 10^5$: a) single static airfoil; b) 3-confined airfoils rotating at $\lambda = 1.0$

The simulation of the three-straight-bladed rotor reveals another interacting vortex flow which can trigger off the drag dynamic stall phenomenon when the TSR is of order unity. In contrast to the free wake of a single airfoil blade, the closed wake formed in the upwind zone of VAWT is a QPD detaching from the blade due to the flow curvature effect at a certain azimuth angle. After this location, the free QPD strongly interacts with blade and changes the pressure distribution around the airfoil, which has a significant impact on the aerodynamic forces. Drag dynamic stall is an intrinsic feature of blade in Darrieus motion which can be described as a two step process: outset of a vortex doublet structure as the blade approaches its downwind passage ($\theta = 180^\circ$) when TSR is of order unity followed by the interaction between the separated boundary-layers flow on the suction side of the airfoil and the vortex doublet which shifts inwards due to the flow curvature effect. This features as can be seen in Fig. 6b is well captured by 2D URANS model and a two transport equation transition model.

Figure 6b presents a chronology of the static pressure fields at different azimuth angles θ of a blade in Darrieus motion at $\lambda = 1.0$, superimposed on the instantaneous stream lines to depict the complicated vortex structures during the stall process. In the early stage of the upwind phase, $\theta = 45^\circ (\alpha_D = 22.5^\circ)$ a long separation bubble can be detected on the upper surface (here it is not shown). Considering the low Reynolds number circumstances in this study, this bubble is actually the so-called laminar separations bubble (LSB) in which the flow turbulence intensity is significantly enhanced and this causes a turbulent boundary layer to appear after the LSB. The LSB grows in size and it travels towards the trailing edge of the airfoil as increases and at $\theta = 60^\circ (\alpha_D = 30^\circ)$ the LSB becomes turbulent beginning from the trailing edge. At $\theta = 75^\circ (\alpha_D = 37.5^\circ)$, the turbulent separation bubble (TSB) which has covered the whole the whole suction surface begins to turn into a VD and at $\theta = 90^\circ (\alpha_D = 45^\circ)$ the TSB was completely turn into the VD. At this instance, the VD covers the whole suction surface and C_L is at its maximum value. Further as the blade moves, the VD is degenerated into a concentrating leading edge-vortex (CV), while the weaker trailing edge-vortex is convected away in the downwind movement. At, $\theta = 120^\circ (\alpha_D = 60^\circ)$ CV detaches and is come localized in the vicinity of the upper surface of blade. At this instance C_D is at its maximum value, after which the drag coefficient drops roughly. The CV moves with the blade in the leeward movement and after the downwind passage ($\theta = 180^\circ; \alpha_D = 90^\circ$) the suction surface switches to opposed side of the blade when the wind favors the convection of the concentrated vortex away from the

airfoil, so that the drag dynamic stall ceases and drag increases.

This entity embedded in the flow field plays the role of "focus" which squeezes the streamlines around the airfoil when it passes across from upwind to downwind and gradually accelerate flow on the upper surface producing a lower pressure and, thus, the flow around blade behaves like as inviscid one. Therefore, the rough switch of the angle of attack at $\lambda = 1$ and $\theta = 180^\circ$ seen in Fig. 2c, actually becomes a smooth process via blade-vortex wake interaction which has the ability to produce a continuous pressure variation at the downwind passage.

Figure 7 shows the comparison of the lift and drag coefficients from the wind tunnel tests [8], as well as the 2D URANS simulations for a single airfoil blade and three straight-bladed rotor at $Re_c = 10^5$. The over predicted results seen in figure 7 are consistent with the observations found [13], [14] that the 2D models is not adequate for predicting unsteady flow structures with large-scale separations around airfoils at relatively high AOAs. However, the used model does not attempt to model with fidelity the wake vortices, but it is used as a computational tool for the understanding of the different aerodynamic behaviors of airfoils as isolated one and confined other.

When the blade, operating in a Darrieus motion, at , TSR ≈ 1.0 passes through azimuth angle of 120° a vortex doublet structure shifts inwards, interferes with the flow around blade producing a pressure drop along the suction side of airfoil, and the blade is in drag stall, leading to a sudden drop in drag coefficient, see Fig. 7. With the increase of azimuth angle, the vortex doublet moves away from the suction surface of blade and it is convected in the leeward movement when it is at azimuth angle of 180° . Further downstream, the flow penetrated from the pressure side of blade into the suction side and the angle of attack began to increase, and therefore a reversed flow occurs at trailing edge, which has a significant impact on the aerodynamic forces, namely the drag coefficient roughly increases during downwind stroke.

Figure 8 shows the tangential force coefficient in terms of azimuth angle and its average value for the three straight-bladed rotor at TSR = 1.0. This positive value indicates the self-starting capability of the Darrieus rotor for high unsteadiness, $c/D = 0.24$, explained by drag dynamic stall rising.

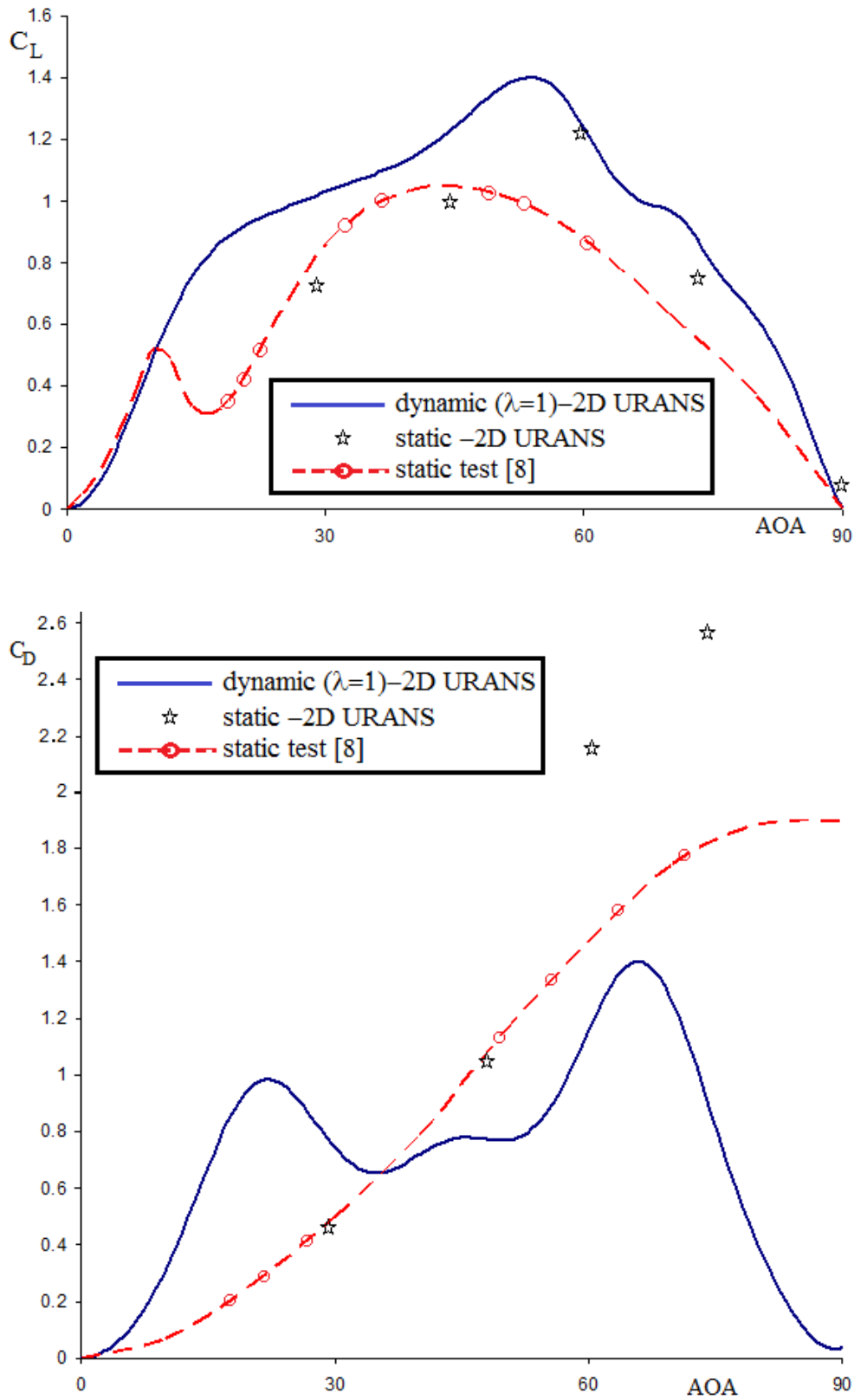


Figure 7: Static and dynamic lift and drag coefficients of NACA0018 airfoil for $Re_c = 10^5$

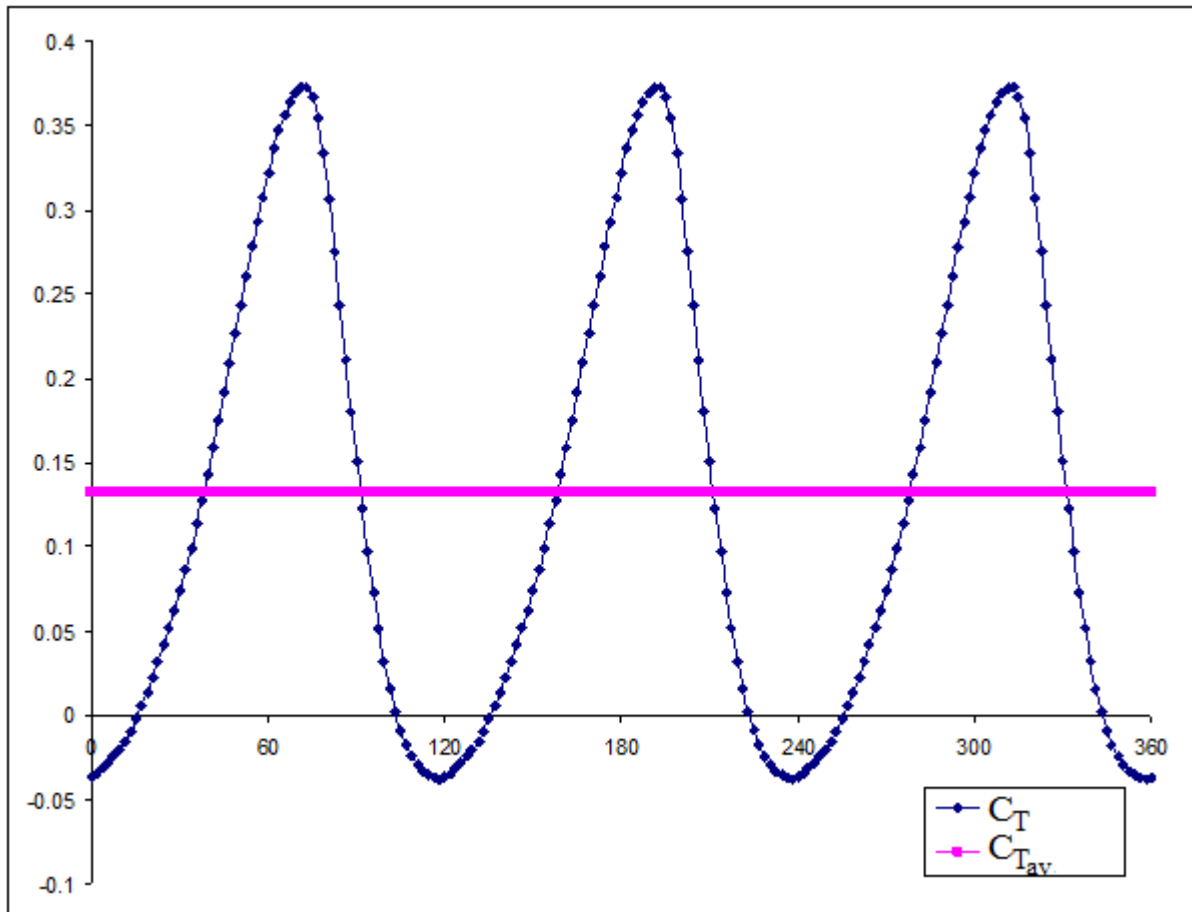


Figure 8 : Curve of tangential force coefficient of 3 straight bladed VAWT for $\lambda = 1.0$

V. CONCLUSIONS

- This paper investigates computationally the VAWT starting process occurring at $TSR=1.0$, when the blades experience high angles of attack and large incidence variations. The computed streamlines pattern reveals a vortex doublet structure produced by the incidence change in the vicinity of the downwind passage of the blade which affects the separation of the boundary-layer flow on the upper surface of airfoil. The result of this blade-vortex interaction leads mainly to the significant reduction of the aerodynamic drag. It is called drag dynamic stall produced by a special configuration interaction, which is an intrinsic feature of blades in Darrieus motion.
- The drag dynamic stall process is triggered by a certain unsteadiness level inside rotor and promotes the shift of operating modes of VAWTs from mixed lift-drag driving to full lift driving, and thereby produces the continuous thrust production when TSR exceeds the value one. However, the shift to full lift-driven state is not a guarantee of further acceleration and it is possible the rotor will be

locked in the dead band ($1 \leq \lambda \leq 1.5$) due to a large area of high angle of attack and insufficient thrust production. In this case other parameters as blade thickness and turbine solidity can be altered for overcoming this drawback.

- Concerning the used computational approach it is remarked that though the 2D URANS model is not adequate for predicting accurately unsteady flow structures with large-scale separation, however, for
- present Re_c it has been able to capture the main features of the drag dynamic stall phenomenon here identified.

VI. ACKNOWLEDGMENTS

This work was realized through the Partnership programme in priority domains – PN II, developed with support from ANCS CNDI – UEFISCDI, project no. PN-II-PT-PCCA-2011-32-1670.

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GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: I
NUMERICAL METHODS

Volume 14 Issue 1 Version 1.0 Year 2014

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 2249-4596 & Print ISSN: 0975-5861

THA-A Hybrid Approach for Rule Induction System using Rough Set Theory, Genetic Algorithm and Boolean Algebra

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GJRE-I Classification : FOR Code: 010101, 230101p



Strictly as per the compliance and regulations of:



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Tribikram Pradhan ^α, Harsh Anand ^σ & Akul Goyal ^ρ

Abstract- The major process of discovering knowledge in database is the extraction of rules from classes of data. One of the major obstacles in performing rule induction from training data set is the inconsistency of information about a problem domain. In order to deal with this problem, many theories and technology have been developed in recent years. Among them the most successful ones are decision tree, fuzzy set, Dempster-Shafer theory of evidence. Unfortunately, all are referring to either prior or posterior probabilities. The rough set concept proposed by Pawlak is a new mathematical approach to inconsistent, vagueness, imprecision and uncertain data. In this paper we have proposed a hybridized model THA (Training dataset on hybrid approach) which combines rough set theory, genetic algorithm and Boolean algebra for discovering certain rules and also induce probable rules from inconsistent information. The experimental result shows that the projected method induced maximal generalized rules efficiently. The hybridized model was validated using the data obtained from observational study.

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I. INTRODUCTION

The major process of discovering knowledge in database is the extraction of rules from classes of data. In order to automate this problem, many inductive learning methods are introduced and applied to extract knowledge from databases such as decision tree learning [1], the technology employs a decision tree to achieve the learning function. The rough sets concept proposed by Pawlak in 1982 [2] is a new mathematical approach to imprecision, vagueness and uncertain. The rough sets philosophy is founded on the assumption that with every object of the universe of discourse we associated, some information objects characterized by the same information are indiscernible in the view of the available information about them. The indiscernibility relation is the mathematical basis of rough sets theory. Any set of all indiscernible objects is called an elementary set and forms a basis granule of knowledge about the universe [14][18]. Any union of elementary sets is referred to as a precise set, otherwise the set is

rough. Each rough set has boundary line cases. With any rough sets a pair of precise sets- called the lower and the upper approximation of the rough sets is associated [7]. In recent 20 years, rough sets approach seems to be of fundamental importance to artificial intelligence (AI) and cognitive sciences and has been successfully applied many real life problems in medical diagnosis engineering [3], banking [3], finances [4] and others. By coupling rough sets theory with genetic algorithms (GA's), it is able to enhance search speed, induce decision rules from inconsistent information and this paper presents a hybrid approach that integrated rough sets theory, GA's and Boolean algebra for rule induction.

II. GENETIC ALGORITHMS (GA'S)

GA's have been established as a viable technique for search, optimization, machine learning, and other problems. Theoretical developments by Holland and De Jong have laid the foundation of GA's [9]. GA's have been theoretically and empirically proven to provide robust search in complex space. The genetic algorithm consisting of a number of iteration process to make the population evolve [6]. Each iteration consists of the following steps:

- **Selection:** The first step consists of selecting individuals for reproduction [12]. This selection is done randomly with a probability depending on the relative fitness of the individuals so that best ones are often chosen for reproduction than poor ones.
- **Reproduction:** In the second step, offspring are bred by the selected individuals. For generating new chromosomes, the algorithm can use both recombination and mutation.
- **Evaluation:** Then the fitness of the new chromosomes is evaluated.
- **Replacement:** During the last step, individuals from the old population are killed and replaced by the new ones. Then, the genetic algorithm loops over an iteration process to make the population evolve. Figure 1 depicts the life cycle of GA's.

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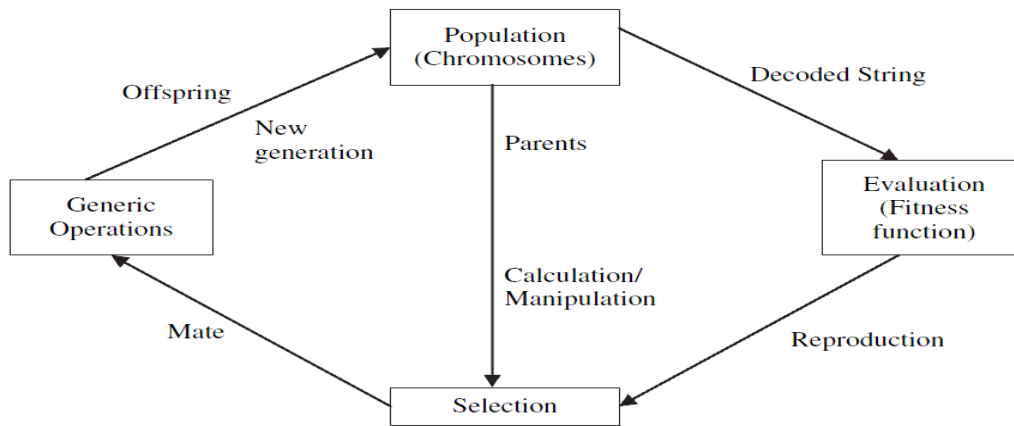


Figure 1 : Genetic algorithm cycle

Finally, the reproduction step involves the creation of offspring chromosomes by using two genetic operators- mutation and cross-over. The most important part of the genetic algorithm is cross-over where we have to randomly select a cross site and swaps the genes of two parent chromosomes to produce two new offspring chromosomes. This can be easily represented

using a pair of chromosomes encoded with two binary strings and cross site is denoted by “|”.

For example:

Chromosome1=10011|01100010110

Chromosome2=01101|11011000011

Offspring 1=1001111011000011

Offspring 2=0110101100010110

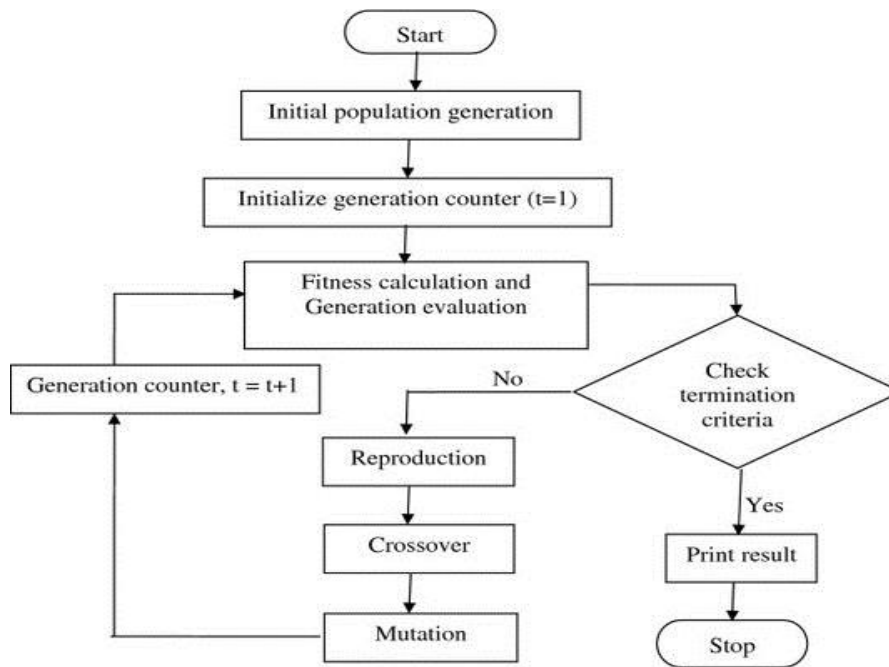


Figure 2 : Flow chart of genetic algorithm

After the crossover operation we have to perform mutation. This is used to prevent the solution converges prematurely. For a chromosome encoded as a binary string, genes are randomly selected to undergo mutation operation where 1 is changed to 0 and vice versa.

For example, in our above example

Mutated Offspring1= 1001110011000011

Mutated Offspring2= 0110100100010110

III. ROUGH SET THEORY

a) Information System

Knowledge representation in Rough sets theory is done via information system.

Definition1 : An information system can be defined as a 4-tuples as follows

$S = \{U, A, V, F\}$ where,

U represents finite set of objects. $U = \{x_1, x_2, \dots, x_n\}$

A is a finite set of attributes (a_1, a_2, \dots, a_n)

V is the domain of each attribute of A.

F is a total function that defines the following application
 $U \times A \rightarrow V$.

Definition 2 : In an information system, S the attributes A are further classified into disjoint sets of condition attributes C and decision attributes D.

i.e. $\Omega = C \cup D$ and $\emptyset = C \cap D$.

All the information represented by a table known as Information System which encompasses a number of rows and columns corresponding to the number objects and attributes.

Table 1 : Information System

Object U	Condition Attribute		Decision Attribute
	A	B	C
x_1	1	0	0
x_2	0	1	1
x_3	1	1	0
x_4	0	0	0
x_5	1	0	1
x_6	0	0	1
x_7	1	1	1

In Table 1 there are 7 objects, 2 conditions attribute A and B and 1 decision attribute C. For example, in object x_5 the condition attributes are 1 and 0, and its decision attribute is 1. Imprecise information causes indiscernibility of objects. This indiscernibility relation is called equivalence relation on the set of object U. In our above example a conflict (inconsistency) exists between object x_1 and x_5 because they are indiscernible by condition attribute A and B and have different decision attribute (C).

Equivalence classes of relation are called Elementary set in S. Any finite union of elementary set is known as Definable Set. The decision elementary set are called concepts. For example, the above table shows the decision attribute is having 2 types of values which is 0 and 1. Hence, 2 types of concept will be shaped. For example, C_1 and C_2 .

C_1 is having the decision attribute 0 and C_2 is having values 1.

$C_1 = \{x_1, x_3, x_4\}$

$C_2 = \{x_2, x_5, x_6, x_7\}$

Definition 3 : In information system, each subset of attributes A Ω determines a binary relation as follows:

$IND(A) = (\{x, y\} \cup x \cup y, \forall a \in A, a(x) = a(y))$.

IND(A) is an equivalence relation on the sets U and is called an indiscernible relation.

Definition 4 : Rough sets theory offers a powerful means to deal with the inconsistency problems. It uses a set of lower and upper approximations as its main vehicles for problem solving.

a) There are five regions of interest, such as $R^*(X)$, $R(X)$, $POSr(X)$, $BNr(X)$ and $NEGr(X)$.

- b) The set $BNr(X) = R^*(X) - R(X)$ will be referred as the R-boundary region of X. If the boundary region of X is the empty set, i.e. $BNr(X) = \emptyset$, then the set X will be called crisp with respect to R; in the opposite case, i.e. if $BNr(X) \neq \emptyset$, the set X will be referred as rough with respect to R.
- c) In the same way, $POSr(X)$ and $NEGr(X)$ are defined as follows:

- $NEGr(X) = U - R^*(X)$ certainly non-member of X
- $POSr(X) = R(X)$

IV. THA: THE HYBRIDIZED MODEL

There are so many inductive learning systems such as ID3, ID4 and ID5 are not capable of handling inconsistent information about training data set effectively. After that Grzymala-Busse designed one system called as LERS which can deal with inconsistent information as well as training data set. But in LERS it's very difficult to maintain a huge training data set. And also the rules induced by LERS are very complicated and very difficult to understand. So in this paper we have proposed a hybridized approach known as THA (Training Data Set on Hybrid Approach), which a combination of Rough set theory, Genetic Algorithm and Boolean algebra.

- Rough Set Theory can handle inconsistent training data set and also missing values.
- Genetic Algorithm based search engine can induce probable decision rules.
- Finally, Boolean operations can simplify the probable decisions rules.

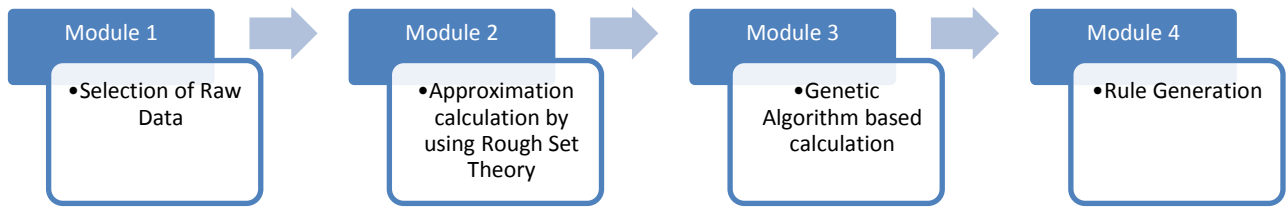


Figure 3 : Framework for the hybridized approach THA

The framework for the hybridized approach THA is depicted in Figure 3. Basically, it consists of 4 modules such as selection of raw data, rough set analyzer, performance evaluation of genetic algorithm, simplification by Boolean operation to generate rule.

The knowledge collected from the process or experts is forwarded to the hybridized model, THA for classification and generation of rules.

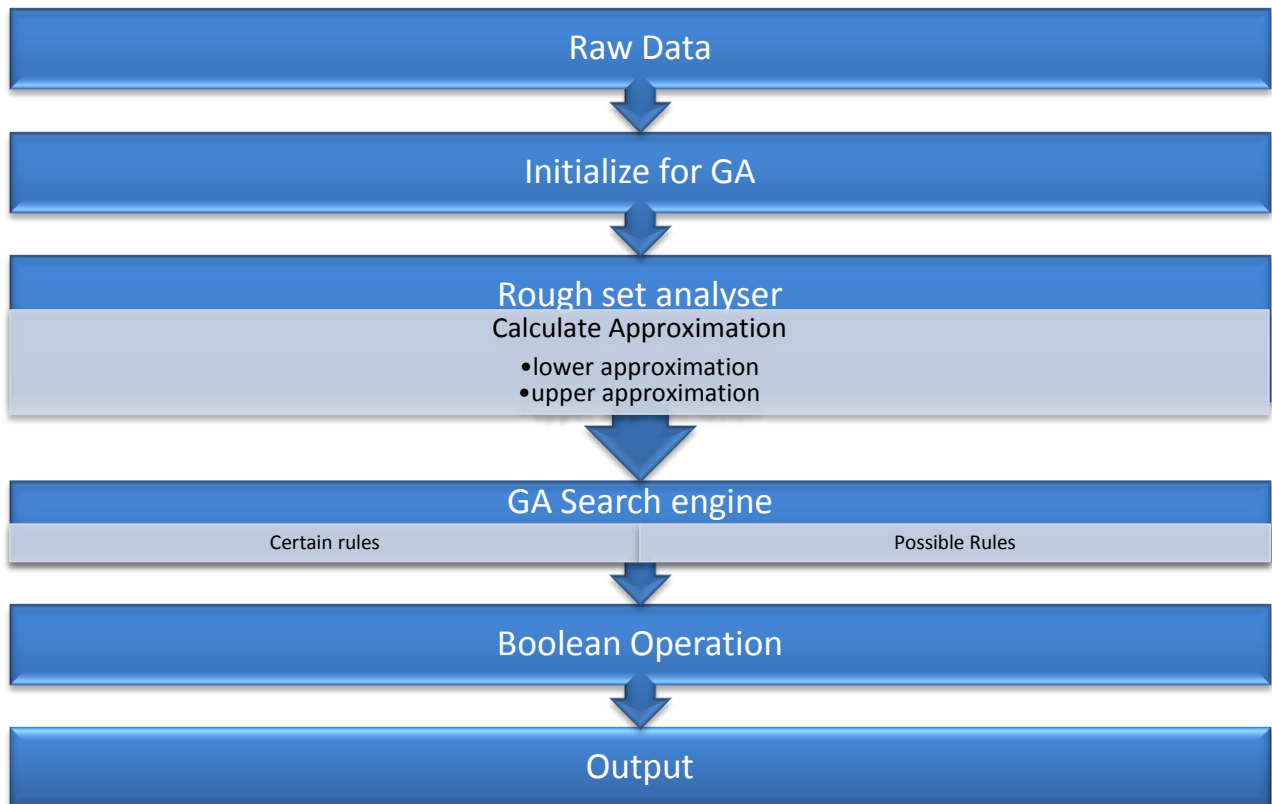


Figure 4 : Lifecycle of Hybridized model- THA

In selection of raw data module such as incomplete and inconsistent data, are feed into the preprocessor which automatically computes all the parameters such as population size, length of the chromosomes, number of generation and the probability of cross-over and mutation necessary for genetic algorithm.

The rough set analyzer performs two tasks such as consistency check on the training data set and the classification of object using rough set theory. For inconsistent data we have to perform three more sub task, namely identification of types of attributes, concept forming and types of approximation are to be

computed. Approximation analysis consists of two types of approximation namely lower and upper approximation. After that we have to classify the training data set into two more subset such as certain training data set and possible training data set.

After the completion of approximation analysis genetic algorithm will take both certain and possible data set. Generally genetic algorithm performs reproduction, cross-over and mutation to extract certain and possible rules from the training set. Finally, Boolean operations are used to simplify the probable decision rules generated by genetic algorithms. The Boolean operators such as union and intersection are used to

simplify the rules. During these operations, redundant rules are to be removed, whereas related rules are to be clustered and generalized during simplification. For every possible rule we have to identify the reliability index, which is defined as the ratio of the number of observations that are correctly classified by possible rules and the number of observation whose condition attributes are covered by the same rule in the original training data set.

$$\text{Reliability index} = \frac{\text{observation of possible rules}}{\text{observation of original data}}$$

Where, observation of possible rule is the number of observations that are correctly classified by a

possible rule and observation of original data is the number of observations with the condition attribute covered by the same rule in the original data set. Now we can easily classify the inconsistent training data set correctly.

V. EXPERIMENTATION

THA is validated from the data obtained from a case study which monitors water quality parameters for drinkable water standards. The data in the Table 2 shows different parameters along with their range of values suitable, based on which we can determine the quality of water. For simplicity, we have taken two or three values for each water quality parameters.

Table 2 : Water quality parameters

Condition	Range	Values
Colour	[5-25]	(12, 15)
P.H. Value	[6.5-8.5]	(7, 7.5, 8)
Sulphate	[200-400]	(220, 300, 375)
Total Hardness	[300-600]	(300, 400)
Turbidity	[5-10]	(6, 8)
Fluoride	[1.0-1.5]	(1.0, 1.2)

Table 3 shows the observation of water sample from different areas along with the decision state. Here there are eighteen observations, five condition attributes

(Colour, pH value, Sulphur, Turbidity and Fluoride) and one decision attribute state which represents whether the water is drinkable or not.

Table 3 : water quality parameters for drinkable water standards

Observation	Colour	pH value	sulphur	turbidity	Fluoride	state
1	15	7.5	300	8	1	D
2	12	8	200	6	1.2	UD
3	12	8	375	6	1.2	UD
4	15	8	375	8	1.2	D
5	12	7	300	8	1	D
6	12	8	200	6	1.2	D
7	15	7	375	6	1	D
8	15	7.5	375	6	1	UD
9	15	8	375	8	1.2	UD
10	12	7.5	200	8	1.2	D
11	15	7.5	300	8	1	D
12	12	8	375	6	1	UD
13	12	7	375	8	1.2	D
14	15	7.5	300	8	1.2	D
15	12	7	200	6	1.2	UD
16	15	8	375	8	1.2	UD
17	15	7.5	300	8	1	UD
18	12	8	200	6	1.2	D

In the above table D stands for drinkable and UD stands for undrinkable.

In order to process the information, we need to depict the parameters in the form of integers. This is

done using the following descriptor scheme show in Table4.

Table 4 : Transformation scheme

Colour	12	0
	15	1
pH value	7	0
	7.5	1
	8	2
Sulphate	200	0
	300	1
	375	2
Turbidity	6	0
	8	1
Fluoride	1	0
	1.2	1
State	D	0
	UD	1

The transformed result is shown in Table 5.

Table 5 : Transformation water quality parameters for drinkable water standards

Observation	Colour	pH value	Sulphur	Turbidity	Fluoride	State
1	1	1	1	1	0	0
2	0	2	0	0	1	1
3	0	2	2	0	1	1
4	1	2	2	1	1	0
5	0	0	1	1	0	0
6	0	2	0	0	1	0
7	1	0	2	0	0	0
8	1	1	2	0	0	1
9	1	2	2	1	1	1
10	0	1	0	1	1	0
11	1	1	1	1	0	0
12	0	2	2	0	0	1
13	0	0	2	1	1	0
14	1	1	1	1	1	0
15	0	0	0	0	1	1
16	1	2	2	1	1	1
17	1	1	1	1	0	1
18	0	2	0	0	1	0

As observed from the above table, the observation (1, 11, 17), (2, 6, 18) and (4, 9, 16) contradict each other, hence we need to perform approximation and concept forming through rough set analyzer. The two decision states, are characterized by C_1 (state=drinkable) and C_2 (state=non drinkable). Applying the rough set theory, we calculate the lower and upper approximations for the concept C_1 and C_2 . As observed from the above table 5, concepts C_1 and C_2 are represented by the following sets:

$$C_1 = \{1, 4, 5, 6, 7, 10, 11, 13, 14, 18\}$$

$$C_2 = \{2, 3, 8, 9, 12, 15, 16, 17\}$$

The lower and upper approximations of the above two concepts are respectively given by

$$\underline{R}(C_1) = \{5, 7, 10, 13, 14\}$$

$$\overline{R}(C_1) = \{1, 2, 4, 5, 6, 7, 9, 10, 11, 13, 14, 16, 17, 18\}$$

$$\underline{R}(C_2) = \{3, 8, 12, 15\}$$

$$\overline{R}(C_2) = \{1, 2, 3, 4, 6, 8, 9, 11, 12, 15, 16, 17, 18\}$$

Then boundary region of the above two states are given by

$$\text{BNR}(C1) = \neg(C1) - R(C1) = \{1, 2, 4, 6, 9, 11, 16, 17, 18\}$$

$$\text{BNR}(C2) = \neg(C2) - R(C2) = \{1, 2, 4, 6, 9, 11, 16, 17, 18\}$$

Now we have to encode the coded data of table 5 by using such a scheme which is called Chromosomes encoding scheme.

Chromosome coding for water parameters are given below.

Table 6 : Chromosome coding for water quality parameters

Bit 3,6,9,12,15; operator	0=Less than or equal to(\leq) 1=Greater than or equal to (\geq)
Bit 1 and 2:Colour	00=12 01=15
Bit 4 and 5: PH value	00=7 01=7.5 10=8
Bit 7 and 8: Sulphate	00=200 01=300 10=375
Bit 10 and 11:Turbidity	00=6 01=8
Bit 13 and 14:Flouride	00=1 10=1.2
Bit 16:State	0=D 1=UD

$$\text{fitness of a chromosome} = \left(\frac{\text{number of objects classified correctly by the rule}}{\text{number of objects covered by the condition of the rule}} \right)^2$$

For example, if a rule (representation of chromosome) can correctly classify five objects in a training data sets, and if there are six objects having the same condition attributes- value pairs as the said rules, then the fitness value of this chromosome is $(5/6)^2 = 0.6944$. The square operator appeared in the fitness function is to ensure rapid convergence. It is used to suppress bad chromosomes with low fitness scores and promotes the creation of good chromosomes with high fitness scores. Thus the above fitness function favours rules that can classify objects correctly. Furthermore, it also satisfies more consistency and completeness criteria, which are of great importance to the evaluation of the rule. A rule is said to be consistent if it covers no negative sample, this is, no object in the training data set violating the rule; and it is said to be complete if the rule is able to cover all the positive sample that satisfy the condition of the rule in the training data set. As previously mentioned, after evaluating the fitness values of chromosomes with above average fitness values are selected for reproduction. As for cross-over and mutation, the respective probabilities are fixed at 0.85 and 0.01. With a higher probability of cross-over,

The GA based search engine is then used to extract rules from certain possible training data set obtained by rough set analyzer then we have to randomly generate 120 chromosomes to form an initial population of possible solution i.e., chromosomes. These chromosomes are coded using the scheme depicted in Table 6.

For chromosome represented the corresponding hybrid approach follows traditional binary string representation and its corresponding crossover and mutation operators. Using this scheme each chromosome is expressed as a binary string i.e., a string containing "0" and "1".

After using the scheme a classification rule can be easily represented by 16 bit chromosome. For example,

If (colour ≥ 15) and (pH value ≥ 7.5) and (sulphate ≥ 300) and (turbidity ≥ 8) and (fluoride ≥ 1) then state will be drinkable can be coded as 0110110110110010.

Other than choosing a good schema for chromosome representation, it is important to define a reasonable fitness function that rewards the kind of chromosomes. Basically the purpose of the GA's is to extract rules that maximize the probability of classifying the objects correctly. Thus, the fitness value of a chromosome can be described by its reliability, or in other words, the probability to classify objects in a training data set correctly. Mathematically, the fitness function used in this work is expressed as

offspring chromosomes that maintain the genetic traits of the parent chromosomes can be generated easily. This allows chromosomes with higher fitness values, that is, better solutions, to be discovered. A lower probability of mutation prevents the search for optimal solutions to degenerate into a random one.

The rule set induced by GA based search engine may contain rules with identical fitness values. Some of these rules can be combined to form a more general or concise rule using Boolean algebra. The rule pruner is assigned to detect and solve the redundancy problem.

Rule 1:

If colour ≥ 1 and pH ≥ 1 and fluoride ≥ 1 then state will be drinkable.

Rule 2:

If colour ≥ 0 and pH ≥ 1 and fluoride ≥ 0 then state will be drinkable.

Here from the Boolean algebra point of view Rule1 is a subset of Rule2. And the resultant rule will be If colour ≥ 0 and pH ≥ 1 and fluoride ≥ 0 then state will be drinkable.

The results generated by hybrid approach are shown in next section.

Two sets of rules are available. As already mentioned, the value recorded in the parentheses following each certain and possible rule represents the completeness and the reliability indices, respectively. All

the indices are represented in fraction form, with the numerator corresponding to the number of correctly classified observations whose condition attributes are covered by the rule. Analysis shows that the rules induced by THA are simple, reasonable and logical.

VI. INDUCED RULE SET

Possible Rules:

- Rule 1: If (Color \geq 15) & (pH \geq 7.5) & (Sulphate \geq 300) & (Fluoride \geq 1) then state = Drinkable
- Rule 2: If (Color \geq 15) & (pH \geq 7.5) & (Sulphate \geq 300) & (Turbidity \geq 8) then state = Drinkable
- Rule 3: If (Color \geq 15) & (pH \geq 8) & (Sulphate \geq 375) & (Turbidity \geq 8) then state = Un-Drinkable
- Rule 4: If (Color \geq 15) & (Sulphate \geq 375) & (Turbidity \geq 6) then state = Un-Drinkable
- Rule 5: If (Color \geq 15) & (pH \geq 8) & (Sulphate \geq 375) & (Fluoride \geq 1.2) then state = Un-Drinkable
- Rule 6: If (Color \geq 12) & (pH \geq 8) & (Sulphate \geq 200) & (Turbidity \geq 6) then state = Drinkable
- Rule 7: If (Color \geq 12) & (pH \geq 7) then state = Drinkable
- Rule 8: If (pH \geq 7.5) & (Sulphate \geq 300) & (Fluoride \geq 1) then state = Drinkable
- Rule 9: If (pH \geq 8) & (Sulphate \geq 200) & (Fluoride \geq 1.2) then state = Drinkable
- Rule 10: If (Color \geq 12) & (Sulphate \geq 375) then state = Drinkable
- Rule 11: If (Sulphate \geq 300) & (Turbidity \geq 8) & (Fluoride \geq 1) then state = Drinkable
- Rule 12: If (Sulphate \geq 375) & (Turbidity \geq 6) & (Fluoride \geq 1) then state = Un-Drinkable
- Rule 13: If (Color \geq 12) & (pH \geq 8) & (Sulphate \geq 200) & (Fluoride \geq 1.2) then state = Drinkable

Certain Rules:

- Rule 1: If (Color \geq 12) & (pH \geq 8) & (Sulphate \geq 375) & (Turbidity \geq 6) & (Fluoride \geq 1.2) then state = Un-Drinkable
- Rule 2: If (Color \geq 12) & (pH \geq 7) & (Sulphate \geq 300) & (Turbidity \geq 8) & (Fluoride \geq 1) then state = Drinkable
- Rule 3: If (Color \geq 15) & (pH \geq 7) & (Sulphate \geq 375) & (Turbidity \geq 6) & (Fluoride \geq 1) then state = Drinkable
- Rule 4: If (Color \geq 15) & (pH \geq 7.5) & (Sulphate \geq 375) & (Turbidity \geq 6) & (Fluoride \geq 1) then state = Un-Drinkable
- Rule 5: If (Color \geq 12) & (pH \geq 7.5) & (Sulphate \geq 200) & (Turbidity \geq 8) & (Fluoride \geq 1.2) then state = Drinkable
- Rule 6: If (Color \geq 12) & (pH \geq 8) & (Sulphate \geq 375) & (Turbidity \geq 6) & (Fluoride \geq 1) then state = Un-Drinkable
- Rule 7: If (Color \geq 12) & (pH \geq 7) & (Sulphate \geq 375) & (Turbidity \geq 8) & (Fluoride \geq 1.2) then state = Drinkable
- Rule 8: If (Color \geq 15) & (pH \geq 7.5) & (Sulphate \geq 300) & (Turbidity \geq 8) & (Fluoride \geq 1.2) then state = Drinkable
- Rule 9: If (Color \geq 12) & (pH \geq 7) & (Sulphate \geq 200) & (Turbidity \geq 6) & (Fluoride \geq 1.2) then state = Un-Drinkable

VII. DISCUSSION

The above experimentation shows that the hybridized approach THA is able to induce rule under uncertainty. We can infer that the hybrid approach can be used for inductive learning under uncertainty. This hybrid approach uses the strength of rough set theory along with efficient GA based search engine and Boolean algebra. In the above experimentation, the GA based search engine reaches its saturation within 60 generations for both possible and certain data sets. This hybrid approach is compared with other inductive learning technique and the result is shown in Table 7. The certain rules that are generated by this approach

are identical to those produced by ID3. Finally the rules generated by this system are simple and concise as compared to those produced by LERS.

Table 7 : Comparison between ID3, LERS, RCLASS and Hybridized approach-THA

Technique	Dealing with uncertainty and inconsistency	Simple and concise rule induction	Extracting complete rules
ID3	No	Yes	For consistent data set
LERS	Yes	No	Not evaluated
RClass	Yes	Yes	No
THA	Yes	Yes	Yes

In Figure 5, we have taken the population size as 120 and the number of generation as 60. In every generation we have identified the average fitness function and plotted the graph accordingly. And based on the average fitness function we have selected the

chromosomes with higher fitness values. In this experiment, we have selected those chromosomes which have fitness function values more than 0.68. We have discarded all the chromosomes whose fitness function is less than 0.68.

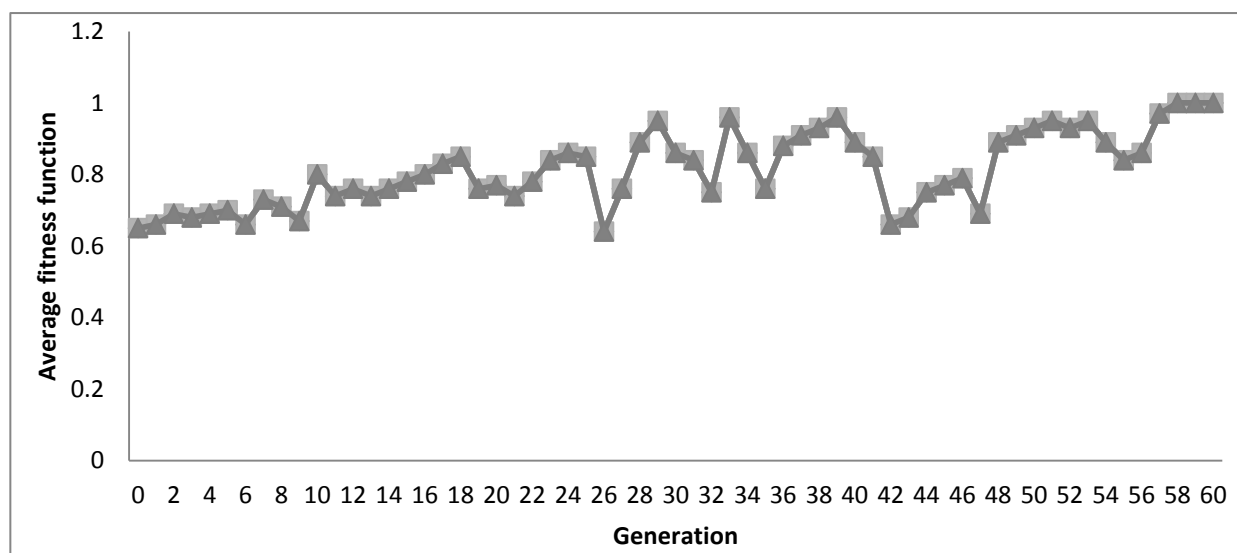


Figure 5 : Performance of Genetic Algorithm based search engine

The rules generated by THA have the following advantages:

- Easy to comprehend
- Easy to deduce and examine
- Easy to validate and cross-check

This hybrid approach THA can handle uncertainty or inconsistent information in real world industrial application such as Data Mining. It is an important area which frequently requires generation of diagnostic rules to be induced from large database.

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GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: I
NUMERICAL METHODS

Volume 14 Issue 1 Version 1.0 Year 2014

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Fourier Spectral Methods for Numerical Solving of the Kuramoto-Sivashinsky Equation

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GJRE-I Classification : *FOR Code: 010106, 230116*



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I. INTRODUCTION

Fourier analysis occurs in the modeling of time-dependent phenomena that are exactly or approximately periodic. Examples of this include the digital processing of information such as speech; the analysis of natural phenom such as earthquakes; in the study of vibrations of spherical, circular or rectangular structures; and in the processing of images. In a typical case, Fourier spectral methods write the solution to the partial differential equation as its Fourier series. Fourier series decomposes a periodic real-valued function of real argument into a sum of simple oscillating trigonometric functions (*sines, cosines*), that can be recombined to obtain the original function. Substituting this series into the partial differential equation gives a system of ordinary differential equations for the time-dependent coefficients of the trigonometric terms in the series then we choose a time-stepping method to solve those ordinary differential equations

II. FOURIER SERIES

The Fourier series of a smooth and periodic real-valued function $f(x) \in [0; 2L]$ with period $2L$ is

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \quad (1)$$

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Since the basis functions $\sin\left(\frac{n\pi x}{L}\right)$ and $\cos\left(\frac{n\pi x}{L}\right)$ are orthogonal the coefficients are given by

$$a_n = \frac{1}{L} \int_0^{2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad n = 0, 1, 2, \dots \quad (2)$$

$$b_n = \frac{1}{L} \int_0^{2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad n = 1, 2, \dots \quad (3)$$

Fourier series can be expressed neatly in complex form as follows

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left[\frac{a_n}{2} \left(e^{\frac{in\pi x}{L}} + e^{-\frac{in\pi x}{L}} \right) + \frac{b_n}{2i} \left(e^{\frac{in\pi x}{L}} - e^{-\frac{in\pi x}{L}} \right) \right] \quad (4)$$

If we define

$$c_0 = \frac{a_o}{2}, c_n = \frac{a_n - ib_n}{2}, c_{-n} = \frac{a_n + ib_n}{2} \quad (5)$$

Then

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}} \quad (6)$$

where the coefficients c_n can be determined from the formulas of a_n and b_n as

$$c_n = \frac{1}{2L} \int_0^{2L} f(x) e^{-\frac{in\pi x}{L}} dx \quad (7)$$

III. DISCRETE FOURIER TRANSFORM

In many applications, particularly in analyzing of real situations, the function $f(x)$ to be approximated is known only on a discrete set of "sampling points" of x . Hence, the integral (7) cannot be evaluated in a closed form and Fourier analysis cannot be applied directly. It then becomes necessary to replace continuous Fourier analysis by a discrete version of it. The linear discrete Fourier transform of a periodic (discrete) sequence of complex values $u_0, u_1, \dots, u_{N_t-1}$ with period u_{N_t} , is a sequence of periodic complex values $\tilde{u}_0, \tilde{u}_1, \dots, \tilde{u}_{N_t-1}$ defined by

$$\tilde{u}_k = \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} u_j e^{\frac{-2ij\pi k}{N_\tau}} \quad (8)$$

The linear inverse transformation is

$$u_j = \sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{2ij\pi k}{N_\tau}} \quad (9)$$

The most obvious application of discrete Fourier analysis consists in the numerical calculation of Fourier coefficients. Suppose we want to approximate a real-valued periodic function $f(x)$ defined on the interval $[0; 2L]$ that is sampled with an even number N_τ of grid points

$$x_j = jh \quad h = \frac{2L}{N_\tau} \quad j = 0, 1, \dots, N_\tau - 1 \quad (10)$$

by its Fourier series. First we compute approximate values of the Fourier coefficients

$$\tilde{c}_k \approx \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} f(x_j) e^{\frac{-2ij\pi k}{N_\tau}} \quad (11)$$

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \dots & \omega^{N_\tau-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \dots & \omega^{2(N_\tau-1)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & \omega^{N_\tau-1} & \omega^{2(N_\tau-1)} & \omega^{3(N_\tau-1)} & \dots & \omega^{(N_\tau-1)(N_\tau-1)} \end{pmatrix}$$

Similarly, the inverse discrete Fourier transform has the form

$$u_j = M^*_{kj} \tilde{u}_k \quad k, j = 0, 1, \dots, N_\tau - 1 \quad (13)$$

Where $M^*_{kj} = \omega^{*kj}$ and $\omega^* = e^{\frac{2i\pi}{N_\tau}}$ where ω^* is complex conjugate of ω

$$M^* = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & \omega^* & \omega^{*2} & \omega^{*3} & \dots & \omega^{*N_\tau-1} \\ 1 & \omega^{*2} & \omega^{*4} & \omega^{*6} & \dots & \omega^{*2(N_\tau-1)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & \omega^{*N_\tau-1} & \omega^{*2(N_\tau-1)} & \omega^{*3(N_\tau-1)} & \dots & \omega^{*(N_\tau-1)(N_\tau-1)} \end{pmatrix}$$

The FFT algorithm reduces the computational work required to carry out a discrete Fourier transform by reducing the number of multiplications and additions of (13), computational time is reduced from $O(N_\tau^2)$ to $O(N_\tau \log N_\tau)$.

Because the discrete Fourier transform and its inverse exhibit periodicity with period N_τ , i.e. $\tilde{u}_k + N_\tau = \tilde{u}_k$ (this property results from the periodic nature of $e^{\frac{2\pi ijk}{N_\tau}}$), it makes no sense to use more than N_τ terms in the series, and it suffices to calculate one full period. The Fourier series formed with the approximate coefficients is

$$\tilde{f}(x) \approx \sum_{k=-N_\tau/2+1}^{N_\tau/2} \tilde{c}_k e^{\frac{-2ij\pi k}{N_\tau}} \quad (11')$$

The function $\tilde{f}(x)$ not only approximates, but actually interpolates $f(x)$ at the sampling grid points x_j

In matrix form, the discrete Fourier transform (8) can be written as

$$\tilde{u}_k = \frac{1}{N_\tau} M_{kj} u_j \quad k, j = 0, 1, \dots, N_\tau - 1 \quad (12)$$

Where $M_{kj} = \omega^{kj}$ and $\omega = e^{\frac{-2i\pi}{N_\tau}}$

To apply spectral methods to a partial differential equation we need to evaluate derivatives of functions. Suppose that we have a periodic real-valued function $f(x) \in [0; 2L]$ with period $2L$ that is discretized with an even number N_τ of grid points, so that the grid

size $h = \frac{2L}{N_\tau}$. The complex form of the Fourier series representation of $f(x)$ is

$$\tilde{f}(x) \approx \sum_{k=-N_\tau/2+1}^{N_\tau/2} \tilde{c}_k e^{\frac{i\pi kx}{L}}$$

At $k = \frac{N_\tau}{2}$ the above series gives a term $\tilde{c}_{N_\tau/2} e^{\frac{i\pi N_\tau x}{2L}}$, which alternates between $\pm \tilde{c}_{N_\tau/2}$ at the

grid point $x_j = jh$, $j = 0, 1, \dots, N_\tau - 1$, and since it cannot be differentiated, we should set its derivative to be zero at the grid points. The numerical derivatives of the function $f(x)$ can be illustrated as a matrix multiplication. For the first derivative, we multiply the Fourier coefficients (11) by the corresponding differentiation matrix for an even number N_τ of grid points.

$$\Lambda_1 = \text{Diag} \left(0, 1, 2, 3, \dots, \frac{N_\tau}{2} - 1, 0, -\left(\frac{N_\tau}{2} - 1\right), \dots, -3, -2, -1 \right) \frac{i\pi}{L}$$

This matrix has non-zero elements only on the diagonal. For an odd number N_τ of grid points the

differentiation matrix corresponding to the first derivative is diagonal with elements.

$$\left(0, 1, 2, 3, \dots, \frac{N_\tau}{2} - 1, 0, -\left(\frac{N_\tau}{2} - 1\right), \dots, -3, -2, -1 \right) \frac{i\pi}{L}$$

Then, we compute an inverse discrete Fourier transform using (11') to return to the physical space and deduce the first derivative of $f(x)$ on the grid. Similarly, taking the second derivative corresponds to

the multiplication of the Fourier coefficients (11) by the corresponding differentiation matrix for an even number N_τ of grid points.

$$\Lambda_2 = \text{Diag} \left(0, 1, 4, 9, \dots, \left(\frac{N_\tau}{2} - 1\right)^2, \left(\frac{N_\tau}{2}\right)^2, \left(\frac{N_\tau}{2} - 1\right)^2, \dots, 9, 4, 1 \right) \left(\frac{i\pi}{L}\right)^2$$

In general, in case of an even number N_τ of grid points approximating the m -th numerical derivatives of a grid function $f(x)$ corresponds to the multiplication of the Fourier coefficients (11) by the corresponding differentiation matrix which is diagonal

with elements $\left(\frac{ik\pi}{L}\right)^m$

for

$$k = 0, 1, 2, 3, \dots, \frac{N_\tau}{2} - 1, \frac{N_\tau}{2}, -\left(\frac{N_\tau}{2} - 1\right), \dots, -3, -2, -1$$

with the exception that for odd derivatives we set the derivative of the highest mode $k = \frac{N_\tau}{2}$ to be zero.

IV. EXPONENTIAL TIME DIFFERENCING

The family of exponential time differencing schemes. This class of schemes is especially suited to semi-linear problems which can be split into a linear part which contains the stiffest part of the dynamics of the problem, and a nonlinear part, which varies more slowly

than the linear part. Exponential time differencing schemes are time integration methods that can be efficiently combined with spatial approximations to provide accurate smooth solutions for stiff or highly oscillatory semi-linear partial differential equations. In this paper I will present the derivation of the explicit Exponential time differencing schemes for arbitrary order following the approach in [12], [2], [4] and presents the explicit Runge-Kutta versions of these schemes constructed by Cox and Matthews [12].

We consider for simplicity a single model of a stiff ordinary differential equation

$$\frac{du(t)}{dt} = cu(t) + F(u(t), t) \quad (e) \quad \text{where } F(u(t), t) \text{ is the nonlinear forcing term.}$$

To derive the s -step Exponential time differencing schemes, we multiply through by the integrating factor e^{-ct} and then integrate the equation over a single time step from $t = t_n$ to $t = t_{n+1} = t_n + \Delta t$ to obtain.

$$u(t_{n+1}) = u(t_n) e^{c\Delta t} + e^{c\Delta t} * \int_0^{\Delta t} F(u(t_n + \tau), t_n + \tau) d\tau \quad (e_1)$$

The next step is to derive approximations to the integral in equation (e₁). This procedure does not introduce an unwanted fast time scale into the solution and the schemes can be generalized to arbitrary order.

If we apply the Newton Backward Difference Formula, we can write a polynomial approximation to $F(u(t_n + \tau), t_n + \tau)$ in the form

$$\begin{aligned} F(u(t_n + \tau), t_n + \tau) &\approx G(t_n, t) = \sum_{m=0}^{s-1} (-1)^m \binom{-\tau / \Delta t}{m} * \nabla^m G_n(t_n) \approx \\ &\approx \sum_{m=0}^{s-1} (-1)^m \binom{-\tau / \Delta t}{m} * \underbrace{\sum_{k=0}^m (-1)^m \binom{m}{k} F(u(t_{n-k}), t_{n-k})}_{\nabla^m G_n(t_n)} \end{aligned} \quad (e_2)$$

$$k! \binom{m}{k} = (m-1)(m-2)\dots(m-k+1), m = 1, \dots, s-1 \text{ note that } 0! \binom{m}{0} = 1$$

If we substitute (e₂) into (e₁) we get

$$\begin{aligned} u(t_{n+1}) &= u(t_n) e^{c\Delta t} + e^{c\Delta t} * \int_0^{\Delta t} \sum_{m=0}^{s-1} (-1)^m \binom{-\tau / \Delta t}{m} * \nabla^m G_n(t_n) d\tau \\ u(t_{n+1}) - u(t_n) e^{c\Delta t} &= \Delta t \sum_{m=0}^{s-1} (-1)^m * \int_0^1 e^{c\Delta t(1-\tau / \Delta t)} \binom{-\tau / \Delta t}{m} * \nabla^m G_n(t_n) d(\tau / \Delta t) \end{aligned} \quad (e_3)$$

$$\text{We will indicate the integral by } \mathcal{G}_m = \int_0^1 e^{c\Delta t(1-\tau / \Delta t)} \binom{-\tau / \Delta t}{m} d(\tau / \Delta t) \quad (e_4)$$

If we substitute (e₂) and (e₄) into (e₃) we get

$$u(t_{n+1}) = u(t_n) e^{c\Delta t} + \Delta t \sum_{m=0}^{s-1} (-1)^m * \mathcal{G}_m * \sum_{k=0}^m (-1)^m \binom{m}{k} F(u(t_{n-k}), t_{n-k}) \quad (e_5)$$

Which represent the general generating formula of the exponential time differencing schemes of order s

The first-order exponential time differencing scheme is obtained by setting $s=1$

$$u_{n+1} = u_n e^{c\Delta t} + (e^{c\Delta t} - 1) F_n / c$$

The second-order exponential time differencing scheme is obtained by setting $s=2$

$$u_{n+1} = u_n e^{c\Delta t} + \left\{ (c\Delta t + 1) e^{c\Delta t} - 2c\Delta t - 1 \right\} F_n + \left\{ -e^{c\Delta t} + c\Delta t + 1 \right\} F_{n-1} / (c^2 \Delta t) :$$

By setting $s=2$ we get the fourth-order exponential time differencing scheme

$$u_{n+1} = u_n e^{c\Delta t} + (\Theta_1 F_n - \Theta_2 F_{n-1} + \Theta_3 F_{n-2} - \Theta_4 F_{n-3}) / (6c^4 \Delta t^3)$$

$$\Theta_1 = (6c^3 \Delta t^3 + 11c^2 \Delta t^2 + 12c\Delta t + 6) e^{c\Delta t} - 24c^3 \Delta t^3 - 2611c^2 \Delta t^2 - 18c\Delta t - 6$$

$$\Theta_2 = (18c^2 \Delta t^2 + 30c\Delta t + 18) e^{c\Delta t} - 36c^3 \Delta t^3 - 57c^2 \Delta t^2 - 48c\Delta t - 18$$

$$\Theta_3 = (6c^2 \Delta t^2 + 24c\Delta t + 18) e^{c\Delta t} - 24c^3 \Delta t^3 - 42c^2 \Delta t^2 - 42c\Delta t - 18$$

$$\Theta_4 = (2c^2 \Delta t^2 + 6c\Delta t + 6) e^{c\Delta t} - 6c^3 \Delta t^3 - 11c^2 \Delta t^2 - 12c\Delta t - 6.$$

V. ON THE STABILITY OF ETDRK4 METHOD

The stability analysis of the ETDRK4 method is as follows (see [21,24] or [12]). For the nonlinear ODE

$$\frac{du(t)}{dt} = cu(t) + F(u(t), t) \quad (3.1)$$

with $F(u(t), t)$ the nonlinear part, we suppose that there exists a fixed point u_0 this means that $cu_0 + F(u_0, t) = 0$. Linearizing about this fixed point, if $u(t)$ is the perturbation of u_0 and $\delta = F'(u_0, t)$ then

where

$$\ell_0 = e^y$$

$$\ell_1 = \frac{-4}{y^3} + \frac{8e^{\frac{y}{2}}}{y^3} - \frac{8e^{\frac{3y}{2}}}{y^3} + \frac{4e^{2y}}{y^3} - \frac{1}{y^2} + \frac{4e^{\frac{y}{2}}}{y^2} - \frac{6e^y}{y^2} + \frac{4e^{\frac{3y}{2}}}{y^2} - \frac{e^{2y}}{y^2}$$

$$\ell_2 = \frac{-8}{y^4} + \frac{16e^{\frac{y}{2}}}{y^4} - \frac{16e^{\frac{3y}{2}}}{y^4} + \frac{8e^{2y}}{y^4} - \frac{5}{y^3} + \frac{12e^{\frac{y}{2}}}{y^3} - \frac{10e^y}{y^3} + \frac{4e^{\frac{3y}{2}}}{y^3} - \frac{e^{2y}}{y^3} - \frac{1}{y^2} + \frac{4e^{\frac{y}{2}}}{y^2} - \frac{3e^y}{y^2},$$

$$\ell_3 = \frac{4}{y^5} - \frac{16e^{\frac{y}{2}}}{y^5} + \frac{16e^y}{y^5} + \frac{8e^{\frac{3y}{2}}}{y^5} - \frac{20e^{2y}}{y^5} + \frac{8e^{\frac{5y}{2}}}{y^5} - \frac{2}{y^4} + \frac{-10e^{\frac{y}{2}}}{y^4} + \frac{16e^y}{y^4} - \frac{12e^{\frac{3y}{2}}}{y^4} + \frac{6e^{2y}}{y^4} - \frac{2e^{\frac{5y}{2}}}{y^4} - \frac{2e^{\frac{y}{2}}}{y^3} + \frac{4e^y}{y^3} - \frac{2e^{\frac{3y}{2}}}{y^3}$$

$$\ell_4 = \frac{8}{y^6} - \frac{24e^{\frac{y}{2}}}{y^6} + \frac{16e^y}{y^6} + \frac{16e^{\frac{3y}{2}}}{y^6} - \frac{24e^{2y}}{y^6} + \frac{8e^{\frac{5y}{2}}}{y^6} + \frac{6}{y^5} + \frac{-18e^{\frac{y}{2}}}{y^5} + \frac{20e^y}{y^5} - \frac{12e^{\frac{3y}{2}}}{y^5} + \frac{6e^{2y}}{y^5} + \frac{2e^{\frac{5y}{2}}}{y^5} + \frac{2}{y^4} - \frac{6e^{\frac{y}{2}}}{y^4} + \frac{6e^y}{y^4} - \frac{2e^{\frac{3y}{2}}}{y^4}$$

An important remark: computing $\ell_0, \ell_1, \ell_2, \ell_3, \ell_4$ by the above expressions suffers from numerical instability for y close to zero. Because of that,

$$\frac{du(t)}{dt} = cu(t) + \delta u(t) \quad (3.2)$$

and the fixed point $u_0(t)$ is stable if $\text{Re}(c + \delta) < 0$.

The application of the ETDRK4 method to (3.2) leads to a recurrence relation involving u_n and u_{n+1} .

Introducing the previous notation $x = \delta h$ and $y = ch$, and using the Mathematica® algebra package, we obtain the following amplification factor

$$\frac{u_{n+1}}{u_n} = r(x, y) = \ell_0 + \ell_1 x + \ell_2 x^2 + \ell_3 x^3 + \ell_4 x^4 \quad (3.3)$$

for small y , instead of them, we will use their asymptotic expansions.

$$\ell_1 = 1 + y + \frac{1}{2}y^2 + \frac{1}{6}y^3 + \frac{13}{320}y^4 + \frac{7}{960}y^5 + O(y^6)$$

$$\ell_2 = \frac{1}{2} + \frac{1}{2}y + \frac{1}{4}y^2 + \frac{247}{2880}y^3 + \frac{131}{5760}y^4 + \frac{479}{96768}y^5 + O(y^6)$$

$$\ell_3 = \frac{1}{6} + \frac{1}{6}y + \frac{61}{720}y^2 + \frac{1}{36}y^3 + \frac{1441}{241920}y^4 + \frac{67}{120960}y^5 + O(y^6)$$

$$\ell_4 = \frac{1}{24} + \frac{1}{32}y + \frac{7}{640}y^2 + \frac{19}{11520}y^3 - \frac{311}{64512}y^4 - \frac{479}{860160}y^5 + O(y^6)$$

We make two observations:

- As $y \mapsto 0y$, our approximation becomes

$$r(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$$

which is the stability function for all the 4-stage Runge–Kutta methods of order four.

- Because c and δ may be complex, the stability region of the

ETDRK4 method is four-dimensional and therefore quite difficult to represent. Unfortunately, we do not know any expression for $|r(x, y)| = 1$ we will only be able to plot it. The most common idea is to study it for each particular case; for example, assuming c to be fixed and real in [21] or that both c and δ are pure imaginary numbers in [24].

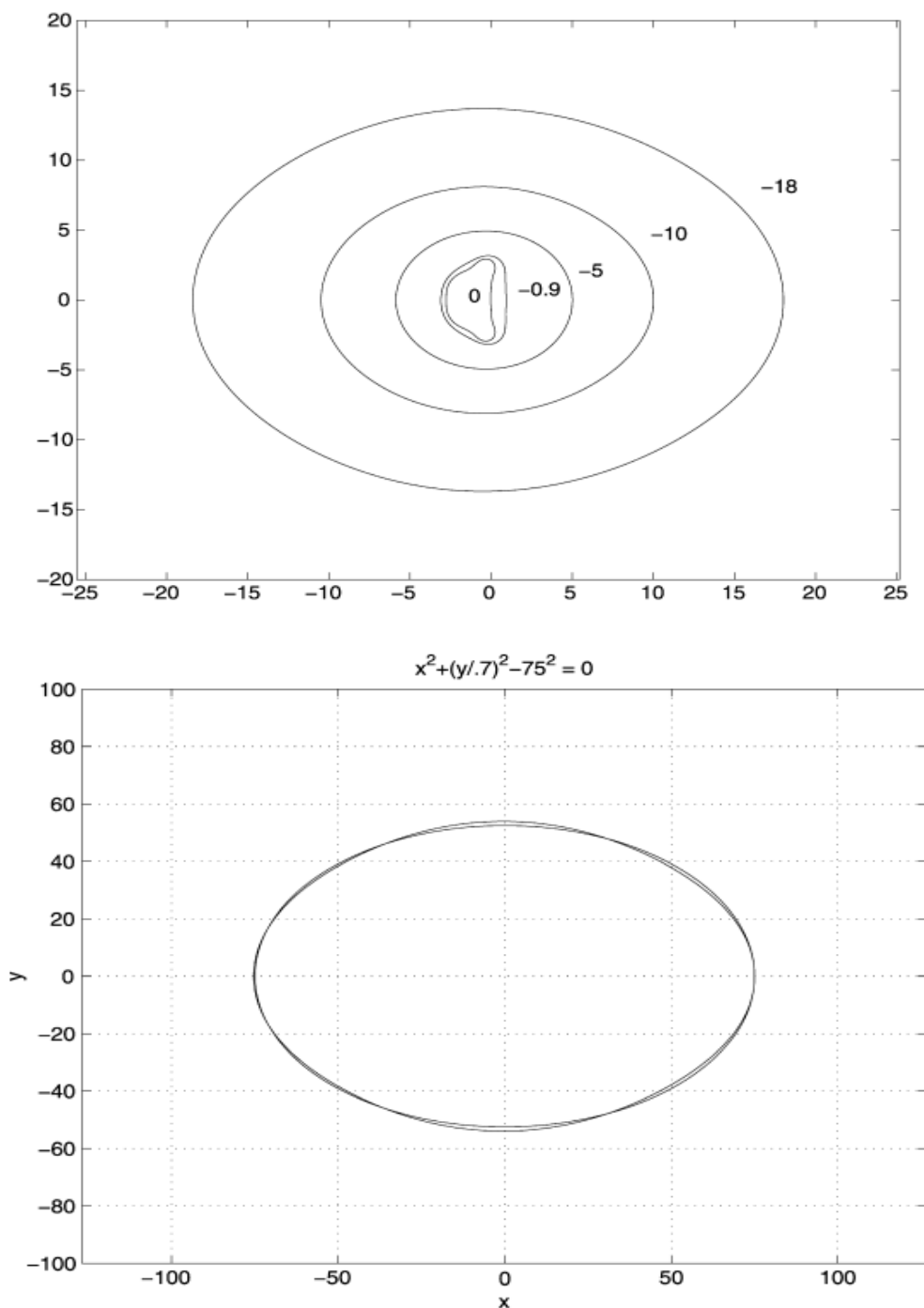


Figure 2 : Experimental boundaries and ellipse for $y = -75$

For dissipative PDEs with periodic boundary conditions, the scalars c that arise with a Fourier spectral method are negative. Let us take for example Burger's equation

$$u_t = \varepsilon u_{xx} - \left(\frac{1}{2}u^2\right)_x \quad x \in [-\pi, \pi] \quad \text{where } 0 < \varepsilon \ll 1 \quad (3.4)$$

Transforming it to the Fourier space gives

$$\tilde{u}_t = -\varepsilon \zeta^2 \tilde{u} - \frac{i\zeta}{2} \tilde{u}^2 \quad \forall \zeta \quad (3.5)$$

where $\forall \zeta$ is the Fourier wave-number and the coefficients $c = -\varepsilon \zeta^2 < 0$, span over a wide range of values when all the Fourier modes are considered. For high values of $|\zeta|$ the solutions are attracted to the slow manifold quickly because $c < 0$ and $|c| \ll 1$.

In *Figure. 1* we draw the boundary stability regions in the complex plane x for $y=0, -0.9, -5, -10, -18$. When the linear part is zero ($y=0$), we recognized the stability region of the fourth-order Runge-Kutta methods and, as $y \mapsto -\infty$, the region grows. Of course, these regions only give an indication of the stability of the method.

In fact, for $y < 0, |y| \ll 1$ the boundaries that are observed approach to ellipses whose parameters have been fitted numerically with the following result.

$$(\text{Re}(x))^2 + \left(\frac{\text{Im}(x)}{0.7}\right)^2 = y^2 \quad (3.6)$$

In *Figure. 2* we draw the experimental boundaries and the ellipses (3.6) with $y = -75$. The spectrum of the linear operator increases as ζ^2 , while the eigenvalues of the linearization of the nonlinear part lay on the imaginary axis and increase as ζ . On the other hand, according to (3.6), when $\text{Re}(x)=0$, the intersection with the imaginary axis $\text{Im}(x)$ increases as $|y|$, i.e., as ζ^2 . Since the boundary of stability grows faster than, the ETD RK4 method should have a very good behavior to solve Burger's equation, which confirms the results of paper [6].

By substituting (e_9) into (e_{10}) we get

$$u_{n+1} = u_n e^{c\Delta t} + \{((c\Delta t - 2)e^{c\Delta t} + c\Delta t + 2) + 2(e^{c\Delta t} - c\Delta t - 1)F(F(a_n, t_n + \Delta t/2))\}/(c^2\Delta t) \quad (e_{11})$$

By setting $s=4$ an fourth-order Runge Kutta exponential time differencing scheme is obtained as follows

$$a_n = u_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)F_n / c$$

$$b_n = u_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)F(a_n, t_n + \Delta t/2)_n / c$$

VI. EXPONENTIAL TIME DIFFERENCING RUNGE-KUTTA METHODS

Generally, for the one-step time-discretization methods and the Runge-Kutta methods, all the information required to start the integration is available. However, for the multi-step time-discretization methods this is not true. These methods require the evaluations of a certain number of starting values of the nonlinear term $F(u(t), t)$ at the n -th and previous time steps to build the history required for the calculations. Therefore, it is desirable to construct exponential time differencing methods that are based on Runge-Kutta methods.

Based in [12] and [3], Putting $s=1$ in equation (e_5) to get

$$a_n = u_n e^{c\Delta t} + (e^{c\Delta t} - 1)F_n / c \quad (e_6)$$

The term a_n approximates the value of u at $t_n + \Delta t$

The next step is to approximate F in the interval $t_n \leq t \leq t_{n+1}$ with

$$F = F_n + (t - t_n)(F(a_n, t_n + \Delta t) - F_n) / \Delta t + O(\Delta t^2) \quad (e_7)$$

and substitute into (e_1) yield

$$u_{n+1} = a_n + (e^{c\Delta t} - c\Delta t - 1)(F(a_n, t_n + \Delta t) - F_n) / (c^2\Delta t) \quad (e_8)$$

Equation (e_8) represent the first-order Runge Kutta exponential time differencing scheme

In a similar way, we can also form the second-order Runge Kutta exponential time differencing scheme

$$a_n = u_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)F_n / c \quad (e_9)$$

As we can see equation (e_9) is formed by taking half a step of (e_6)

The next step is to approximate F in the interval $t_n \leq t \leq t_{n+1}$ with

$$F = F_n + \frac{(t - t_n)}{\Delta t/2} (F(a_n, t_n + \Delta t/2) - F_n) + O(\Delta t^2) \quad (e_{10})$$

By substituting (e_9) into (e_{10}) we get

$$u_{n+1} = u_n e^{c\Delta t} + \{((c\Delta t - 2)e^{c\Delta t} + c\Delta t + 2) + 2(e^{c\Delta t} - c\Delta t - 1)F(F(a_n, t_n + \Delta t/2))\}/(c^2\Delta t) \quad (e_{11})$$

By setting $s=4$ an fourth-order Runge Kutta exponential time differencing scheme is obtained as follows

$$a_n = u_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)F_n / c$$

$$b_n = u_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)F(a_n, t_n + \Delta t/2)_n / c$$

$$c_n = a_n e^{c\Delta t/2} + (e^{c\Delta t/2} - 1)(2F(b_n, t_n + \Delta t/2) - F_n) / c$$

$$u_{n+1} = u_n e^{c\Delta t} + \{(c^2 \Delta t^2 - 3c\Delta t + 4)e^{c\Delta t} - c\Delta t - 4\} F_n + 2((c\Delta t - 2)e^{c\Delta t} + c\Delta t + 2)(F(a_n, t_n + \Delta t/2) + F(b_n, t_n + \Delta t/2)) \\ ((-c\Delta t + 4)e^{c\Delta t} - c^2 \Delta t^2 + -3c\Delta t - 4)F(c_n, t_n + \Delta t) / (c^2 \Delta t) \quad (e_{12})$$

In general, the exponential time differencing Runge-Kutta method (e_{12}) has classical order four, but Hochbruck and Ostermann[11] showed that this method suffers from an order reduction. They also presented numerical experiments which show that the order reduction, predicted by their theory, may in fact arise in practical examples. In the worst case, this leads to an order reduction to order three for the Cox and Matthews method (e_{12}) [12]. However, for certain problems, such as the numerical experiments conducted by Kassam and Trefethen[13],[6] for solving various one-dimensional diffusion-type problems, and the numerical results obtained in for solving some dissipative and dispersive PDEs, the fourth-order convergence of the fourth-order Runge Kutta exponential time differencing method [12] is confirmed numerically.

Finally, we note that as $c \rightarrow 0$ in the coefficients of the s -order exponential time differencing Runge-Kutta methods, the methods reduce to the corresponding order of the Runge-Kutta schemes.

V. THE KURAMOTO-SIVASHINSKY EQUATION

The Kuramoto-Sivashinsky equation, is one of the simplest PDEs capable of describing complex behavior in both time and space. This equation has been of mathematical interest because of its rich dynamical properties. In physical terms, this equation describes reaction diffusion problems, and the dynamics of viscous-fluid films flowing along walls.

Kuramoto-Sivashinsky equation in one space dimension can be written

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) \frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^4 u(x,t)}{\partial x^4} \quad (14)$$

Equation (14) can be written in integral form if we introduce

If we substitute into (14) we get

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) \frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^4 u(x,t)}{\partial x^4} \Leftrightarrow \frac{\partial u_j(x,t)}{\partial t} = -u_j(x,t) \frac{\partial u_j(x,t)}{\partial x} - \frac{\partial^2 u_j(x,t)}{\partial x^2} - \frac{\partial^4 u_j(x,t)}{\partial x^4} \\ \Leftrightarrow \sum_{k=0}^{N_\tau-1} \frac{d\tilde{u}_k}{dt} e^{\frac{ik\pi x}{L}} = - \left[\left(\sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \right) * \left(\sum_{k=0}^{N_\tau-1} \frac{ik\pi}{L} \tilde{u}_k e^{\frac{ik\pi x}{L}} \right) \right] - \left[\sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^2 \tilde{u}_k e^{\frac{ik\pi x}{L}} \right] - \left[\sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^4 \tilde{u}_k e^{\frac{ik\pi x}{L}} \right]$$

By simplifying (16), (16.1), (16.2), (16.3), (16.4), (16.5) and note that

$$u(x,t) = \frac{\partial \zeta(x,t)}{\partial t}$$

then

$$\frac{\partial \zeta(x,t)}{\partial t} = -\frac{1}{2} \left(\frac{\partial \zeta(x,t)}{\partial x} \right)^2 - \frac{\partial^2 \zeta(x,t)}{\partial x^2} - \frac{\partial^4 \zeta(x,t)}{\partial x^4} \quad (15)$$

or in form

$$u_t + \nabla^4 u + \nabla^2 u + \left| \nabla u \right|^2 / 2 = 0$$

The Kuramoto-Sivashinsky equation with $2L$ periodic boundary conditions in Fourier space can be written as follows

$$u_j(x) = \sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16)$$

$$u_j(x,t) = \sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16.1)$$

$$\frac{\partial u_j(x,t)}{\partial t} = \sum_{k=0}^{N_\tau-1} \frac{d\tilde{u}_k}{dt} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16.2)$$

$$\frac{\partial u_j(x,t)}{\partial x} = \sum_{k=0}^{N_\tau-1} \frac{ik\pi}{L} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16.3)$$

$$\frac{\partial^2 u_j(x,t)}{\partial x^2} = \sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^2 \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16.4)$$

$$\frac{\partial^4 u_j(x,t)}{\partial x^4} = \sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^4 \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (16.5)$$

$$\tilde{u}_k = \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} u_j e^{-2ijk/N_\tau}, \quad (i)^2 = -1, (i)^4 = 1 \quad x_j = jh, h = \frac{2L}{N_\tau}$$

Equation (14) can be written as follows

$$\frac{\partial \tilde{u}_k(t)}{\partial t} = (k^2 - k^4) \tilde{u}_k(t) - \frac{ik}{2} \varpi_k \quad (17)$$

$$\text{Where } \varpi_k = \frac{1}{2L} \int_{-L}^L u^2(x, t) e^{\frac{ik\pi x}{L}} dx = \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} u_j^2(x, t) e^{-2ijk/N_\tau} = \text{FFT}[u(t)^2]$$

In final form will be

$$\frac{\partial \tilde{u}_k(t)}{\partial t} = (k^2 - k^4) \tilde{u}_k(t) - \frac{ik}{2} \text{FFT}[u(t)^2] \quad (18)$$

Equation has strong dissipative dynamics, which arise from the fourth order dissipation $\frac{\partial^4 u(x, t)}{\partial x^4}$ term that provides damping at small scales. Also, it includes

the mechanisms of a linear negative diffusion $\frac{\partial^2 u(x, t)}{\partial x^2}$ term, which is responsible for an instability of modes with large wavelength, i.e. small wave-numbers. The nonlinear advection/steepening $u(x, t) \frac{\partial u(x, t)}{\partial x}$ term in the equation transforms energy between large and small scales.

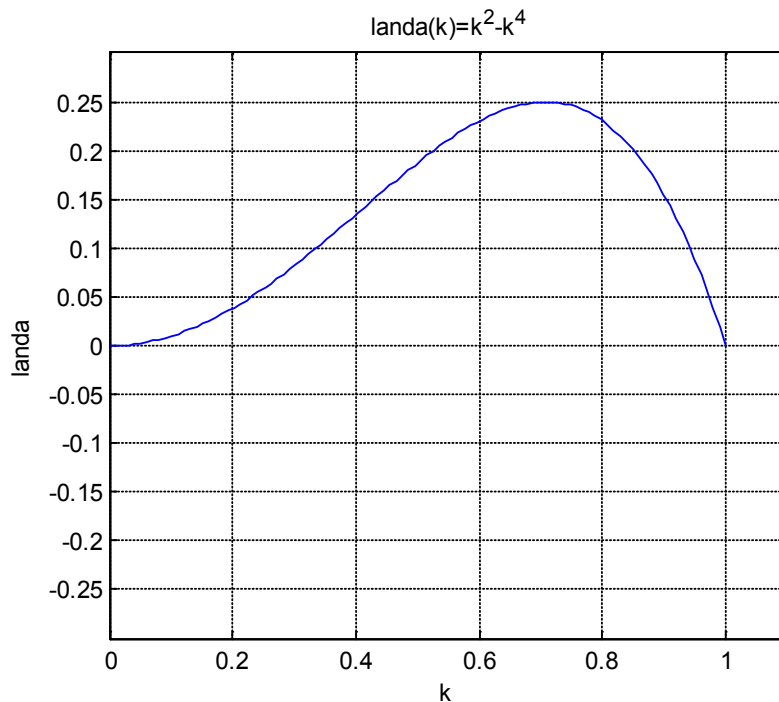


Figure 3: The growth rate $\lambda(k)$ for perturbations of the form $e^{\lambda t} e^{ikx}$ to the zero solution of the Kuramoto-Sivashinsky (K-S) equation

The zero solution of the K-S equation is linearly unstable (the growth rate $\lambda(k) > 0$ for perturbations of the form $e^{\lambda t} e^{ikx}$ to modes with wave-numbers $|k| = \frac{2\pi}{\mathcal{G}} < 1$ for a wavelength \mathcal{G} and is damped for modes with $|k| > 1$ see Figure. 3 these modes are coupled to each other through the non-linear term.

The stiffness in the system (17) is due to the fact that the diagonal linear operator $(k^2 - k^4)$, with the elements, has some large negative real eigenvalues that represent decay, because of the strong dissipation, on a time scale much shorter than that typical of the nonlinear term. The nature of the solutions to the the Kuramoto-Sivashinsky equation varies with the system size of linear operator. For large size of linear operator,

enough unstable Fourier modes exist to make the system chaotic. For small size of linear operator, insufficient Fourier modes exist, causing the system to approach a steady state solution. In this case, the exponential time differencing methods integrate the system very much more accurately than other methods since the the exponential time differencing methods assume in their derivation that the solution varies slowly in time.

$$u_2(x) = 1.7 \cos\left(\frac{x}{2}\right) + 0.1 \sin\left(\frac{x}{2}\right) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$$

When evaluating the coefficients of the exponential time differencing and the exponential time differencing Runge Kutta methods via the "Cauchy integral" approach [5],[6] we choose circular contours of radius $R = 1$. Each contour is centered at one of the elements that are on the diagonal matrix of the linear part

VI. NUMERICAL RESULT

For the simulation tests, we choose two periodic initial conditions

$$u_1(x) = e^{\cos\left(\frac{x}{2}\right)}, x \in [0, 4\pi]$$

of the semi-discretized model. We integrate the system (17) using fourth-order Runge Kutta exponential time differencing scheme using $N_\tau = 64$ with time-step size $\Delta t = 2e - 10$.

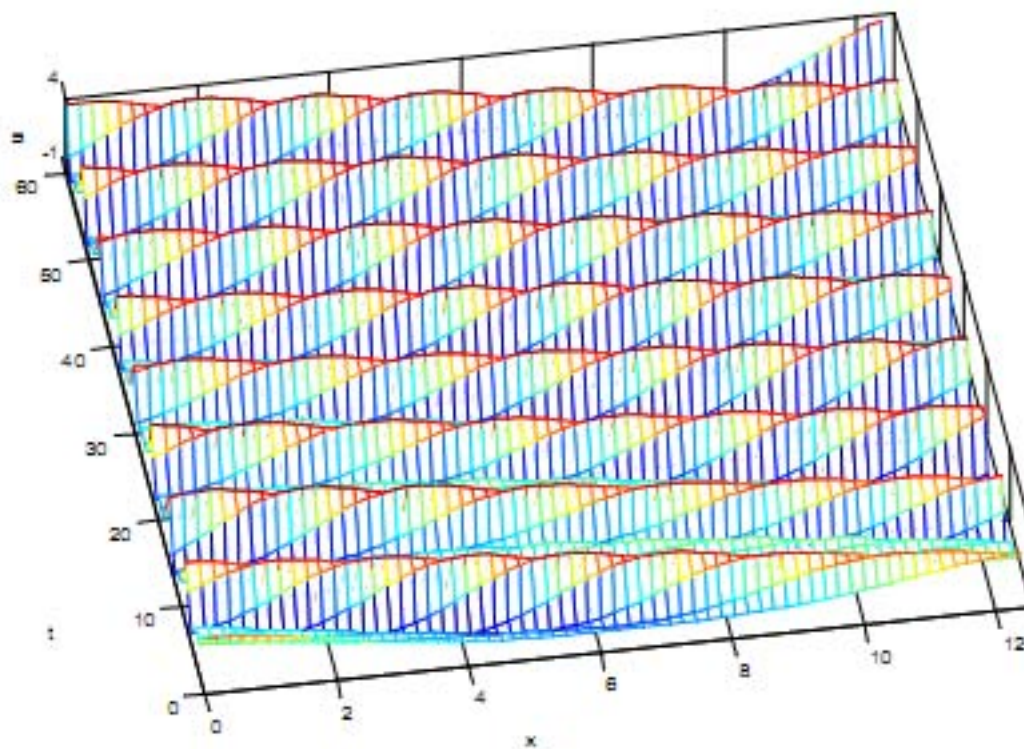


Figure 1 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = e^{\cos\left(\frac{x}{2}\right)}, x \in [0, 4\pi]$

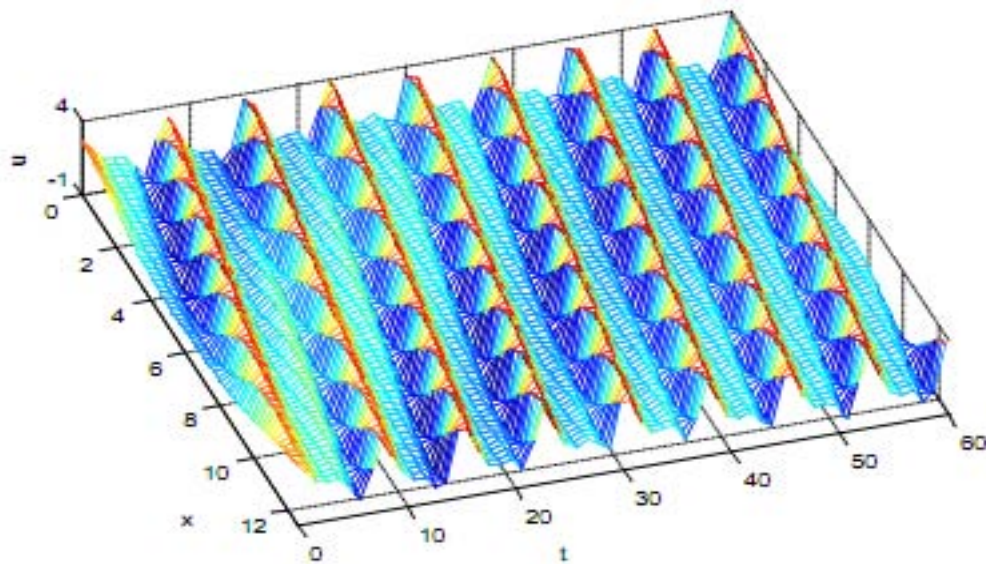


Figure 2 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = e^{\cos(\frac{x}{2})}, x \in [0, 4\pi]$

The solution, in the figure 1 with the initial condition $u_1(x) = e^{\cos(\frac{x}{2})}, x \in [0, 4\pi]$ with $N_\tau = 64$ and time-step size $\Delta t = 2e - 10$, appears as a mesh plot and

shows waves propagating, traveling periodically in time and persisting without change of shape.

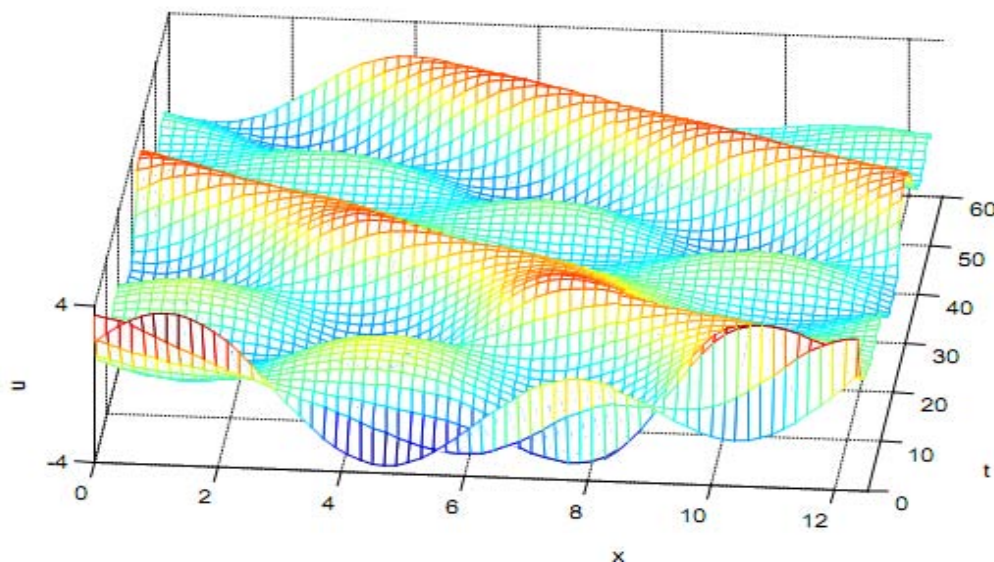


Figure 3 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_2(x) = 1.7 \cos(\frac{x}{2}) + 0.1 \sin(\frac{x}{2}) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$

In the figure 2 with the initial condition $u_2(x) = 1.7 \cos(\frac{x}{2}) + 0.1 \sin(\frac{x}{2}) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$ with $N_\tau = 64$ and time-step size $\Delta t = 2e - 10$, the

solution appears as a mesh plot and shows waves propagating, traveling periodically in time and persisting without change of shape.

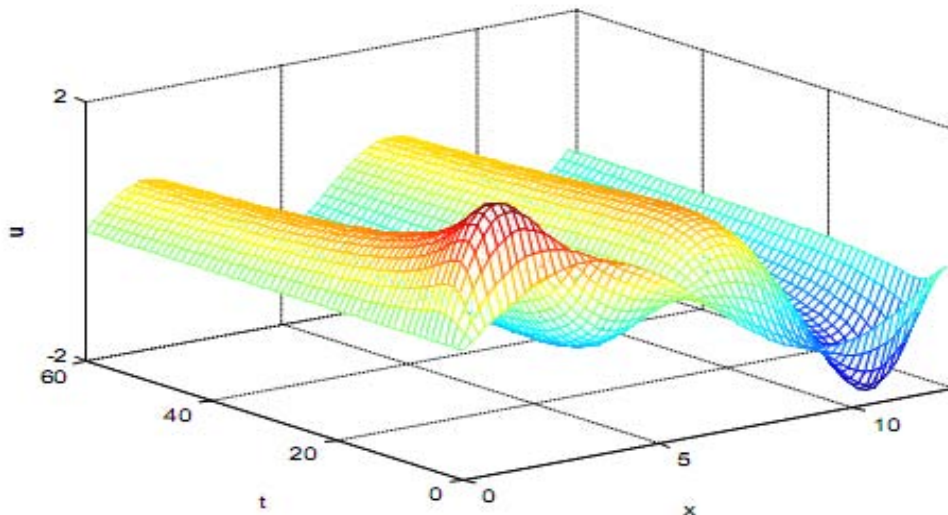


Figure 4 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = \sin(x/2), x \in [0, 4\pi]$

In the figure 3 with the initial condition $u_1(x) = \sin(x/2), x \in [0, 4\pi]$ with $N_x = 64$ and time-step size $\Delta t = 2e-10$, the solution appears more clear as a mesh plot and shows waves propagating, traveling periodically in time and persisting without change of shape.

VII. CONCLUSIONS

In this paper, the main objective of this study was for finding the solution of one dimensional semilinear fourth order hyperbolic *Kuramoto-Sivashinsky* equation, describing reaction diffusion problems, and the dynamics of viscous-fluid films flowing along walls. In order to achieve this, we applied Fourier spectral approximation for the spatial discretization. In addition, we evaluated the coefficients of the exponential time differencing and the exponential time differencing –fourth order Runge Kutta methods via the “Cauchy integral”. Some typical examples have been demonstrated in order to illustrate the efficiency and accuracy of the exponential time differencing methods technique in this case. For the simulation tests, we chose periodic boundary conditions and applied Fourier spectral approximation for the spatial discretization. In addition, we evaluated the coefficients of the Exponential Time Differencing Runge-Kutta methods via the “Cauchy integral” approach. The equations can be used repeatedly with necessary adaptations of the initial conditions.

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GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: I
NUMERICAL METHODS

Volume 14 Issue 1 Version 1.0 Year 2014

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Comparison of Capability Analysis of Cumulative Cardiac Thoracic Ratio (CTR) Outputs

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Abstract- This study investigated the Capability Process Analysis of cumulative Cardiac Thoracic Ratio (CTR) during Radiological Chest Examination using MX.4 Radiological Diagnostic Machine (DRM) at the Fate Medical Foundation Radiological Department, Auchi. The data for the study are classified as raw and simulated CTR values. Statistical process control was investigated to address process stability and capability analysis was performed for the two processes. The pattern of the means of the raw and simulated values was investigated using normal probability plots and empirical CDF functions. The raw computed CTR values and simulated CTR values confirmed that the system is operating under 1.0 – 1.3 sigma level for the raw CTR values. Around 28-39% of the raw CTR values obtained fall outside the specification limits. In addition, for all the cumulative raw CTR values suggested that the process is off centered and is towards the lower specification limit. Further study should be conducted on large repeated experimental CTR sample to ascertain the reliability of this study. Fellow up study of patients should be undertaken by the cardiologist to reduce the possible health risk associated with high CTR.

Keywords: *algorithms, capability plot, CTR, mx.4 DRM x-ray, heart failure.*

GJRE-I Classification : *FOR Code: 230199*



Strictly as per the compliance and regulations of:



Comparison of Capability Analysis of Cumulative Cardiac Thoracic Ratio (CTR) Outputs

Adekeye, K.S. ^α & Shaib, I.O ^σ

Abstract This study investigated the Capability Process Analysis of cumulative Cardiac Thoracic Ratio (CTR) during Radiological Chest Examination using MX.4 Radiological Diagnostic Machine (DRM) at the Fate Medical Foundation Radiological Department, Auch. The data for the study are classified as raw and simulated CTR values. Statistical process control was investigated to address process stability and capability analysis was performed for the two processes. The pattern of the means of the raw and simulated values was investigated using normal probability plots and empirical CDF functions. The raw computed CTR values and simulated CTR values confirmed that the system is operating under 1.0 – 1.3 sigma level for the raw CTR values. Around 28-39% of the raw CTR values obtained fall outside the specification limits. In addition, for all the cumulative raw CTR values suggested that the process is off centered and is towards the lower specification limit. Further study should be conducted on large repeated experimental CTR sample to ascertain the reliability of this study. Follow up study of patients should be undertaken by the cardiologist to reduce the possible health risk associated with high CTR.

Keywords: algorithms, capability plot, CTR, mx.4 DRM x-ray, heart failure.

I. INTRODUCTION

Advance knowledge has made the study of process capability analysis not limited to the industry or manufacturing process only but is gaining overwhelming application in other fields of human endeavour especially in medicine for the evaluation of health care performance such as surgical site control, infection rate, response of patient to change in treatment in the hospital, outbreak of epidemic and performance of a forecasting system related to medical studies such as heart false positive radiological examination. This study looks at the process monitoring of CTR output measurements and check its state of stability for abnormality detection.

In medicine, chest radiography is commonly called chest X-ray (CXR). It is a projection of radiography of the chest use to diagnose conditions affecting the chest, its contents and nearly structure. Ribeiro, Jose, Renato, Roberto, Francisco, Domingo, and Beatriz (2012) observed that chest radiography is

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among the most common films taken to diagnose many conditions. Like all methods of radiography, chest radiography employs ionizing radiation in the form of x-rays to generate images of the chest (Ribeiro, et al. (2012).

This research is motivated by the real life application of process capability analysis in the output of Cardio Thoracic Ratio of chest X-ray measurements in the examination of radiological process to establish capability analysis of the CTR experimental values. The aim of this study is to determine the capability analysis of Radiological CTR experimental values and the simulated values. The Specific objectives of the research are:

- To do capability analysis for experimental (Raw) and Simulated Cardiac Thoracic Ratio (CTR) values.
- To compare the capability analysis of the experimental Cardiac Thoracic Ratio (CTR) data (Raw values) and the simulated Cardiac Thoracic Ratio (CTR) data.
- To examine the significant difference in the variance of Cardiac Thoracic Ratio (CTR) data of raw and simulated CTR values.

II. LITERATURE REVIEW

The most commonly and widely used indices are C_p (Juan 1974), C_{pk} (Kane 1986), C_{pm} (Hsiang and Taguchi 1985) and C_{pmk} (Choiward and Owen 1970; Pearn and Kotz and Chen 1994-95) and their generalization for non-normal process suggested (Pearn and Kotz, 1995; Pearn and Chen 1995). Mukherjee (1995) studied conceptual approaches to process capability analysis. A number of new approaches to process capability analysis have been attempted and experimented (Carr 1991; Flaig 1996). Another index is given by Boyles (1994), when researcher or quality control officer is confronted with processes described by a characteristic whose values are discrete. Therefore, in such cases none of these indices can be used. The indices suggested so far whose assessment is meaningful regardless of whether the studied process in discrete or continuous are those suggested by Yeh and Bhaltachiya (1998). Borges and Ho (2001), Perakis and Kekalaki (2002; 2005) and James (1998) devised control

charts for clinical process improvement and sample size determination for discrete and continuous processes.

When the process parameters μ and σ are known, a kind of contour plots of the index C_{PK} , called process performance chart, was introduced to understand capability of a process. To also compare the indices c_p, c_{pk} and c_{pmk} (Boyles 1991) used contour plots called (μ, σ) - plots of these indices as functions of the process parameter (μ, σ) . Deleryd and Vannman (1999) and Vannman (2001) used contour plots, called (δ, γ) - plots, as functions of the process parameters to illustrate the restrictions that the different indices in the $c_p(u, v)$ - family impose on the process parameter (μ, σ) . When the process parameters μ and σ are unknown and need to be estimated, Deleryd and Vannman (1990) and Vannman (2001) developed what they called the (δ, γ) - plot and the confidence rectangle plots. These plots are tools to draw inference about the process capability based on a random studied quality characteristic.

In this study, evaluation of cumulative capability characteristics of the experimental CTR values (Raw) and Simulated CTR values using uniform distribution are investigated.

III. DISTRIBUTION PROCESS INDEX APPLICATION

In real life application, calculation of proposed capability index boils down to computation of the process yield. To evaluate the process yield, it is necessary to apply a curve fitting method to approximate the quality characteristic distribution $f(x)$. Polansky (1999) used non-parametric approach particularly Kernel density estimation to estimate process yield for both univariate as well as multivariate quality characteristics. Ciarlini, Gigli and Regoliosi (1999) used bootstrap methodology to estimate failed probabilities even in regions not supported by data with accuracy. Independent of the sample variances is useful when data are not nearly normal. The Pearson distribution was implemented (Clement 1989), the Johnson distribution was suggested (Chou and Polansky 1996; Chou, Polansky and Mason 1998; Polansky et al., 1998). Burr distribution was used to describe non-normal process data (Castagholia 1996).

In practice, one may often be faced with processes whose distributions are far from being normal. In this capability study the index and the assumption that the underlying distribution of the examined process is a non-normal form and in particular, exponential. Gunter (1989) observed the

experimental distribution arises frequently in industrial processes and were explained in the article (Yeh and Bhattachayya 1998). The normal and exponential process index is achievable for continuous process however; they are useless when the process is discrete. Poison process index C_{pk} is used in the assessment of discrete process. The properties of are examined in the case where the studied process is described by a poison distribution characteristic with parameter $m > 0$. The uniform process index is achievable for continuous process however; it is useful when the process was discrete. Uniform process index C_{pk} is used in the assessment of discrete process. The properties of C_{pk} are examined in the case where the studied process is described by a uniform distribution characteristic with some parameter a and b (Maiti *et al.*, 2009).

In this study chart such as histogram with normal distribution is used to detect the trend behaviour of the CTR distribution outlier for abnormal CTR values. Uniformly simulated data will be compared with the raw CTR values based on capability analysis and variance. Uniform distribution process is simulated to compare with the raw CTR value of chest radiological examination in this study.

IV. SIMULATION TECHNIQUE

Simulation provides a method for checking your understanding of the world around you and helps us to produce better results faster.

a) A Study Simulation

In the study of Cardiac Thoracic Ration of Chest X-ray films examination, the raw values of cardiac and thoracic measure shall be computed to obtain the CTR value of patients that undergo the Chest X-ray examination as:

$$CTR = \frac{C_v}{T_v} \quad (1)$$

where C_v is the cardiac value and the T_v is the thoracic value of the measurements. If the $CTR=0.5$, the reading is said to be normal with boundary allowances of 0.45 and 0.55 for error of readings accommodation. Hence, the tolerance values are $USL=0.55$ and $LSL=0.45$ with the target value

$$T = \left(\frac{USL + LSL}{2} \right) = 0.5. \quad (2)$$

The study employs simulation technique using uniform distribution process between $F(b)=0.71$ and $F(a)=0.43$ with 5 numbers of subgroups for 150 random numbers making a total of 750 simulated patients' CTR values for the study.

V. DESIGN AND IMPLEMENTATION OF SIMULATION

The simulation use in this study follows a uniform distribution process which ranges from 0.43 to 0.71 with 5 number of variable as subgroup measurements for 150 sample random number all together making 750 observations. Excel application package is the implementation medium used for the random number generation.

VI. VARIANCE CTR RAW AND SIMULATED PROCESSES COMPARISON

Bartlet 'b'-statistic is assumed as test-statistic that is distributed approximately as χ^2 - distribution when samples are independently drawn from normal population (Singha, 2002). We test that $H_0 : \sigma_r^2 = \sigma_s^2$ and $H_0 : \sigma_r^2 \neq \sigma_s^2$ to determine equality of variances (Gomez and Kwanchai, 1984) of both raw and simulated CTR values of Chest X-ray measurement. Comparison of the variances of the raw CTR and Simulated CTR value is carried out in this study to investigate the process equality of variances. In this study, the variance of the CTR raw and simulated values are computed and tested for homogeneity based on the Bartlet Test 'b' statistic. The algorithm for the procedure is described by the following algorithm steps (A4).

VII. RESEARCH METHOD

The source of data for the analysis is primary through raw computation and computer simulation using uniform distribution. The raw data are generated

b) *Probability Plot of RCTrv and SCTrv*

through the measurement values of the cardiac and thoracic of films output of Chest X-ray of patients from the radiological machine process. The ratios of the measurements are computed to obtain various CTR values over time. Inspection Coding Sheet (ICS) is used to randomly generate the samples for the study. Limits are set equal to 3sigma as $\bar{x} \pm 3\hat{\sigma}$ for both upper and lower limit (USL and LSL) and tolerance limit was established by $T \pm 0.05$ for the raw and simulated CTR values. $T = 0.5$ is based on the specification criteria for non-sensitivity analysis (specificity) while statistical process control is investigated to address process stability. Capability analysis is performed for the two processes. The pattern of the means of the raw and simulated values are detected using exploratory data Analysis (EDA) approach like normal probability plots, empirical CDF functions and Box-plot. In addition, homogeneity of variance of the two processes is investigated based on Bartlet's 'b' statistic. The analysis of data is performed electronically with the aid of statistical software MINITAB version 16.0.

VIII. DATA ANALYSIS AND RESULT

This aspect focuses on exploring data analysis behaviour pattern of Raw and Simulated Cardiac Thoracic Ratio (CTR) values. It also discusses control chart graphs, process capability analysis and the process variance comparison using Bartlet 'b' statistic.

a) *Exploratory Data Analysis of RCTrv and SCTrv*

The result of normality plots, cumulative probability density and box plot descriptive analysis results are summary as follow.

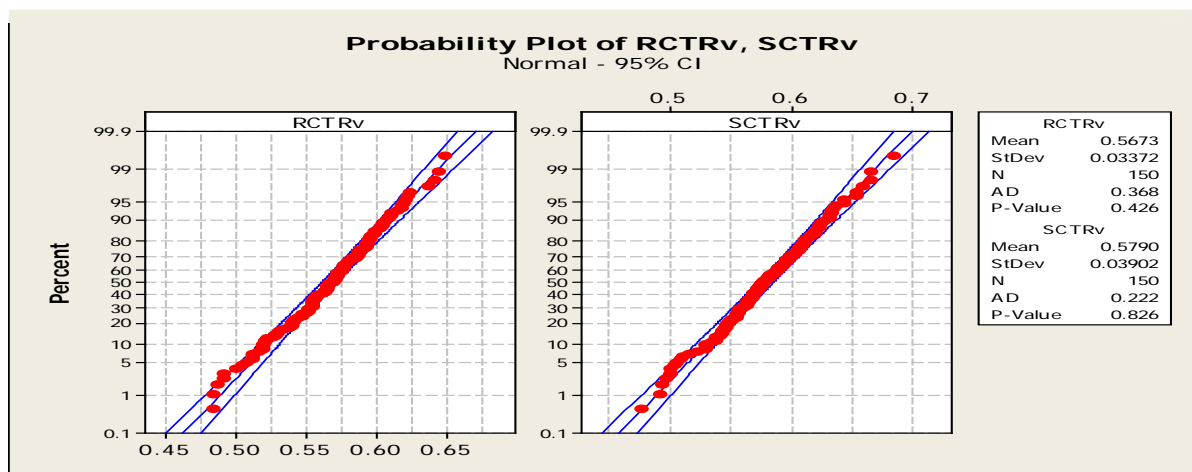


Figure 1 : Normality Plot of RCTrv and SCTrv

The probability plots of raw and simulated CTR values illustrated in the fig. above shows that both the raw and simulated CTR values follow linear pattern demonstrating normality trend with Simulated CTR value

perfectly fit in the trend. On the average raw cardio thoracic ratio (RCTR) and simulated CTR values are 0.38 and 0.37 with standard deviation values of 0.033 and 0.039 based on the 150 total samples. The probability

values for RCTRV and SCTRV (0.428 and 0.828) suggest strong evidence of accepting that the raw value and the

simulated CTRv are normally distributed as the P-values are greater than 0.05 critical value at 5%.

a) *Empirical CDF of RCTRV and SCTRV*

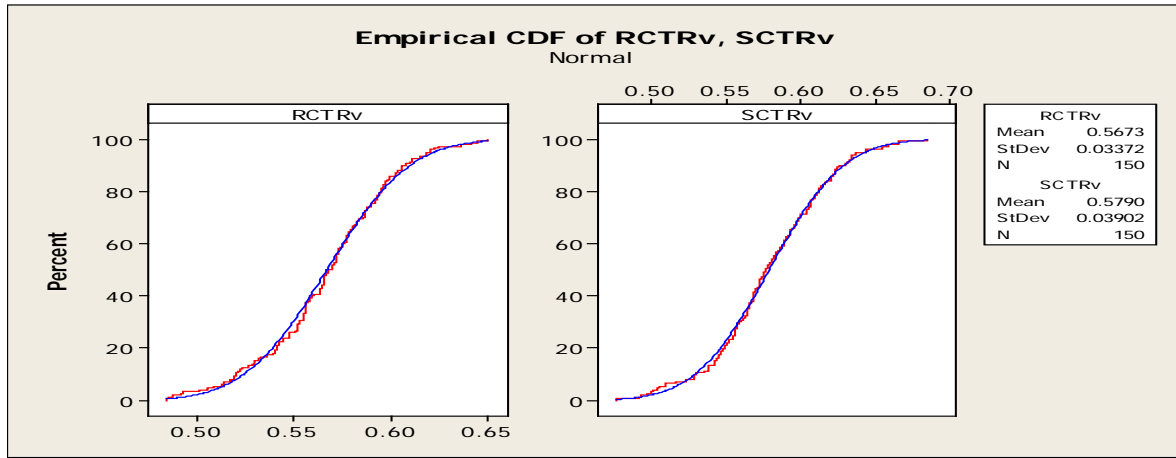


Figure 2 : Cumulative CDF Plot of RCTRV and SCTRV

The probability plots of raw and simulated CTR values illustrated in the fig. above shows that both the raw and simulated CTR values follow linear pattern demonstrating normality trend with Simulated CTR value

perfectly fit in the trend. On the average raw cardio thoracic ratio (RCTR) and simulated CTR values are 0.38 and 0.37 with standard deviation values of 0.033 and 0.039 based on the 150 total samples.

b) *Boxplot of RCTRV and SCTRV*

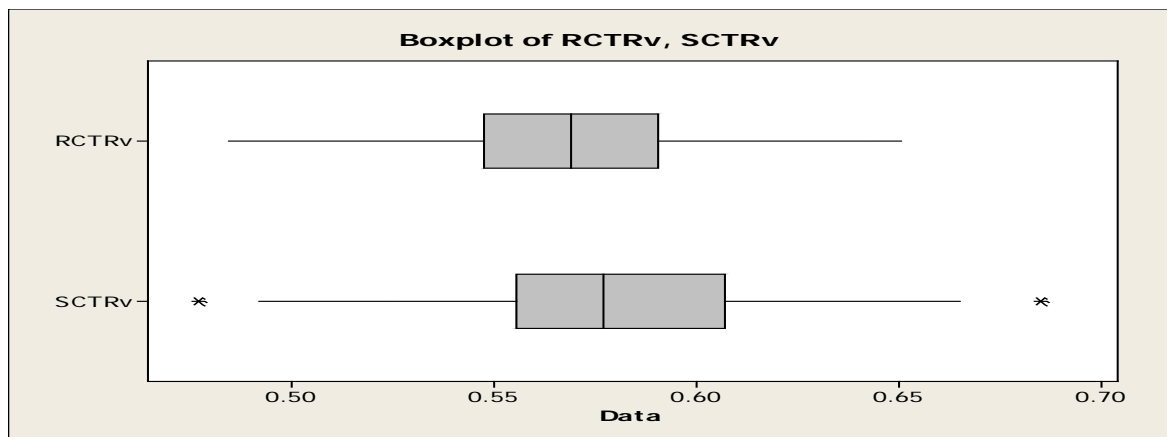


Figure 3 : Boxplot of RCTRV and SCTRV

The Boxplot of RCTRV and SCTRV illustrate non deviation in the RCTRV but deviation exists in the SCTRV because of the existence of the spike (whiskers of dispersion). This confirms that there is likelihood of more deviation from the 0.5 CTR standard in the SCTRV compare to the RCTRV.

c) Process capability Analysis of CTR Measurements of Raw Data N=150

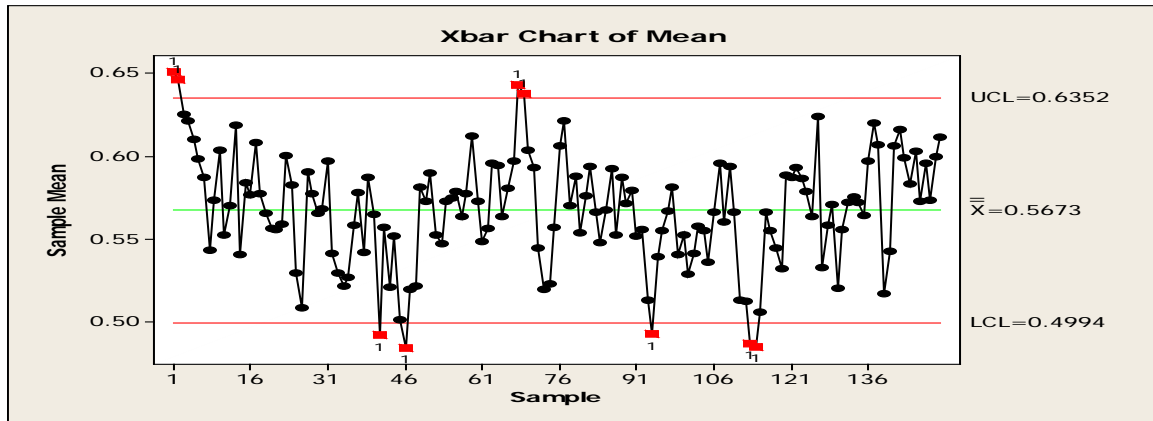


Figure 1a

From the fig1a, the aggregate observation of process statistical stability and under control with 150 samples indicates that all points of the raw CTR values are falling within control limit confirming the predicted trend of sensitivity.

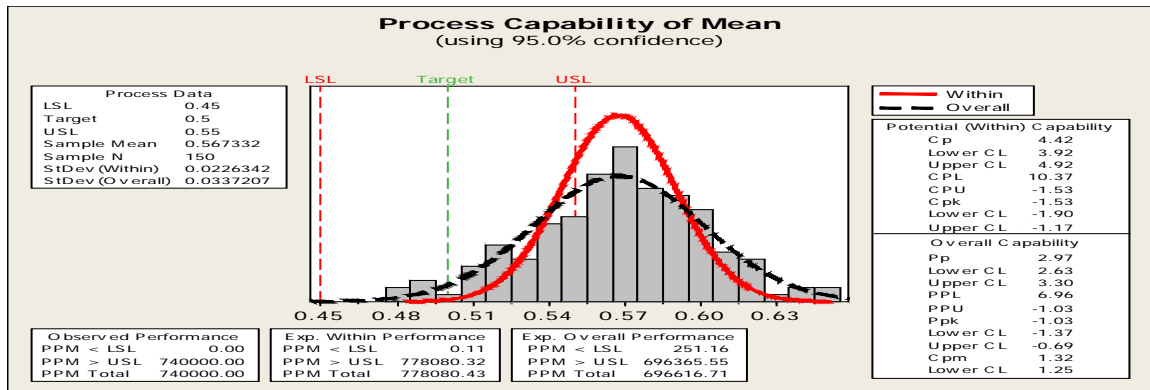


Figure 1b

For sample 150, the mean estimated is 0.5654 where the within and overall standard deviation are 0.0222 and 0.0227, $C_p = 4.42$, $C_{pk} = -1.52$, $C_{pm} = 1.22$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The

percentage of the specification band that the process uses up is $P = (1/C_p) * 100 = 22.6\%$. This indicates that the process is using about 22.6% of the specification band.

d) Process capability Analysis Simulated CTR value Uniform Distribution N=150

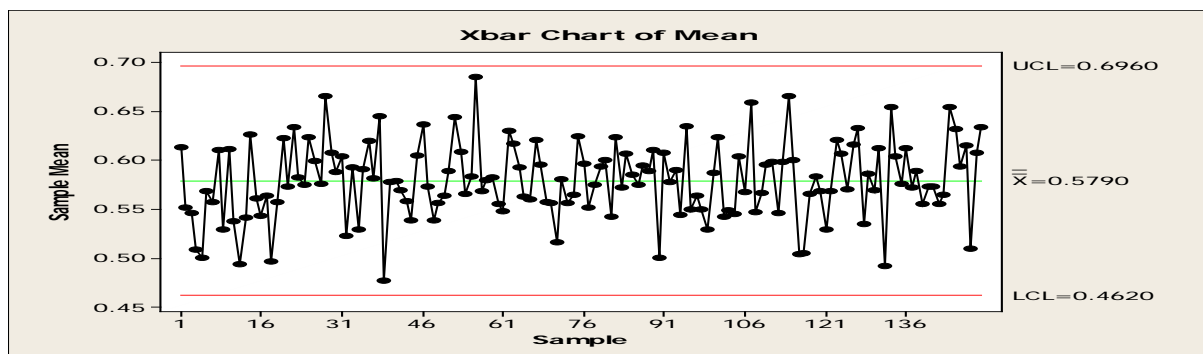


Figure 2a

From the fig2a, cumulative 150 samples all points of the simulated CTR values are falling within control limits implying process stable and follow a predictable trend.

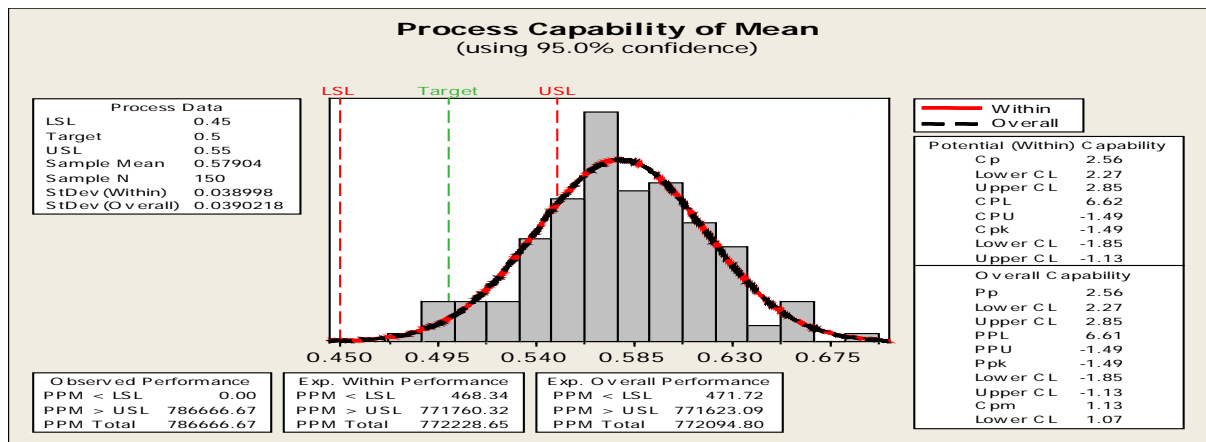


Figure 2b

For cumulative sample 150, the mean estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p) * 100 = 39.1\%$. This implies

that the process is using about 39.1% of the specification band. Therefore, the values of $C_p = 2.56$ and $P_p = 2.56$ are equal therefore the process has little between subgroup variability. The empirical analysis results and findings of process capability analysis of raw and simulated CTR values are summarized in the table below:

Table 1 : Summary of Findings of cumulative sample process capability analysis
Raw Cardiac Thoracic Ratio Value (RCTRv)

n	25	50	75	100	125	150
μ	0.5654	0.5658	0.5745	0.5681	0.5648	0.5654
σ	0.0323	0.0276	0.0275	0.0226	0.0229	0.0227
C_p	4.42	2.99	4.40	4.27	4.55	4.42
C_{pk}	-2.14	-1.27	-2.16	-1.58	-2.15	-1.52
C_{pm}	1.07	1.21	1.19	1.12	1.26	1.22
P_p	2.09	2.66	2.66	2.97	2.94	2.97
$\left(\frac{1}{C_p}\right)$	0.226	0.334	0.227	0.234	0.2180	0.2260
$P = \left(\frac{1}{C_p}\right) * 100$	22.6%	33.4%	22.7%	23.4%	22%	22.6%
$C_p \neq P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$

Source: Results extracted from Minitab 16.0

For cumulative sample 150, the mean estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process

uses up is $P = (1/C_p) * 100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Values of C_p and P_p are barely equal hence there is substantial between subgroup variability.

Table 2 : Summary of Findings of cumulative sample process capability analysis
Simulated Cardiac Thoracic Ratio Value (SCTRV)

n	25	50	75	100	125	150
μ	0.5651	0.5724	0.5768	0.5771	0.5771	0.5790
σ	0.0422	0.0413	0.0400	0.0280	0.0280	0.0227
C_p	2.21	2.38	2.56	2.62	2.61	2.56
C_{pk}	-2.14	-1.08	-1.28	-1.42	-1.42	-1.49
C_{pm}	1.07	1.19	1.15	1.16	1.16	1.22
P_p	2.09	2.66	2.66	2.97	2.94	2.97
$\left(\frac{1}{C_p}\right)$	0.4516	0.4201	0.3908	0.3821	0.3827	0.3907
$P = \left(\frac{1}{C_p}\right) * 100$	45.2%	42%	39.1%	38.2%	38.2%	39.1%
$C_p \neq P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$	$C_p < P_p$

Source: Results extracted from Minitab 16.0

For cumulative sample 150, the mean estimate is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p) * 100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Values of C_p and P_p are barely equal hence there is substantial between subgroup variability.

For the total sample 150, the mean value estimated is 0.5790 where the within and overall standard deviation are 0.0289 and 0.0290, $C_p = 2.56$, $C_{pk} = -1.49$, $C_{pm} = 1.12$ since the $C_{pk} < C_p$, the process is off centred and is toward the lower specification limits. The percentage of the specification band that the process uses up is $P = (1/C_p) * 100 = 39.1\%$. This implies that the process is using about 39.1% of the specification band. Hence, the values of $C_p = 2.56$ and $P_p = 2.56$ and are equal then the process has no subgroup variability. The average estimated value of CTR is 0.57 which is 0.02 higher than the upper specification limit. True sensitivity analysis value of about 59.9% is confirmed fail points among the examined patients while the deviation among the sample measures is 0.023. Both C_p and

P_p are near approximate hence there is little between subgroup variability.

e) Bartlet Test 'b' Statistic Computation and Result

The computational result of the Bartlet Test 'b' Statistic value do not exceed the Chi-square value, the variance of the raw and the simulated CTR values have unequal variance.

IX. CONCLUSION

After aggregating all the raw computed CTR values and simulated CTR values obtained, it is empirically confirmed that the system is operating under 1.0 – 1.3 sigma level for the raw CTR values. Around 28-39% of the raw CTR values obtained are falling outside the specification limits and 30-45% of the specification band is being used. In addition, the $C_{pk} < C_p$ for all the cumulative raw CTR values suggesting that the process is off centred and is towards the lower specification limit. Therefore, the points are falling outside the upper specification limit which clearly indicates that the variability in the raw CTR process is very high.

X. RECOMMENDATION

Based on the empirical outputs of capability analysis of radiological result of CTR values (raw and simulated), this study therefore recommends that health awareness campaign on slow death resulting from heart failure as a result of absence of early detection of

abnormal CTR value among patients should be created by the government and health agencies. Patients should be medically advised on the measure to control and maintain stable CTR. Also on how to adopt better management methods which can subsequently prevent possibility of high CTR and further study should be conducted on large repeated experimental scale to ascertain the reliability of this study. Follow up study of patients should be undertaken by the cardiologist to reduce the possible health risk that could result from the CTR.

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APPENDIX

A1: Algorithm Statistical Quality Control Process

Step 1 : Obtain the data design in sample subgroups in row (i) and Column (j) respectively Let subg $i = 1, 2, 3, 4, \dots, n$. for each sample and subg $j = 1, 2, 3, 4, \dots, m$. for each sample subgroups respectively.

Step 2 : Calculated the row total values $\sum_{i=1}^n x_i$ and the row average value of the sample subgroups and the mean of the mean of sample subgroup as:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n x_i \text{ and } \bar{\bar{X}} = \frac{1}{M} \sum_{j=1}^M \bar{X}_j$$

Step 3 : Calculate the sample range and the sample subgroup range;

$$R_i = \text{MaxValue of } x_i - \text{MinValue of } x_i \text{ and } \bar{R}_j = \frac{1}{M} \sum_{j=1}^M R_j$$

Step 4 : Compute the sample variance and standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}$$

Step 5 : Evaluate the limits USL, CL and LSL for the sample mean

$$USL = \bar{\bar{X}} + A_2 \bar{R} \quad LSL = \bar{\bar{X}} - A_2 \bar{R} \quad CL = \bar{\bar{X}}$$

Step 6 : Evaluate the limits USL, LSL and CT for the sample range for = 0.577, , when n=5 from the SQC table readings.

$$USL = D_4 \bar{R} \quad LSL = D_3 \bar{R} \quad CL = \bar{R}$$

$$\text{for } A_2 = 0.577, D_3 = 0.00, D_4 = 2.115 \text{ when}$$

n=5 from the SQC table readings.

Step 7 : Plot the graph of \bar{X} chart and \bar{R} chart using the step 6 and 7 for the plotting conditions against the sample subgroup.

A2: Algorithm Process Capability Analysis

Step 1 : obtain the values of LSL and USL

Step 2 : Compute C_{pu} and C_{pl} as

$$C_{pu} = \frac{USL - \mu}{3\sigma} \text{ and } C_{pl} = \frac{\mu - LSL}{3\sigma}$$

Step 3 : Calculate the $C_p = \frac{USL - LSL}{6\sigma}$

Step 4 : Compute

$$C_{pk} = \text{Min} \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} \\ = \text{Min}(C_{pu}, C_{pl})$$

$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}$$

Step 5 : Calculate

$$T = (USL + LSL) / 2 \quad \mu = \bar{\bar{X}} = \frac{1}{M} \sum_{j=1}^M \bar{X}_j$$

Step 6 : Define $k = \frac{|T - \mu|}{(USL - LSL) / 2}$

Step 7 : Evaluate $C_{pk} = C_p (1 - k)$

When $LSL \leq USL$, $0 \leq k \leq 1$, $k = 0$ i.e. the process average equals target value T , then $C_{pk} = C_p$ if $\mu = LSL$ or $\mu = USL$ then $k = 1$ and $C_{pk} = 0$

Step 8 : If $T = \frac{(LSL - USL)}{2}$, we compute

$$C_{pm} = \frac{USL - LSL}{6\sigma} = \frac{USL - LSL}{6\sigma \sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}}$$

Step 9 : If $T \neq \frac{(LSL - USL)}{2}$, compute

$$C_{pm}^* = \text{Min} \left\{ \frac{USL - T}{3\sigma'}, \frac{T - LSL}{3\sigma'} \right\}$$

$$\hat{C}_{pm} = \frac{USL - LSL}{6\sqrt{s^2 + \left(\bar{X} - T\right)^2}}$$

A3 Simulation Algorithms

Step 1 : Open Excel Window

Step 2 : Click on Data > Select Data Analysis

Step 3 : Click on Random Number Generation

Step 4 : Type 5 in the number of variable column then 150 in the number of random column

Step 5 : Click on the distribution combo box and select uniform in the pull down menu

Step 6 : Type in the between column 0.43 and 0.71 values

Step 7 : Click Ok

Step 8 : The simulated CTR value appears in row and column format End.

A4: Algorithm Bartlet Test 'b' Statistic

Step 1 : obtain the values of Raw and Simulated CTR values

Step 2 : Compute s_1^2 and s_2^2 variances of the CTR

$$s_1^2 = \frac{1}{n} \sum_{i=1}^n \left(X_i - \bar{X} \right)^2 \text{ and } s_2^2 = \frac{1}{n} \sum_{i=1}^n \left(X_i - \bar{X} \right)^2$$

Step 3 : Calculate the $S^2 = \frac{\sum (n_i - 1) S_i^2}{N - K}$ $i = 1, 2, \dots$

Step 4 : Compute

$$Q = (n - k) \log S^2 - \sum (n_i - 1) \log S_i^2$$

Step 5 : Calculate

$$H = 1 + \frac{1}{3} (k - 1) \left[\sum \frac{1}{(n_i - 1)} - \frac{1}{(N - K)} \right]$$

Step 6 : Compute b

A4 : Bartlet 'b' Statistic computational results for Raw and Simulated CTR values

Raw CTR	Variance	Simulated CTR	Variance
s_1^2	0.674	s_2^2	1.193
$\log s_1^2$	-0.172	$\log s_2^2$	0.0765
$S^2 = \frac{\sum (n_i - 1) S_i^2}{N - K} = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{N - K}$ $= \frac{(5 - 1)(0.674) + (5 - 1)(1.193)}{1500 - 5} = 0.0515$			
$Q = (n - k) \log S^2 - \sum (n_i - 1) \log S_i^2 = (5 - 2) \log(0.0515) - [(4)(0.674) + (4)(1.193)]$ $Q = (3)(-1.2882) - [7.4675] = -11.3321$			
$H = 1 + \frac{1}{3} (k - 1) \left[\sum \frac{1}{(n_i - 1)} - \frac{1}{(N - K)} \right] = 1 + \frac{1}{3} (2 - 1) \left[\frac{1}{(4)(4)} - \frac{1}{(150 - 5)} \right]$ $H = 1 + \frac{1}{3} \left[\frac{1}{16} - \frac{1}{(145)} \right] = 1.33(0.0625 - 0.006896) = 0.07395$			
$b = 2.3026 \frac{Q}{H} = 2.3026 \left(\frac{-11.3321}{0.07395} \right) = -352.85$			

$$\chi^2_{\alpha} \text{ with } (k-1) = \chi^2_{0.05, (2-1)} = 3.38$$

Decision Rule

* If b value exceeds χ^2 value, both processes have equal variance.

* If b value does not exceed χ^2 value, both processes do not have equal variance.

APPENDIX B B3

Raw Computed Data										Simulated Cardio-Thoracic Ratio Using Uniform Distribution									
Radiographic Films Readings of Chest X-ray										Radiographic Films Readings of Chest X-ray									
For congestive Heart Failure Cardiomegaly Conditions										For congestive Heart Failure Cardiomegaly Conditions									
Cadio-Thoracic Ratio					Variance					Cadio-Thoracic Ratio									
Sample	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Total	Mean	Range	Sample	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Total	Mean	Range	Variance	
1	0.62	0.47	0.71	0.7	0.76	3.25	0.65	0.29	0.013	1	0.64	0.65	0.59	0.66	0.52	3.07	0.61	0.14	0.003
2	0.65	0.7	0.73	0.59	0.56	3.23	0.65	0.17	0.005	2	0.65	0.57	0.64	0.47	0.43	2.76	0.55	0.21	0.010
3	0.67	0.51	0.74	0.57	0.63	3.13	0.63	0.23	0.008	3	0.55	0.64	0.56	0.54	0.44	2.73	0.55	0.19	0.005
4	0.51	0.63	0.78	0.68	0.52	3.11	0.62	0.27	0.013	4	0.6	0.56	0.43	0.48	0.48	2.55	0.51	0.17	0.005
5	0.72	0.53	0.6	0.69	0.51	3.05	0.61	0.2	0.009	5	0.64	0.42	0.47	0.42	0.54	2.5	0.5	0.22	0.009
6	0.63	0.52	0.73	0.55	0.57	2.99	0.6	0.21	0.007	6	0.45	0.47	0.63	0.68	0.61	2.84	0.57	0.22	0.010
7	0.6	0.56	0.71	0.61	0.46	2.94	0.59	0.25	0.008	7	0.49	0.68	0.45	0.55	0.62	2.79	0.56	0.23	0.009
8	0.53	0.52	0.51	0.59	0.57	2.71	0.54	0.09	0.001	8	0.48	0.67	0.72	0.64	0.54	3.05	0.61	0.24	0.010
9	0.54	0.63	0.68	0.54	0.48	2.87	0.57	0.2	0.006	9	0.45	0.51	0.54	0.44	0.7	2.65	0.53	0.26	0.011
10	0.71	0.49	0.64	0.61	0.56	3.02	0.6	0.21	0.007	10	0.62	0.7	0.52	0.51	0.72	3.06	0.61	0.22	0.010
11	0.61	0.48	0.5	0.52	0.66	2.76	0.55	0.18	0.006	11	0.64	0.62	0.51	0.46	0.47	2.69	0.54	0.18	0.007
.
.
137	0.59	0.62	0.64	0.72	0.53	3.1	0.62	0.19	0.005	137	0.55	0.54	0.68	0.47	0.62	2.86	0.57	0.21	0.006
138	0.66	0.62	0.61	0.58	0.57	3.03	0.61	0.08	0.001	138	0.53	0.67	0.73	0.48	0.54	2.95	0.59	0.25	0.011
139	0.57	0.5	0.46	0.52	0.53	2.59	0.52	0.11	0.002	139	0.68	0.62	0.51	0.43	0.53	2.78	0.56	0.24	0.010
140	0.6	0.57	0.55	0.5	0.5	2.71	0.54	0.1	0.002	140	0.56	0.58	0.59	0.54	0.6	2.86	0.57	0.06	0.001
141	0.58	0.57	0.74	0.64	0.5	3.03	0.61	0.24	0.008	141	0.47	0.65	0.61	0.63	0.5	2.87	0.57	0.18	0.007
142	0.66	0.54	0.71	0.66	0.51	3.08	0.62	0.2	0.007	142	0.51	0.54	0.61	0.66	0.46	2.77	0.55	0.2	0.006
143	0.6	0.58	0.61	0.68	0.52	3	0.6	0.16	0.003	143	0.67	0.55	0.55	0.46	0.59	2.82	0.56	0.21	0.006
144	0.63	0.48	0.55	0.67	0.58	2.91	0.58	0.19	0.005	144	0.65	0.69	0.67	0.56	0.7	3.27	0.65	0.15	0.003
145	0.73	0.53	0.56	0.67	0.52	3.01	0.6	0.21	0.009	145	0.56	0.7	0.71	0.6	0.59	3.16	0.63	0.15	0.005
146	0.73	0.5	0.56	0.55	0.52	2.86	0.57	0.23	0.008	146	0.73	0.52	0.53	0.49	0.69	2.97	0.59	0.24	0.012
147	0.66	0.52	0.57	0.62	0.62	2.98	0.6	0.15	0.003	147	0.46	0.63	0.71	0.55	0.72	3.07	0.61	0.26	0.012
148	0.57	0.53	0.59	0.56	0.63	2.87	0.57	0.1	0.001	148	0.54	0.56	0.52	0.44	0.49	2.55	0.51	0.12	0.002
149	0.62	0.57	0.6	0.59	0.61	3	0.6	0.05	0.000	149	0.61	0.47	0.66	0.59	0.7	3.04	0.61	0.23	0.008
150	0.58	0.53	0.59	0.75	0.62	3.06	0.61	0.22	0.007	150	0.71	0.56	0.63	0.57	0.69	3.17	0.63	0.16	0.005

Source: Chest X-ray Radiological Readings 2011

Source: Chest X-ray Radiological Simulation Readings 2011



An Exponential Time Differencing Method for the Kuramoto-Sivashinsky Equation

By Gentian Zavalani

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Abstract- The spectral methods offer very high spatial resolution for a wide range of nonlinear wave equations, so, for the best computational efficiency, it should be desirable to use also high order methods in time but without very strict restrictions on the step size by reason of numerical stability. In this paper we study the exponential time differencing fourth-order Runge-Kutta (ETDRK4) method; this scheme was derived by Cox and Matthews in [S.M. Cox, P.C. Matthews, Exponential time differencing for stiff systems, J. Comp. Phys. 176 (2002) 430–455] and was modified by Kassam and Trefethen in [A. Kassam, L.N. Trefethen, Fourth-order time stepping for stiff PDEs, SIAM J. Sci. Comp. 26 (2005) 1214–1233]. We compute its amplification factor and plot its stability region, which gives us an explanation of its good behavior for dissipative and dispersive problems. We apply this method to the Kuramoto-Sivashinsky Equation obtaining excellent results.

Keywords: *kuramoto-sivashinsky, etd, ETDRK4, stiff systems, integrating factor.*

GJRE-I Classification : *FOR Code: 230199, 010301p*



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Abstract- The spectral methods offer very high spatial resolution for a wide range of nonlinear wave equations, so, for the best computational efficiency, it should be desirable to use also high order methods in time but without very strict restrictions on the step size by reason of numerical stability. In this paper we study the exponential time differencing fourth-order Runge–Kutta (ETDRK4) method; this scheme was derived by Cox and Matthews in [S.M. Cox, P.C. Matthews, Exponential time differencing for stiff systems, J. Comp. Phys. 176 (2002) 430–455] and was modified by Kassam and Trefethen in [A. Kassam, L.N. Trefethen, Fourth-order time stepping for stiff PDEs, SIAM J. Sci. Comp. 26 (2005) 1214–1233]. We compute its amplification factor and plot its stability region, which gives us an explanation of its good behavior for dissipative and dispersive problems. We apply this method to the Kuramoto-Sivashinsky Equation obtaining excellent results.

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I. INTRODUCTION

The spectral methods have been shown to be remarkably successful when solving time-dependent partial differential equations (PDEs). The idea is to approximate a solution $u(x, t)$ by a finite sum

$$\psi(x, t) = \sum_{k=0}^N \kappa_k(t) \varphi_k(x)$$

Where the function class $\varphi_k(x)$, $k = 0, 1, 2, \dots, N$ will be trigonometric for x -periodic problems and, otherwise, an orthogonal polynomial of Jacobi type, with Chebyshev polynomial being the most important special case. To determine the expansion coefficients $\kappa_k(t)$, we will focus on the pseudospectral methods, where it is required that the coefficients make the residual equal zero at as many (suitably chosen) spatial points as possible. Three books [15, 17] and [19] have been contributed to supplement the classic references [18] and [16].

When a time-dependent PDE is discretized in space with a spectral simulation, the result is a coupled system of ordinary differential equations (ODEs) in time: it is the notion of the method of lines and the resulting set of ODEs is stiff; the stiffness problem may be even

exacerbated sometimes, for example, using Chebyshev polynomials. The linear terms are primarily responsible for the stiffness with rapid exponential decay of some modes (as with a dissipative PDE) or a rapid oscillation of some modes (as with a dispersive PDE). Therefore, for a time-dependent PDE which combines low-order nonlinear terms with higher-order linear terms it is desirable to use higher-order approximation in space and time. The outline of this paper is as follows. In Section 2 we describe the ETDRK4 (Exponential Time Differencing fourth-order Runge–Kutta) method by Cox and Matthews in [12] and the modification proposed by Kassam and Trefethen in [6]. We discuss the stability of the ETDRK4 method in Section 3. In Sections 4 and 5 we test the method for the Kuramoto-Sivashinsky equation in one space dimensions and, finally, In Sections 6 we summarize our conclusions.

II. EXPONENTIAL TIME DIFFERENCING FOURTH-ORDER RUNGE–KUTTA METHOD

The numerical method considered in this paper is an exponential time differencing (ETD) scheme. These methods arose originally in the field of computational electrodynamics [20]. Since then, they have recently received attention in [21] and [22], but the most comprehensive treatment, and in particular the ETD with Runge–Kutta time stepping, is in the paper by Cox and Matthews [12]. The idea of the ETD methods is similar to the method of the integrating factor (see, for example, [15] or [19]) we multiply both sides of a differential equation by some integrating factor, then we make a change of variable that allows us to solve the linear part exactly and, finally, we use a numerical method of our choice to solve the transformed nonlinear part.

When a time-dependent PDE in the form

$$u_t = \mathcal{L}u + \mathcal{N}(u, t) \quad (2.1)$$

where \mathcal{L} and \mathcal{N} are the linear and nonlinear operators respectively, is discretized in space with a spectral method, the result is a coupled system of ordinary differential equations (ODEs),

$$u_t = \mathcal{L}u + \mathcal{N}(u, t) \quad (2.2)$$

Multiplying (2.2) by the term $e^{-\mathcal{L}t}$, known as the integrating factor, gives

$$e^{-\mathcal{L}t} u_t - e^{-\mathcal{L}t} \mathcal{L}u = e^{-\mathcal{L}t} \mathcal{N}(u, t) \quad (2.3)$$

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and with the new variable $v = e^{-Lt}u$, we find the transformed equation

$$v_t = e^{-Lt}N(e^{Lt}v, t) \quad (2.4)$$

Where the linear term is gone; now we can use a time stepping method of our choice to advance in time. However, the integrating factor methods can also be a trap, for example, to model the formation and dynamics of solitary waves of the KdV equation (see Chapter 14 of [15]). A second drawback is the large error constant. In the derivation of the ETD methods, following

[21], instead of changing the variable, we integrate (2.3) over a single time step of length, getting

$$u_{n+1} = e^{Lh}u_n + e^{Lh} \int_0^h e^{-L\tau} N(u(t_n + \tau), t_n + \tau) d\tau \quad (2.5)$$

The various ETD methods come from how one approximates the integral in this expression. Cox and Matthews derived in [12] a set of ETD methods based on the Runge-Kutta time stepping, which they called ETDRK methods. In this paper we consider the ETDRK4 fourth-order scheme with the formulae

$$a_n = e^{Lh/2}u_n + L^{-1}(e^{Lh/2} - I)N(u_n, t_n)$$

$$b_n = e^{Lh/2}u_n + L^{-1}(e^{Lh/2} - I)N(a_n, u_n, t_n + \frac{h}{2})$$

$$b_n = e^{Lh/2}a_n + L^{-1}(e^{Lh/2} - I) \left[2N(b_n, t_n + \frac{h}{2}) - N(u_n, t_n) \right]$$

$$u_{n+1} = e^{Lh}u_n + h^{-2}L^{-3} \left\{ \left[-4I - hL + e^{Lh}(4I - 3hL + (hL)^2) \right] N(u_n, t_n) \right.$$

$$+ 2[2I + hL + e^{Lh}(-2I + hL)] \left(N(a_n, t_n + \frac{h}{2}) + N(b_n, t_n + \frac{h}{2}) \right)$$

$$\left. + \left[-4I - 3hL - (hL)^2 + e^{Lh}(4I - hL) \right] N(c_n, t_n + \frac{h}{2}) \right\}$$

More detailed derivations of the ETD schemes can be found in [12].

Unfortunately, in this form ETDRK4 suffers from numerical instability when L has eigenvalues close to zero, because disastrous cancellation errors arise. Kassam and Trefethen have studied in [6] these instabilities and have found that they can be removed by evaluating a certain integral on a contour that is separated from zero. The procedure is basically to change the evaluation of the coefficients, which is mathematically equivalent to the original ETDRK4 scheme of [12], but in [23] it has been shown to have the effect of improving the stability of integration in time. Also, it can be easily implemented and the impact on the total computing time is small. In fact, we have always used this idea in our MATLAB® codes.

III. ON THE STABILITY OF ETDRK4 METHOD

The stability analysis of the ETDRK4 method is as follows (see [21,24] or [12]). For the nonlinear ODE

$$\frac{du(t)}{dt} = cu(t) + F(u(t), t) \quad (3.1)$$

With $F(u(t), t)$ the nonlinear part, we suppose that there exists a fixed point u_0 this means that $cu_0 + F(u_0, t) = 0$. Linearizing about this fixed point, if $u(t)$ is the perturbation of u_0 and $\delta = F'(u_0, t)$ then

$$\frac{du(t)}{dt} = cu(t) + \delta u(t) \quad (3.2)$$

and the fixed point $u_0(t)$ is stable if $\text{Re}(c + \delta) < 0$. The application of the ETDRK4 method to (3.2) leads to a recurrence relation involving u_n and u_{n+1} . Introducing the previous notation $x = \delta h$ and $y = ch$, and using the Mathematica® algebra package, we obtain the following amplification factor

$$\frac{u_{n+1}}{u_n} = r(x, y) = \ell_0 + \ell_1 x + \ell_2 x^2 + \ell_3 x^3 + \ell_4 x^4 \quad (3.3)$$

where

$$\ell_0 = e^y$$

$$\ell_1 = \frac{-4}{y^3} + \frac{8e^{\frac{y}{2}}}{y^3} - \frac{8e^{\frac{3y}{2}}}{y^3} + \frac{4e^{2y}}{y^3} - \frac{1}{y^2} + \frac{4e^{\frac{y}{2}}}{y^2} - \frac{6e^y}{y^2} + \frac{4e^{\frac{3y}{2}}}{y^2} - \frac{e^{2y}}{y^2}$$

$$\ell_2 = \frac{-8}{y^4} + \frac{16e^{\frac{y}{2}}}{y^4} - \frac{16e^{\frac{3y}{2}}}{y^4} + \frac{8e^{2y}}{y^4} - \frac{5}{y^3} + \frac{12e^{\frac{y}{2}}}{y^3} - \frac{10e^y}{y^3} + \frac{4e^{\frac{3y}{2}}}{y^3} - \frac{e^{2y}}{y^3} - \frac{1}{y^2} + \frac{4e^{\frac{y}{2}}}{y^2} - \frac{3e^y}{y^2},$$

$$\ell_3 = \frac{4}{y^5} - \frac{16e^{\frac{y}{2}}}{y^5} + \frac{16e^y}{y^5} + \frac{8e^{\frac{3y}{2}}}{y^5} - \frac{20e^{2y}}{y^5} + \frac{8e^{\frac{5y}{2}}}{y^5} - \frac{2}{y^4} + \frac{-10e^{\frac{y}{2}}}{y^4} + \frac{16e^y}{y^4} - \frac{12e^{\frac{3y}{2}}}{y^4} + \frac{6e^{2y}}{y^4} - \frac{2e^{\frac{5y}{2}}}{y^4} - \frac{2e^{\frac{y}{2}}}{y^3} + \frac{4e^y}{y^3} - \frac{2e^{\frac{3y}{2}}}{y^3}$$

$$\ell_4 = \frac{8}{y^6} - \frac{24e^{\frac{y}{2}}}{y^6} + \frac{16e^y}{y^6} + \frac{16e^{\frac{3y}{2}}}{y^6} - \frac{24e^{2y}}{y^6} + \frac{8e^{\frac{5y}{2}}}{y^6} + \frac{6}{y^5} + \frac{-18e^{\frac{y}{2}}}{y^5} + \frac{20e^y}{y^5} - \frac{12e^{\frac{3y}{2}}}{y^5} + \frac{6e^{2y}}{y^5} + \frac{2e^{\frac{5y}{2}}}{y^5} + \frac{2}{y^4} - \frac{6e^{\frac{y}{2}}}{y^4} + \frac{6e^y}{y^4} - \frac{2e^{\frac{3y}{2}}}{y^4}$$

An important remark: computing for small y , instead of them, we will use their asymptotic expansions. $\ell_0, \ell_1, \ell_2, \ell_3, \ell_4$ by the above expressions suffers from numerical instability for y close to zero. Because of that,

$$\ell_1 = 1 + y + \frac{1}{2}y^2 + \frac{1}{6}y^3 + \frac{13}{320}y^4 + \frac{7}{960}y^5 + O(y^6)$$

$$\ell_2 = \frac{1}{2} + \frac{1}{2}y + \frac{1}{4}y^2 + \frac{247}{2880}y^3 + \frac{131}{5760}y^4 + \frac{479}{96768}y^5 + O(y^6)$$

$$\ell_3 = \frac{1}{6} + \frac{1}{6}y + \frac{61}{720}y^2 + \frac{1}{36}y^3 + \frac{1441}{241920}y^4 + \frac{67}{120960}y^5 + O(y^6)$$

$$\ell_4 = \frac{1}{24} + \frac{1}{32}y + \frac{7}{640}y^2 + \frac{19}{11520}y^3 - \frac{311}{64512}y^4 - \frac{479}{860160}y^5 + O(y^6)$$

We make two observations:

- As $y \mapsto 0$, our approximation becomes

$$r(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$$

which is the stability function for all the 4-stage Runge-Kutta methods of order four.

- Because c and δ may be complex, the stability region of the ETDRK4 method is four-dimensional

and therefore quite difficult to represent. Unfortunately, we do not know any expression for $|r(x, y)| = 1$ we will only be able to plot it. The most common idea is to study it for each particular case; for example, assuming c to be fixed and real in [21] or that both c and δ are pure imaginary numbers in [24].

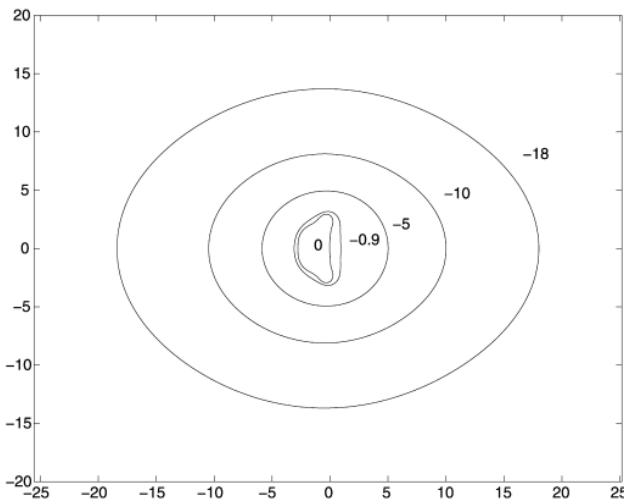


Figure 1 : Boundary of stability regions for several negative

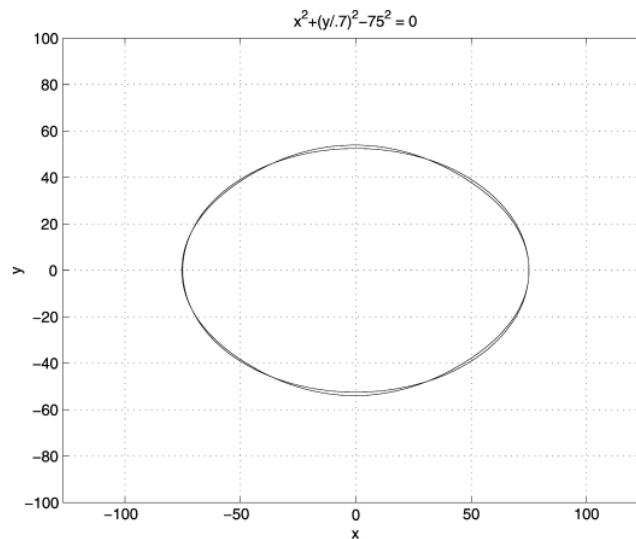


Figure 2 : Experimental boundaries and ellipse for $y = -75$

For dissipative PDEs with periodic boundary conditions, the scalars c that arise with a Fourier spectral method are negative. Let us take for example Burger's equation

$$u_t = \varepsilon u_{xx} - \left(\frac{1}{2}u^2\right)_x \quad x \in [-\pi, \pi] \quad \text{where } 0 < \varepsilon \ll 1 \quad (3.4)$$

Transforming it to the Fourier space gives

$$\tilde{u}_t = -\varepsilon \zeta^2 \tilde{u} - \frac{i\zeta}{2} \tilde{u}^2 \quad \forall \zeta \quad (3.5)$$

where $\forall \zeta$ is the Fourier wave-number and the coefficients $c = -\varepsilon \zeta^2 < 0$, span over a wide range of values when all the Fourier modes are considered. For high values of $|\zeta|$ the solutions are attracted to the slow manifold quickly because $c < 0$ and $|c| \ll 1$.

In Figure 1 we draw the boundary stability regions in the complex plane x for $y=0, -0.9, -5, -10, -18$. When the linear part is zero ($y=0$), we recognized the stability region of the fourth-order Runge-Kutta methods and, as $y \mapsto -\infty$, the region grows. Of course, these regions only give an indication of the stability of the method.

In fact, for $y < 0$, $|y| \ll 1$ the boundaries that are observed approach to ellipses whose parameters have been fitted numerically with the following result.

$$(\operatorname{Re}(x))^2 + \left(\frac{\operatorname{Im}(x)}{0.7}\right)^2 = y^2 \quad (3.6)$$

In Figure 2 we draw the experimental boundaries and the ellipses (3.6) with $y = -75$. The

spectrum of the linear operator increases as ζ^2 , while the eigenvalues of the linearization of the nonlinear part lay on the imaginary axis and increase as ζ . On the other hand, according to (3.6), when $\text{Re}(x) = 0$, the intersection with the imaginary axis $\text{Im}(x)$ increases as $|y|$, i.e., as ζ^2 . Since the boundary of stability grows faster than , the ETDRK4 method should have a very good behavior to solve Burger's equation, which confirms the results of paper [6].

IV. THE KURAMOTO-SIVASHINSKY EQUATION

The Kuramoto-Sivashinsky equation (K-S), is one of the simplest PDEs capable of describing complex behavior in both time and space. This equation has been of mathematical interest because of its rich dynamical properties. In physical terms, this equation describes reaction diffusion problems, and the dynamics of viscous-fluid films flowing along walls.

Kuramoto-Sivashinsky equation in one space dimension can be written

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t) \frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^4 u(x,t)}{\partial x^4} \quad (4.1)$$

or in form

$$u_t + \nabla^4 u + \nabla^2 u + |\nabla u|^2 / 2 = 0$$

Equation (4.1) can be written in integral form if we introduce

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} = -u(x,t) \frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^4 u(x,t)}{\partial x^4} &\Leftrightarrow \frac{\partial u_j(x,t)}{\partial t} = -u_j(x,t) \frac{\partial u_j(x,t)}{\partial x} - \frac{\partial^2 u_j(x,t)}{\partial x^2} - \frac{\partial^4 u_j(x,t)}{\partial x^4} \\ &\Leftrightarrow \sum_{k=0}^{N_\tau-1} \frac{d\tilde{u}_k}{dt} e^{\frac{ik\pi x}{L}} = - \left[\left(\sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \right) * \left(\sum_{k=0}^{N_\tau-1} \frac{ik\pi}{L} \tilde{u}_k e^{\frac{ik\pi x}{L}} \right) \right] - \left[\sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^2 \tilde{u}_k e^{\frac{ik\pi x}{L}} \right] - \left[\sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^4 \tilde{u}_k e^{\frac{ik\pi x}{L}} \right] \end{aligned}$$

By simplifying and note that

$$\tilde{u}_k = \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} u_j e^{-2ijk/N_\tau}, \quad (i)^2 = -1, (i)^4 = 1 \quad x_j = jh, h = \frac{2L}{N_\tau}$$

Equation (4.1) can be written as follows

$$\frac{\partial \tilde{u}_k(t)}{\partial t} = (k^2 - k^4) \tilde{u}_k(t) - \frac{ik}{2} \varpi_k \quad (4.9)$$

$$u(x,t) = \frac{\partial \zeta(x,t)}{\partial t}$$

then

$$\frac{\partial \zeta(x,t)}{\partial t} = -\frac{1}{2} \left(\frac{\partial \zeta(x,t)}{\partial x} \right)^2 - \frac{\partial^2 \zeta(x,t)}{\partial x^2} - \frac{\partial^4 \zeta(x,t)}{\partial x^4} \quad (4.2)$$

The Kuramoto-Sivashinsky equation with 2L periodic boundary conditions in Fourier space can be written as follows

$$u_j(x) = \sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.3)$$

$$u_j(x,t) = \sum_{k=0}^{N_\tau-1} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.4)$$

$$\frac{\partial u_j(x,t)}{\partial t} = \sum_{k=0}^{N_\tau-1} \frac{d\tilde{u}_k}{dt} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.5)$$

$$\frac{\partial u_j(x,t)}{\partial x} = \sum_{k=0}^{N_\tau-1} \frac{ik\pi}{L} \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.6)$$

$$\frac{\partial^2 u_j(x,t)}{\partial x^2} = \sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^2 \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.7)$$

$$\frac{\partial^4 u_j(x,t)}{\partial x^4} = \sum_{k=0}^{N_\tau-1} \left(\frac{ik\pi}{L} \right)^4 \tilde{u}_k e^{\frac{ik\pi x}{L}} \quad (4.8)$$

If we substitute (4.3), (4.4), (4.5), (4.6), (4.7), (4.8) into (4.1) we get

$$\text{Where } \varpi_k = \frac{1}{2L} \int_{-L}^L u^2(x,t) e^{\frac{ik\pi x}{L}} dx = \frac{1}{N_\tau} \sum_{j=0}^{N_\tau-1} u_j^2(x,t) e^{-2ijk/N_\tau} = FFT[u(t)^2]$$

In final form will be

$$\frac{\partial \tilde{u}_k(t)}{\partial t} = (k^2 - k^4) \tilde{u}_k(t) - \frac{ik}{2} \varpi_k \quad (4.9)$$

Equation has strong dissipative dynamics, which arise from the fourth order dissipation $\frac{\partial^4 u(x,t)}{\partial x^4}$

term that provides damping at small scales. Also, it includes the mechanisms of a linear negative diffusion $\frac{\partial^2 u(x,t)}{\partial x^2}$ term, which is responsible for an instability of

modes with large wavelength, i.e. small wave-numbers.

The nonlinear advection/steepening $u(x,t) \frac{\partial u(x,t)}{\partial x}$ term in the equation transforms energy between large and small scales.

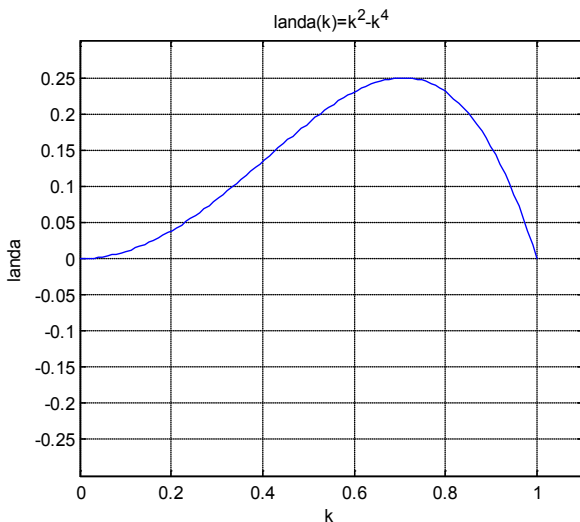


Figure 3 : The growth rate for perturbations of the form $e^{\lambda t} e^{ikx}$ to the zero solution of the Kuramoto-Sivashinsky (K-S) equation

V. NUMERICAL RESULT

For the simulation tests, we choose two periodic initial conditions

$$u_1(x) = e^{\cos\left(\frac{x}{2}\right)}, x \in [0, 4\pi]$$

$$u_2(x) = 1.7 \cos\left(\frac{x}{2}\right) + 0.1 \sin\left(\frac{x}{2}\right) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$$

When evaluating the coefficients of the exponential time differencing and the exponential time differencing Runge- Kutta methods via the "Cauchy integral" approach [5],[6] we choose circular contours of radius $R = 1$. Each contour is centered at one of the

The zero solution of the K-S equation is linearly unstable (the growth rate $\lambda(k) > 0$ for perturbations of the form $e^{\lambda t} e^{ikx}$ to modes with wave-numbers

$|k| = \left| \frac{2\pi}{g} \right| < 1$ for a wavelength g and is damped for modes with $|k| > 1$ see Figure3: these modes are coupled to each other through the non-linear term.

The stiffness in the system (4.9) is due to the fact that the diagonal linear operator $(k^2 - k^4)$, with the elements, has some large negative real eigenvalues that represent decay, because of the strong dissipation, on a time scale much shorter than that typical of the nonlinear term. The nature of the solutions to the Kuramoto-Sivashinsky equation varies with the system size of linear operator. For large size of linear operator, enough unstable Fourier modes exist to make the system chaotic. For small size of linear operator, insufficient Fourier modes exist, causing the system to approach a steady state solution. In this case, the exponential time differencing methods integrate the system much more accurately than other methods since the exponential time differencing methods assume in their derivation that the solution varies slowly in time.

elements that are on the diagonal matrix of the linear part of the semi-discretized model. We integrate the system (4.9) using fourth-order Runge Kutta exponential time differencing scheme using $N_\tau = 64$ with time-step size $\Delta t = 2e - 10$.

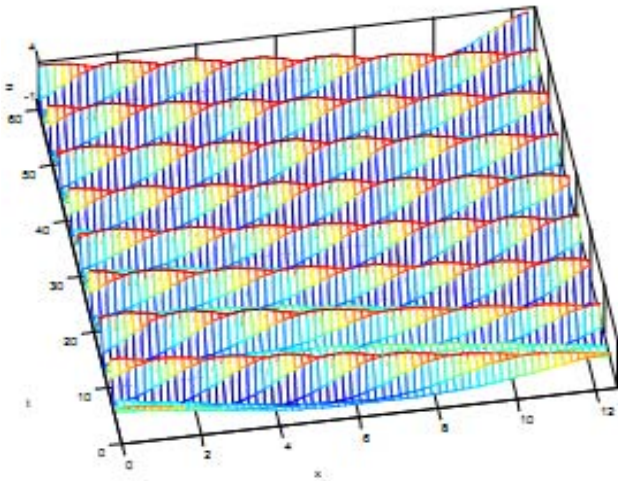


Figure 4 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = e^{\cos(\frac{x}{2})}, x \in [0, 4\pi]$

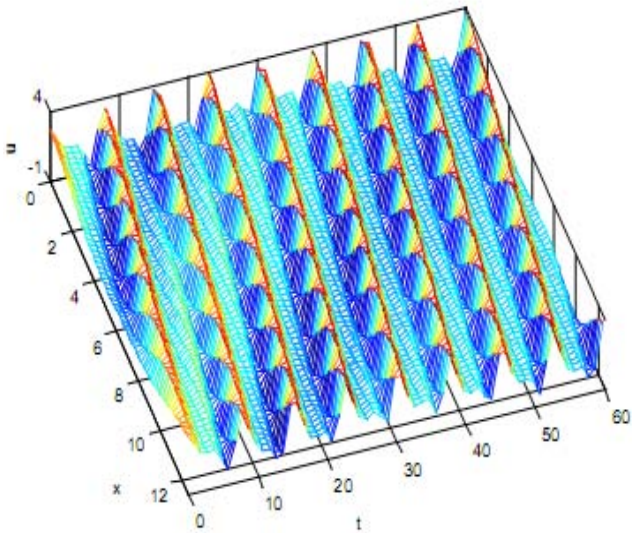


Figure 5 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = e^{\cos(\frac{x}{2})}, x \in [0, 4\pi]$

The solution, in the Figure 5 with the initial condition $u_1(x) = e^{\cos(\frac{x}{2})}, x \in [0, 4\pi]$ with $N_\tau = 64$ and time-step size $\Delta t = 2e - 10$, appears as a mesh plot and shows waves propagating, traveling periodically in time and persisting without change of shape.

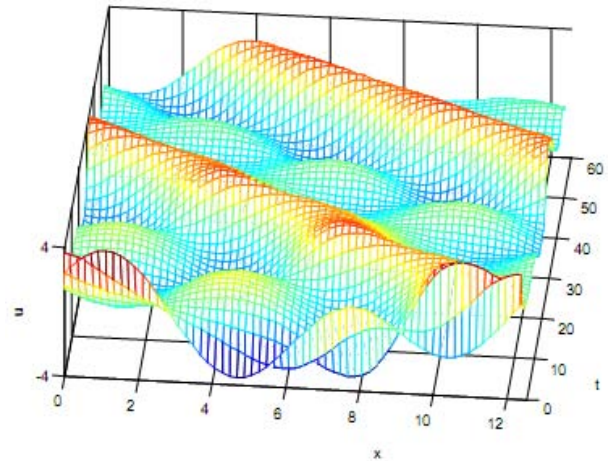


Figure 6 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition

$$u_2(x) = 1.7 \cos\left(\frac{x}{2}\right) + 0.1 \sin\left(\frac{x}{2}\right) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$$

In the Figure 6 with the initial condition $u_2(x) = 1.7 \cos\left(\frac{x}{2}\right) + 0.1 \sin\left(\frac{x}{2}\right) + 0.6 \cos(x) + 2.4 \sin(x), x \in [0, 4\pi]$ with

$N_\tau = 64$ and time-step size $\Delta t = 2e - 10$, the solution appears as a mesh plot and shows waves propagating, traveling periodically in time and persisting without change of shape.

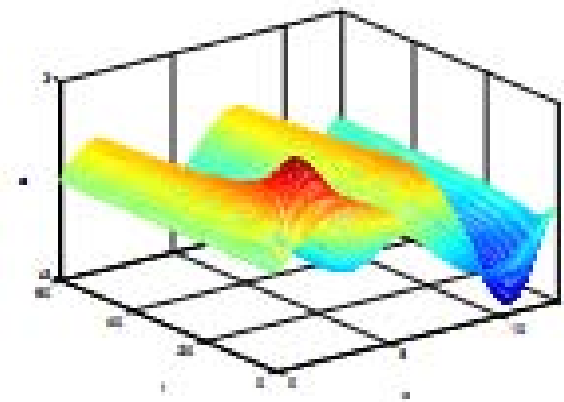


Figure 7 : Time evolution of the numerical solution of the Kuramoto-Sivashinsky up to $t = 60$ with the initial condition $u_1(x) = \sin(x/2), x \in [0, 4\pi]$

In the Figure 7 with the initial condition $u_1(x) = \sin(x/2), x \in [0, 4\pi]$ with $N_\tau = 64$ and time-step size $\Delta t = 2e - 10$, the solution appears more clear as a mesh plot and shows waves propagating, traveling periodically in time and persisting without change of shape.

VI. CONCLUSIONS

The proposers of the ETDRK schemes in [12] concluded that they are more accurate than other methods (standard integrating factor techniques or linearly implicit schemes); they have good stability properties and are widely applicable to nonlinear wave equations. However, Cox and Matthews were aware of the numerical instability for the ETDRK4 method when computing the coefficients. Later, Kassam and Trefethen in [6] modified the ETDRK4 method with very good results. In the opinion of these authors, the modified ETDRK4 is the best by a clear margin compared with others methods. We have computed and studied the numerical stability function of the ETDRK4 methods. In addition, we have applied this method to the Kuramoto-Sivashinsky equation, achieving the excellent results that we have just mentioned. In order to achieve this, we applied Fourier spectral approximation for the spatial discretization. For the simulation tests, we chose periodic boundary conditions and applied Fourier spectral approximation for the spatial discretization. The equations can be used repeatedly with necessary adaptations of the initial conditions.

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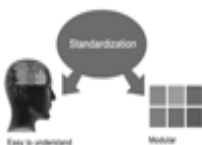
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- (f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;
- (g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.
- (h) Brief Acknowledgements.
- (i) References in the proper form.

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Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

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A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

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- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
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References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

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21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

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27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

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29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

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Topics	Grades		
	A-B	C-D	E-F
<i>Abstract</i>	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
<i>Introduction</i>	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
<i>Methods and Procedures</i>	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
<i>Result</i>	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
<i>Discussion</i>	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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ISSN 9755861

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