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Implementation of Alternative Solutions in Linear Programming Modeling using the Dual Simplex Method and Duality Method from Primal Problem, Establishing Implementation through the Simplex Methodology

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Objective

Show different ranges of alternatives that can provide solution to linear programming problems that may from feasible to optimal or optimal to feasible solution through analysis of their primal and dual models, which enable you to get valuable information for managerial decision making and the search for increased productivity as well as waste minimization maximizing available resources. It is unusual as detailed three most important methodologies at the same time for identifying solving tools in linear programming problems that can achieve large-scale analysis.

I. Introduction

The interest in linear equations goes back to ancient times of the year 1700 BC where the Egyptians left writings on your papyri mathematical problem solved with the use of algebra, later the Babylonians in the year 600 the BC left evidence that came from their work with Cuneiform Inscriptions where writing their participation in the mathematical problem solving equations using second grade, which is inferred taking too elemental use of basic linear Equations additionally they don't know negative numbers, they inferred that values did not exist, there is evidence that problems were resolved in systems with five equations and five unknowns variables.

In Ancient Greece around the year 300 BC the Greeks troubleshooting developed with construction of linear equations that through the application of algebra could be solved, including Theodore from Cyrene and Eudoxus from Cnido consolidated their jobs between geometric advances, worth mentioning that the center of scientific activity occurred in the city of Athens. The School of the Greek mathematician Pythagoras incorporates elements of Babylonian algebra.

Around 1700 as Euler's attachment theory calculations of variations on movement of considerations assuming constant flux densities at any time and strength on surface elements among others. Also Isaac Newton in his treatise of mathematical composition and resolution work wrote his book calculation to find approximate solutions which seeks to find the roots of equations and higher order.

Moreover Guillaume L Hopital Sainte, Lord marquees from Mesme in France in the seventeenth century work on the analysis of the infinitely small and established the rule of L Hopital for analysis and study of mathematical problems within the differential calculus.

Also Joseph Louis Lagrange in the eighteenth century in his "miscellaneous works taurinensa" results obtained by implementing linear equations applied to problems over straight line movement and analysis of the dynamics of their movements.

After World War II ended where George Dantzig job in the Air Force of the United States through the Combat Analysis Branch of Statistical Control. Where he found the problems that lead him to make his great discoveries, considering the progress of the Nobel laureate economist Wassily Leontief in 1947 and met the general problem of linear programming, commented at the time that it started watching the feasible region of a

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geometric space and how the process could be improved movements take place within its endpoints. So it was like in the summer of 1947 could solve the first problem of linear programming in the area of food.

That same year he met Von mathematical Newmann Hungarian mathematician who since 1928 had been working strategy games whose work was published in 1944 (died in 1957) with renowned mathematician Morgenstem Australian economist who established the beginning of game theory consisting to define a logical instrument that assesses the competitive behavior of a rational human being under consideration, then Dantzig perceives the importance of the theory of duality. Because the linear programming model of a player who maximizes your chance of winning will be equivalent to linear programming model other player that minimizes your chance of losing the game. It was observed as if it were equivalent problems began with basic feasible solutions.

II. Methodology

The simplex method is a method that solves linear programming problems, where you try to optimize a maximum function or minimum satisfying a set of constraints embodied in forms of equations such that while meeting other conditions given as fundamental method requirements are met.

Restrictions can be of three types.

(<=) Less than or equal, which are passed form the equality restriction adding a slack variable.

(=) Just which way to go to equal or standard by adding an artificial variable.

(> =) Greater than or equal which are passed to the form of equal or standard form you need to deduct a slack variable and increasing an artificial variable.

Once covered this requirement, it is necessary to empty the information in tabular form where information of the coefficients of the objective function and constraints that accompany the issue is placed. The process is iterative and each stage is verified if the optimal value obtained by checking the line Zj-Cj, sought where if the value is zero or positive will have reached the optimal solution of a maximization problem, moreover if values are all zero or negative will have reached the optimal value for the case of minimization.

a) Maximize case

In the event that the values are negative or zero in the case of maximization have to create a new database inside inverse matrix by performing elementary row operations column, the basis for the minimization problem is updated by removing the most negative value in row Zj-Cj and entering the ratio between the minimum vector of the right side and the elements of the incoming column. Breaking ties arbitrarily.

b) Minimize Case

In the event that the values are positive or zero in the case of minimization have to generate a new database inside inverse matrix by performing elementary row operations column, the basis for the minimization problem is updated by removing the most positive value in row Zj-Cj and entering the ratio between the minimum vector of the right side and the elements of the incoming column. Breaking ties arbitrarily.

$$General, Model$$
$$Max, z = c\ x$$
$$st$$
$$Ax <= b$$
$$With, x >= 0$$

Where "A" is the original matrix with "m" row by "n" columns

b= vector of available Resource in "m" rows

c= coefficient of Known variables in objective function (Maximize or Minimize) in "n" columns

x= nonknown variable also is called decision variable. Therefore, when we try to solve linear programming in simplex table we are making the matrix operations.

The operations performed within the Table simplex matrix can be explained manner as shown below.

$$Zj = Cb * B^{-1} * Aj (1)$$
$$Zj - Cj (2)$$

$$Where;$$
$$Cb = Cofficients, in, base$$
$$B^{-1} = Inverse, Matrix$$
$$Aj = Coefficients, out, base$$
$$Cj = Coeffcients, of, variables, in, objective, function$$
$$Zj = value, for, each, variable, in, optimization, process.$$

In equation 2 is where you can check if it has reached the desired optimality or should continue iterating through the inversion of the matrix to enhance your solution and approach the expected value.

III. Duality Theory

All primal problem is associated with another called dual problem are so called because they both have the same information but some in the form of row and other column addition to exchanging the coefficients of the objective function in the vector on the right side and this in once, in a reciprocal manner. It is assumed that if the primal feasibility is then possible to find the same optimal solution to the primal and the dual.

A primal problem will have "m" equations and "n" variables and the dual problem will be reverse.

$$\begin{array}{ll} Primal, Problem & Dual, Problem \\ Max, z = c\ x & Min, w = b^t y \\ st & st \\ Ax <= b & A^t y >= c^t \\ With, x >= 0 & With, y >= 0 \end{array}$$

Economic interpretation of the dual.

The knowledge of how much profit or cost change with an additional unit of each several resources can be valuable information.

$$Z^* = Cb * B^{-1} * b = Cb * XB = w^* * b$$

Therefore w* is the rate of change of optimal objective function value

Considering Dual $\partial z^* / \partial b = Cb * B^{-1} = w^*$

Finally economically, w * is a vector of shadow prices for the vector b which is available resource.

IV. Dual Simplex Method

The linear programming problem solved with normal simplex method has the basic idea from a feasible basic solution and move through endpoints to reach the optimum point basic solution. But sometimes it can happen that the linear programming problem starts being optimal but far away from feasibility, it can happen when we just change the signs of the objective function as well as the constraints and sense of inequality.

V. Procedure

Step 1. Be sure that the restrictions are in position infeasibility is easy to identify by the negative sign on the right side of the resources available.

Step 2. Ensure that the restrictions are in standard form i.e. in the form of equity using slack variables and artificial depending on the direction of the inequality.

Step 3. Identify the variable that will leave the base which will be one that has the most negative value in the associated resource available (b) column.

Step 4. Identify the variable that enters visualizing the smallest ratio considering the absolute value of the row Zj - Cj between the values of the corresponding row to the more negative variable, it will happen in the case of maximization problems, by other hand in the case minimization problems the most positive ratio is chosen without considering absolute value of row Zj - Cj and elements of the more negative variable that leaves the base.

Step 5. The other elements of the simplex table is updated with elementary row column Operations thus the inverse matrix iteratively updated to display the elements in row Zj - Cj remain all zeros or positive in the case of maximization and are zero or negative for minimization. Do not forget to check that column vector on the right side should be kept positive values associated to the decision variables that provide the solution to the linear programming problem.

a) *Implementation and Experimental results*

Primal Problem

Maxz = 3x1+5x2	
subject to	
x1<=4	
2x2<=12	
3x1+2x2<=18	
x1,x2>=0	

First iteration

Cj	↦	X	3	5	0	0	0
	x	b	X1	X2	H1	H2	H3
0	H1	4	1	0	1	0	0
0	H2	12	0	2	0	1	0
0	H3	18	3	2	0	0	1
Zj		0	0	0	0	0	0
Zj-Cj			-3	-5	0	0	0

Second Iteration

Cj	↦	X	3	5	0	0	0
	x	b	X1	X2	H1	H2	H3
0	H1	4	1	0	1	0	0
5	X2	6	0	1	0	1/2	0
0	H3	6	3	0	0	-1	1
Zj		30	0	5	0	2 1/2	0
Zj-Cj			-3	0	0	2 1/2	0

Third Iteration

Cj	↦	X	3	5	0	0	0
	x	b	X1	X2	H1	H2	H3
0	H1	2	0	0	1	1/3	- 1/3
5	X2	6	0	1	0	1/2	0
0	H3	2	1	0	0	- 1/3	1/3
Zj		36	3	5	0	1 1/2	1
Zj-Cj			0	0	0	1 1/2	1

Solution			**Testing**	
Z*=36			X1+H1=4	2(2)=4
X1*=2			2X2+H2=12	2(6)=12
X2*=6			3X1+2X2+H3=	3(2)+2(6)=18
			Z*=3(2)+5(6)=36	

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b) Implementation and Experimental Results

Dual Problem

Min W = 4Y1+12Y2+18Y3
subject to
Y1+3Y3>=3
2Y2+2Y3>=5
Y1,Y2,Y3>=0

First iteration

Cj	↦	X	4	12	18	0	0	M	M
	X	b	Y1	Y2	Y3	H1	H2	A1	A2
M	A1	3	1	0	3	-1	0	1	0
M	A2	5	0	2	2	0	-1	0	1
Zj		5M	M	2M	5M	-M	-M	M	M
Zj-Cj			M-4	2M-12	5M-18	-M	-M	0	0

Second Iteration

Cj	↦	X	4	12	18	0	0	M	M
	X	b	Y1	Y2	Y3	H1	H2	A1	A2
18	Y3	1	1/3	0	1	- 1/3	0	1/3	0
M	A2	3	- 2/3	2	0	2/3	-1	- 2/3	1
Zj		3M+18	-2/3M+6	2M	18	2/3M-6	-M	-2/3M+6	M
Zj-Cj			-2/3M+2	2M-12	0	2/3M-6	-M	-5/3M+6	0

Third Iteration

Cj	↦	X	4	12	18	0	0	M	M
	X	b	Y1	Y2	Y3	H1	H2	A1	A2
18	Y3	1	1/3	0	1	- 1/3	0	1/3	0
12	A2	1 1/2	- 1/3	1	0	1/3	- 1/2	- 1/3	1/2
Zj		36	2	12	18	-2	-6	2	6
Zj-Cj			-2	0	0	-2	-6	2-M	6-M

Solution			**Testing**	
W*=36			Y1+3Y3-H1+A1=3	0+3(1)-0+0=3
Y1*=0			2Y2+2Y3-H2+A2=5	2(3/2)+2(1)-0+0=5
Y2*=3/2				
Y3*=1			Z*=3(2)+5(6)=36	

c) Implementation and Experimental Results

Dual Simplex Problem

Max Z = -4Y1-12Y2-18Y3
subject to
-Y1-3Y3<=-3
-2Y2-2Y3<=-5
Y1,Y2,Y3>=0

First iteration

Cj		X	-4	-12	-18	0	0
	X	b	Y1	Y2	Y3	H1	H2
0	H1	-3	-1	0	-3	1	0
0	H2	-5	0	-2	-2	0	1
Zj		0	0	0	0	0	0
Zj-Cj			4	12	18	0	0

Second iteration

Cj		X	-4	-12	-18	0	0
	X	b	Y1	Y2	Y3	H1	H2
0	H1	-3	-1	0	-3	1	0
-12	Y2	2 1/2	0	1	1	0	- 1/2
Zj		-30	0	-12	-12	0	6
Zj-Cj			4	0	6	0	6

Third iteration

Cj		X	-4	-12	-18	0	0
	X	b	Y1	Y2	Y3	H1	H2
-18	Y3	1	1/3	0	1	- 1/3	0
-12	Y2	2 1/2	- 1/3	1	0	1/3	- 1/2
Zj		-36	-2	-12	-18	2	6
Zj-Cj			2	0	0	2	6

Solution	Testing	
Z*=36	-Y1-3Y3+H1=-3	-2(0)-3(1)+0=-3
Y1*=0	-2Y2-2Y3+H2=5	-2(3/2)-2(1)-0+0=-5
Y2*=3/2		
Y3*=1	Z*=-4(0)-12(3/2)-18(1)=-36	

VI. Analysis of Results

- You can see at the out put of experimental results are proven solutions using equations models of primal, dual and dual simplex Methods.
- Interesting to see how the values of the dual problem can be found in the Zj-Cj row of the primal model.
- The ability to reach optimal solutions based on different scenarios of linear programming and algebraic conditions required.
- The results are obtained from tabular models which work the basic operations column line, to generate the inverse matrix iteratively.
- You can see how in the implementation of Dual Simplex Method the column associated to vector of right side starts with negative value and in the third table the values associated to column of available resource finished being positive. Is to say we starts being optimum solution but infeasible and after the problem finished being optimum and feasible therefore all the values of decision variable are positive.
- Mathematical models are solved shown three different linear programming techniques and in all cases the solution is reached in Z*=36.

VII. Conclusions

- It is important to show the importance knowing of these important linear programming methods as alternative optimal solutions from a feasible basic solution or when there is already optimality. In everyday life can be in any of these scenarios and certainly this research paper help in making administrative decisions and engineering at the management level as complementary tools, where you can go from one to another depending on the knowledge, skills and interests of your responsible established.

- For young people who are in the process of learning methodology mathematical programming, knowing the different criteria and rules that each of these three methods have will help them understand and manage properly solve their modeling. Also knowing where to find the dual values may allow determining future economic investments without going to perform the method of duality.
- Many real life problems can be treated using the dual simplex algorithm where an initial optimality to the actual feasibility of resources and goals, many goals can be adjusted with the methodologies presented in this investigation is determined.
- The possibility of creating your own code can flow as software tailored look that strengthens the cognitive process in abstract and complex problem to practice problem solving large-scale solutions in polynomial time given.
- Many times we are accustomed to use software that solves the problem of linear programming but we cannot identify the type of methodology used, certainly we lost the opportunity to identify areas of opportunity such as post-optimality analysis and economic interpretation of decision variables as a way to integrate into the management of productive enterprises or services.

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